

Exothermic chemical reaction of magneto-convective nanofluid flow in a square cavity

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ABSTRACT

We investigate the unsteady laminar convective magnetohydrodynamic nanofluids flow in a square cavity driven by an exothermic chemical reaction. Because exothermic chemical reactions are intrinsic in nanofluidic flow applications, we consider this exothermic chemical reaction in the analysis, which is governed by Arrhenius kinetics energy. A water-based nanofluid containing iron oxide (Fe_3O_4) nanoparticles is employed in the simulation. The square cavity is accurately propounded by a combination of suitable heating and flow conditions. The left vertical wall of the enclosure is considered a higher unevenly heated wall, the right vertical wall of the domain is considered a relatively cool constant temperature, and the upper and lower walls are considered insulating walls. Each wall assumes a no-slip condition. The nanofluid governing equations are transformed into the non-dimensional set of equations using similarity analysis and then modified into finite element equations. Galerkin's method in finite element analysis is used to obtain the results of the problem. The results show that the Rayleigh number, the Frank-Kamenetsky number, and the nanosolid volume ratio all have significant effects on the convective flow regime, and the average Nusselt number increases with these parameters. Due to the greater value of the Rayleigh number ($Ra = 10^6$), the average Nusselt number increased to 75.92%, and heat generation due to a strongly exothermic reaction (higher Frank-Kamenetskii number) can blow up the bounded solution. The water- Fe_3O_4 nanofluid achieves a greater rate of heat transfer (maximum 22.65%) than that of the base fluid.

1. Introduction

Nanofluids are a new type of industrial coolant with greater application prospects in various fields including science and engineering. Choi [1] was the first to coin the term "Nanofluid," which refers to a specific type of heat transmission fluid with outstanding thermal characteristics. Nanofluid is made up of common fluids combined with nanometer-sized particles (about 1 to 100 nm in diameter), also known as nanoparticles, which have a remarkably higher heat transfer efficiency than the base fluid [2]. Many researchers are getting interested in working on nanofluids because of the increasing demand of the uses of nanofluids in different areas. At present, analyzing the efficiency of heat transfer, particularly in the industrial arena, is an upward concern for the vast number of researchers in terms of its widespread applications, such as heat exchangers of various machines, cooling systems of electronic devices, nuclear and solar power energy generation, micro and nano-electro systems, space-heating, and other fields [3].

The heat transfer performance of nanofluid depends on some crucial parameters such as particle's size and shape, constructive materials of base fluids, concentration, etc. [4,5], and nanofluids are being used as a momentum to improve heat transmission, according to reports. Many investigators have focused on the properties of nanofluids and heat transfer escalation in various geometrical shapes, namely, triangular by Arani et al. [6] and Mansour et al. [7], triangular having wavy surfaces by Nasrin et al. [8], convergent-divergent nozzle by Zahan et al. [9], square by Al-Balushi et al. [10], trapezoidal by Nasrin et al. [11] and Kareem et al. [12], C-shaped domain by Makulati et al. [13], semi-annulus by Soleimani et al. [14], and annulus by Roy [15].

In industrial applications, natural or free convection analysis in domains or cavities of numerous geometrical shapes plays an imperative role. It is worth noting that various characteristics of fluid flow and heat transfer in square shape geometry combined with the presence of an exothermic chemical reaction, have a significant impact on a wide range of engineering applications, such as reactor of the chemical industry, chemical fractionalization, and oil reposition, humidity relocation in

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Nomenclature		X, Y	cartesian coordinates (dimensionless)
		x, y	cartesian coordinates (dimensional) [m]
		<i>Subscripts</i>	
a	concentration of the reactant [mol/m^3]	h	hotter region
B_0	effect of magnetic field [$\sqrt{\Omega \text{kg}/\text{m}\sqrt{\text{s}}}$]	c	colder region
c_p	specific heat [J/KgK]	bf	base fluid
E	energy of activation [J/mol]	nf	nanofluid
F_k	constant number (Frank- Kamenetskii)	s	solid particle
g	gravitational force [m/s^2]	<i>Greek Symbols</i>	
Ha	Hartmann-number (dimensionless parameter)	τ	dimensionless time
k_0	factor of pre-exponential [$1/\text{s}$]	γ	angle
L	hight and wide of sides [m]	ρ	density [kg/m^3]
MHD	magnetohydrodynamics	κ	thermal or Fluid conductivity [W/mK]or [J/mKs]
Nu_L	Nusselt-number (local) (dimensionless parameter)	φ	nanoparticle's volume fraction
$\overline{Nu}$	Nusselt-number (average) (dimensionless parameter)	σ	electrical conductivity [S/m]
Nu	Nusselt-number (dimensionless parameter)	α	diffusivity parameter (thermal) [m^2/s]
P	pressure-term (dimensionless parameter)	ν	viscosity (kinematic) [m^2/s]
p	dimensional pressure [kg/ms^2]	β	expansion coefficient (thermal) [$1/\text{K}$]
Pr	Prandtl-number (dimensionless parameter)	σ^*	Stefan-Boltzmann constant
q	heat flux	$\rho\beta'$	volumetric expansion of mass
q_r	heat flux (radiative) [kg/s^3]	$\rho\beta$	volumetric expansion of thermal
Q	exothermicity of a reaction ([J/mol])	θ	temperature (dimensionless)
R	Fluid-constant [$\text{J}/(\text{k mol})^{-1}$]	μ	viscosity (dynamic) [kg/ms]
Ra	Rayleigh-number (dimensionless parameter)	ρc_p	capacitance of heat [$\text{J}/\text{m}^3\text{K}$]
R_v	velocity magnitude/ resultant velocity (dimensionless parameter)	λ	conductivity of fluid [W/mK]
T_0	constant value of temperature [K]	ψ	stream function
T	temperature of fluid [K]	Ω	vorticity
T_h	high value of temperature [K]	ξ	vorticity (dimensionless)
T_c	low value of temperature [K]	ε	surface emissivity
U, V	velocity component (dimensionless)		
u, v	velocity component (dimensional) [m/s]		

grain storage, burning, explosions, and a variety of other natural and artificial heat transfer processes. Several studies and computational measurements have been introduced in the last two decades to describe the mechanism of natural or free convection in the square domain. The influence of different Rayleigh numbers, as well as the solid volume fraction of nanoparticles, on heat transmission and fluid flow, was investigated in these studies.

The properties of electrically conducting fluid in the presence of an electromagnetic field are the subject of the dynamics branch known as magnetohydrodynamics (MHD). There are many uses for MHD convective flow and heat transfer in science and engineering, including the extraction of geothermal energy, plasma confinement, crystal formation, thermal insulation of petroleum reserves, and cooling of nuclear reactors. Many scientists work on the magnetohydrodynamic flow of convective fluids (see Chamkha [16], Uddin et al. [17], Islam et al. [18], Priam and Nasrin [19], Rahman and Uddin [20], Malleswaran and Sivasankaran [21], Uddin et al. [22], Chatterjee and Gupta [23], Uddin et al. [24], Habibishandiz and Saghir [25], Yeasmin et al. [26] and so on).

Selimefendigil et al. [27] numerically investigated in a square cavity with the flexible wall of inside moving cylinder occupied by a water-based SiO_2 nanofluid. Their study shows that among various shapes of nanoparticles, the cylindrical shape of nanoparticles provides the best output, and the spherical shape provides lower performances in the analysis. Roy [28] studied the heat transfer characteristics of a square shape enclosure with three geometric shapes: circular, elliptical, and rectangular cylinders, found that the Nusselt number increased linearly at the internal and external cylinders due to the upper volumetric ratio of nanosolid, while the Rayleigh number increased exponentially. Meanwhile, Jani et al. [29] quantitatively investigated magnetohydrodynamics (MHD) natural convection in a square domain

in terms of lower Rayleigh numbers and discovered that the mode of heat transmission transformed conduction to convection owing to rising values of magnetic field parameter (Hartmann number). Lower and higher values of the Hartman number and Rayleigh number, respectively, lead to considerable heat transfer increase, according to Ali et al. [30], and the reverse phenomena is also seen at average temperature. The MHD natural convection with a sectionally heated square cavity was explored by the researchers for this experiment. Hasanuzzaman et al. [31] found that convective heat transfer coefficients of water-based nanofluids (nanosolids include TiO_2 , Al_2O_3 , Al , and Cu) are 64, 66, 6, and 81%, respectively greater than conventional fluid water. The study of Guerrieri et al. [32] correspondingly distinguished almost similar results. Brownian motion in the free convection under the magnetic field influence has a substantial impression on thermal transmission [33]. Kahveci and Ztuna [34] considered the heat transference and the stimulus of the heated segregated hollow. The average rate of heat transmission lowers by as much as 80% when the magnetic field is applied in the x – direction and partition occurs in the middle of the hollow. Hamama et al. [35] discovered that the Prandtl number has a huge influence on heat transfer owing to magnetohydrodynamics free convection in a square shape domain. Ashouri et al. [36] demonstrated the influence of Hartmann number on free convection in a square domain with conducting solid rectangular barriers.

Analyzing heat transfer through chemical reactions of nanofluids flow has imperative practical significance for inventors and engineers because, in several fields of research and manufacturing, it is nearly universal. This phenomenon is essential in the chemical, power, and cooling industries. It is employed in cooling towers, evaporation and drying sector, cooling of the nuclear reactor, outward chemical deposition, petroleum industry, and some heat sources. In addition, other transportation processes are controlled by the buoyancy of the me-

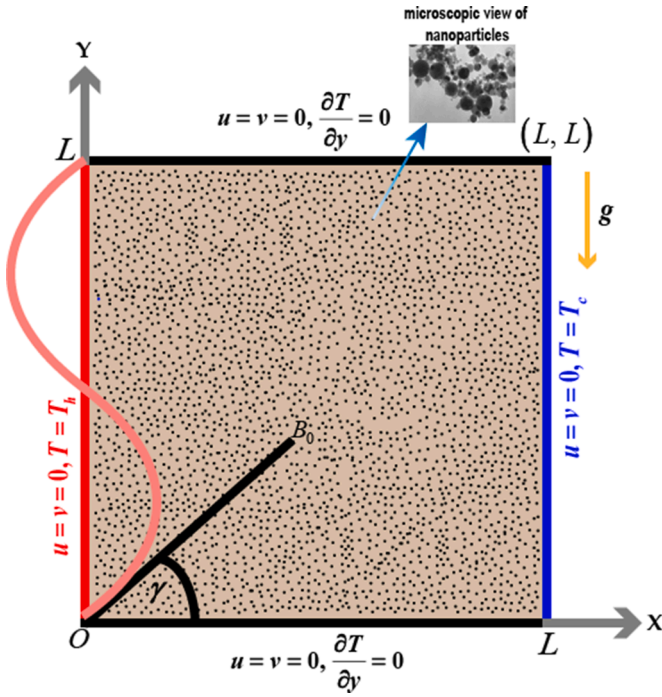


Fig. 1. The flow configurations and the physical system (cross-section).

chanical source. The exothermic surface chemical reaction is an essential feature, which is usually effective in providing heat to the fluid. Arrhenius kinetics is a first-order kinetic equation that represents this chemical reaction. The Arrhenius equation helps to regulate the rate at which the temperature of a chemical structure varies due to a chemical reaction. This relationship relates the pace of reaction with the temperature in a variety of physical processes. The most often used form of the Arrhenius Eq. [37] is, $rate = ak_0 \exp(-\frac{E}{RT})$, $A \rightarrow B$, here B and A , both represent the reactants and R is the fluid constant, E is the activation energy, k_0 is the pre-exponential factor, T is the temperature, and a is the concentration of reactant A . Merkin and Pop [38] investigated how Arrhenius-kinetics influenced stagnation flow by contracting or extending it outward and identified three-branch findings for unitless temperature and concentration, as well as the location of critical points and hysteresis bifurcation. For their experiment, Zhang et al. [39] used a horizontal plane with porous medium and evaluated the influence of heat transport using three different solid particles: Al_2O_3 , Ag, and Cu. Rahman et al. [40] numerically investigated the effectiveness of nanofluid heat transfer in an inclined porous square domain using Arrhenius kinetics and showed Frank-Kamenetskii and Rayleigh numbers were crucial in heat production. The goal of Raees-ul-Haq's [41] study was to examine the influence of chemical processes on heat transmission phenomena in nanofluids, with a focus on a triangular shape. According to the researchers, increasing the heat transmission rate is directly related to growing Frank-Kamenetskii and Rayleigh numbers when the Nusselt number is smaller.

The main purpose of nanofluidic research is to reach an agreement between different research findings based on the results of experiments and simulations. From the evaluation of the current literature review, we can see that there are still some gaps in the field of nanofluidic research, for example,

Table 1

Thermophysical characteristics of water- Fe_3O_4 nanofluid at 25°C.

Objects	$\kappa [Wm^{-1}K^{-1}]$	$\mu [kgm^{-1}s^{-1}]$	$\beta [K^{-1}]$	$\rho [kgm^{-3}]$	$c_p [Jkg^{-1}K^{-1}]$	$\sigma [Sm^{-1}]$
H_2O	0.613	0.001003	21×10^{-5}	997.1	4179	5.5×10^{-6}
Fe_3O_4	80.4	-	20.6×10^{-5}	5180	670	0.112×10^{-6}

- Despite its various commercial and technical uses, little research has been done on Arrhenius kinetically mediated exothermic chemical processes.
- The study of nanofluids with square geometries in the presence of gradient magnetic fields with periodic dips has received little attention.

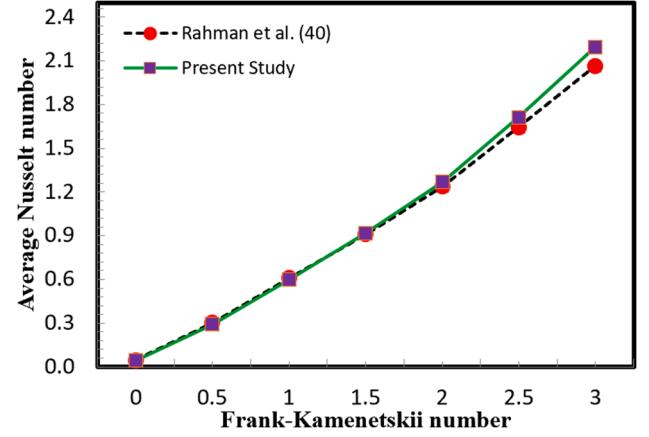


Fig. 2. Code validation in terms of the average Nusselt number (Nu) against Fk between present work and Rahman et al. [40]

Table 2

For the numerous numbers of the grid, the average Nusselt number ($\bar{Nu}$) when $\phi = 0.05$, $Fk = 2$, $Ha = 15$, $Ra = 10^6$, $A = 0$, $T_0 = 25$, and $\tau = 1$ (study case)

No. of elements	$\bar{Nu}$	No. of elements	$\bar{Nu}$
202	12.58224	2540	13.47423
356	13.21483	6578	13.47225
548	13.23273	17038	13.47924
1032	13.42919	26300	13.47812
1504	13.45707		

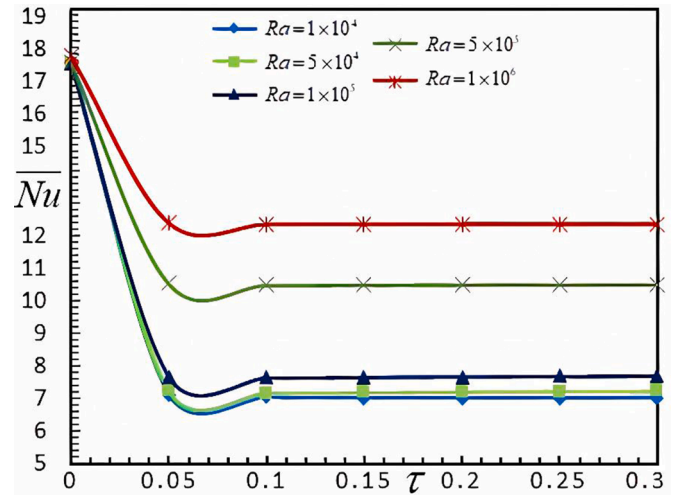


Fig. 3. $\bar{Nu}$ concerning τ and several Ra for $\phi = 0.05$, $Ha = 40$, $Fk = 2$, and $\gamma = 0$

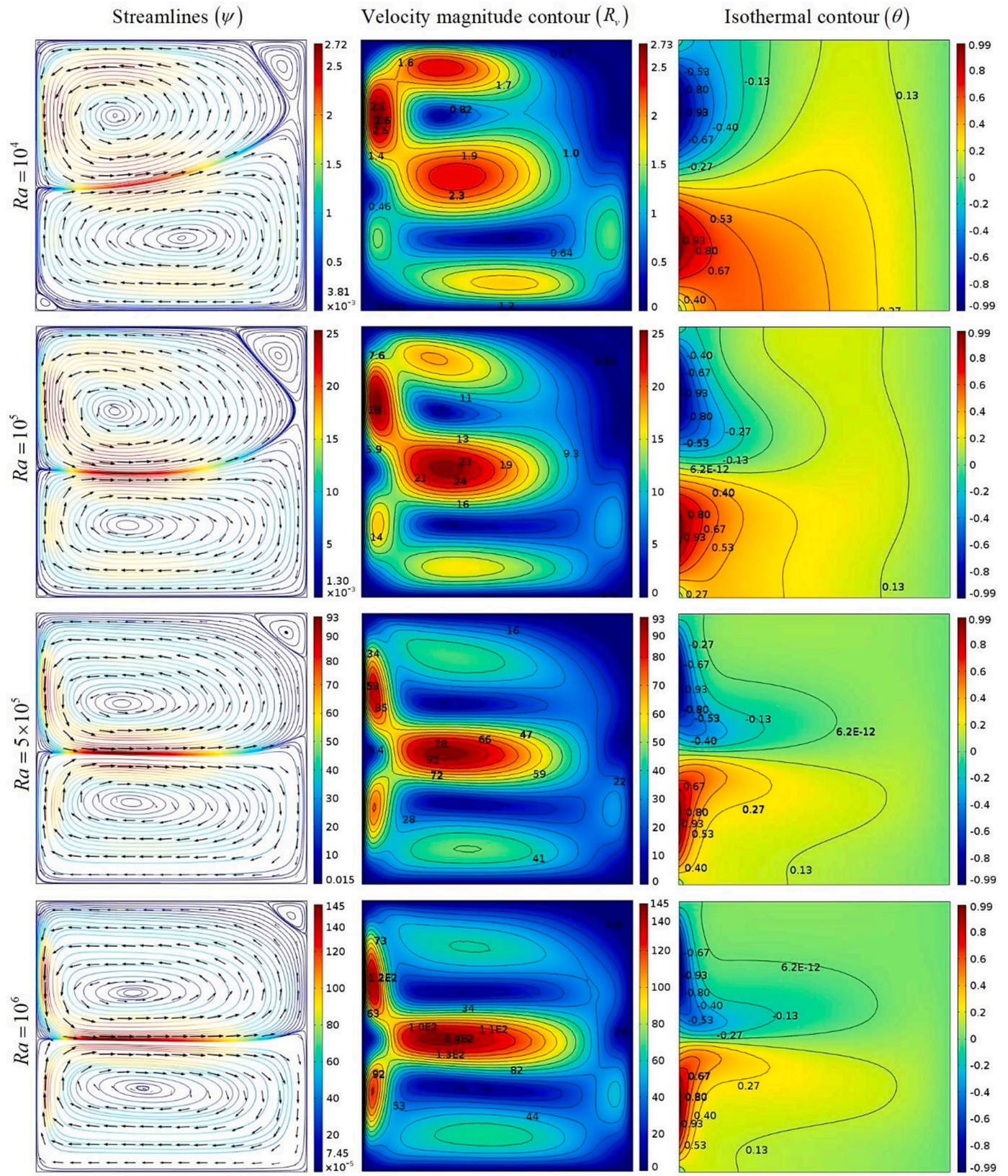


Fig. 4. Streamlines, velocity magnitude contour, and isothermal contour for various values of Rayleigh numbers with $\phi = 0.05$, $Ha = 40$, $F_k = 2$, $\gamma = 0$, $T_0 = 25$ and $\tau = 5$

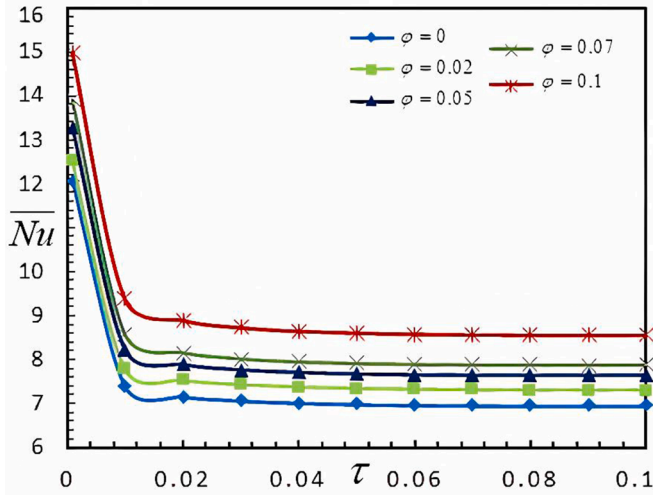


Fig. 5. Mean Nusselt number concerning time and volume fractions for $Ra = 10^5$, $Ha = 40$, $F_k = 2$, and $\gamma = 0$

- While studying various types of nanofluids with different domain shapes, the Frank-Kamenetskii theory number, while influencing heat transfer and fluid flow, has not given sufficient attention to its wide-ranging consequence.
- In terms of the geometrical setting of the current study, relatively little research has been done on the inclination angle of the magnetic field.
- Relatively few studies have been done on magnetic field dips in terms of the geometry settings studied so far. From a simulation-based study, anomalous heat transfer results for magnetic field inclination angles are reported.

The present article numerically investigates the natural or free convection of nanofluid flow and heat transfer of water- Fe_3O_4 nanofluid inside a square-shaped domain considering the exothermic Arrhenius chemical reaction in presence of a magnetic field for an extensive range of Frank-Kamenetskii numbers, Rayleigh numbers, nanoparticle volumetric ratio, Hartmann numbers, and the inclined angle of the magnetic field. The modelling of convective hydromagnetic nanofluid flow inside a productive and specific physical system have a comprehensive prospect in the engineering application. These models include the existence and identification of the critical parameter values, a challenge that arises frequently in combustion systems; for examples, see Scott [42]

Understanding convective processes and energy transformations in chemical thermodynamics, which is often used to anticipate the energy exchanges that occur in chemical reactions, phase transitions, and the creation of solutions, is the main goal of the current study. The problem's physical specifications can give a thorough understanding of the energy exchanges in the process engineering. Because the thermal performances of nanofluids in square geometries combined with the presence of exothermic chemical reactions have significant implications for a wide range of engineering applications, such as chemical industry reactors, chemical fractionation, oil recovery, moisture migration in food storage, combustion, explosions, etc. This research provides some novelties as follows:

- The effect of a thermal Rayleigh number has been quantitatively investigated.
- The effect of exothermic chemical reactions on natural convection has been investigated to see if it can be used for thermal engineering.
- The effects of a tilted magnetic field on heat transfer and fluid motion are being studied.
- The thermal explosion for the Frank-Kamenetskii number has been quantitatively investigated

- The significant impact on convective fluxes and the average Nusselt number for various parameters entered the problem has been analyzed.
- The impacts of the nanoparticles in the base fluid in terms of nanoparticle volume fraction has been investigated.

2. Physical modeling

A square shape of the physical domain having side length L has been placed in a magnetic field. The natural convective, laminar, and incompressible time-dependent nanofluid flow has been deemed inside the square cavity. Fig. 1 depicts the flow formation and the physical design of the problem. The origin of the coordinate is at the lower-left corner of the enclosure, the x-axis is represented by the bottom wall, and the y-axis is normal to it. The low temperature $T = T_c$ is applied to the right vertical sidewalls, while the high temperature $T = T_h$ is applied to the left vertical sidewalls and $T_h > T_c$ is maintained in all circumstances. The upper and lower walls are adiabatic and modeled as $\frac{\partial T}{\partial y} = 0$, and consider the velocity of fluid flow on all walls to be zero (no-slip), which is described as $u = v = 0$. The simulation is run utilizing thermophysical characteristics as input of iron oxide (Fe_3O_4)-water nanofluid. The thermophysical property values are given in Table 1. The Arrhenius chemical processes associated with the existence of a magnetic field are used, but the radiation impact is ignored in the present study because of the simplicity of the solutions of the assumed problem. The momentum equations have been incorporated with the hydromagnetic field and its inclination angle (Uddin and Rahman [43]) to understand the heat transference, and flow dynamics in presence of the magnetic field.

3. Modeling

The two-dimensional unsteady laminar free convection of water- Fe_3O_4 nanofluid inside square shape geometry is considered in this work to evaluate fluid flow and heat transmission. The nanosolids' are regarded as the species of major fluid due to the compatibility of molecules in size as well as the availability of fluidized. The Arrhenius equation is used to examine the exothermic chemical processes within the domain, where the chemical reaction yields a constant rate that is reliant on the absolute temperature. In addition, the first-order chemical reaction rate (non-isothermal) is defined as $k_0 a e^{(-\frac{E}{RT})}$ (Lazarovici et al. [44]). The primary temperature for the problem is assumed to be lower than the incremental temperature. The dimensional form of several governing partial differential equations (momentum, mass, and energy conservation) is shown below.

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0 \quad (1)$$

$$v \frac{\partial u}{\partial y} + u \frac{\partial u}{\partial x} = \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2} \right) - \left(\frac{\partial p}{\partial x} + \frac{\partial u}{\partial t} \right) - \{u \sin^2(\gamma) - v \sin(\gamma) \cos(\gamma)\} B_0^2 \frac{\sigma_{nf}}{\rho_{nf}} \quad (2)$$

$$v \frac{\partial v}{\partial y} + u \frac{\partial v}{\partial x} = \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} \right) + g \frac{(\rho \beta)_{nf}}{\rho_{nf}} (T - T_c) - \left(\frac{\partial p}{\partial y} + \frac{\partial v}{\partial t} \right) - B_0^2 \frac{\sigma_{nf}}{\rho_{nf}} \{v \cos^2(\gamma) - u \sin(\gamma) \cos(\gamma)\} \quad (3)$$

$$v \frac{\partial T}{\partial y} + u \frac{\partial T}{\partial x} = \alpha_{nf} \left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial x^2} \right) + \frac{1}{(\rho c_p)_{nf}} \frac{Q k_0 a}{e^{\frac{E}{RT}}} - \frac{\partial T}{\partial t} \quad (4)$$

here v and u are represents the velocity in y and x axis, ρ and p indicate the density and pressure, thermal diffusivity and expansion coefficient are represented by α and β , T is the temperature and μ is the dynamic

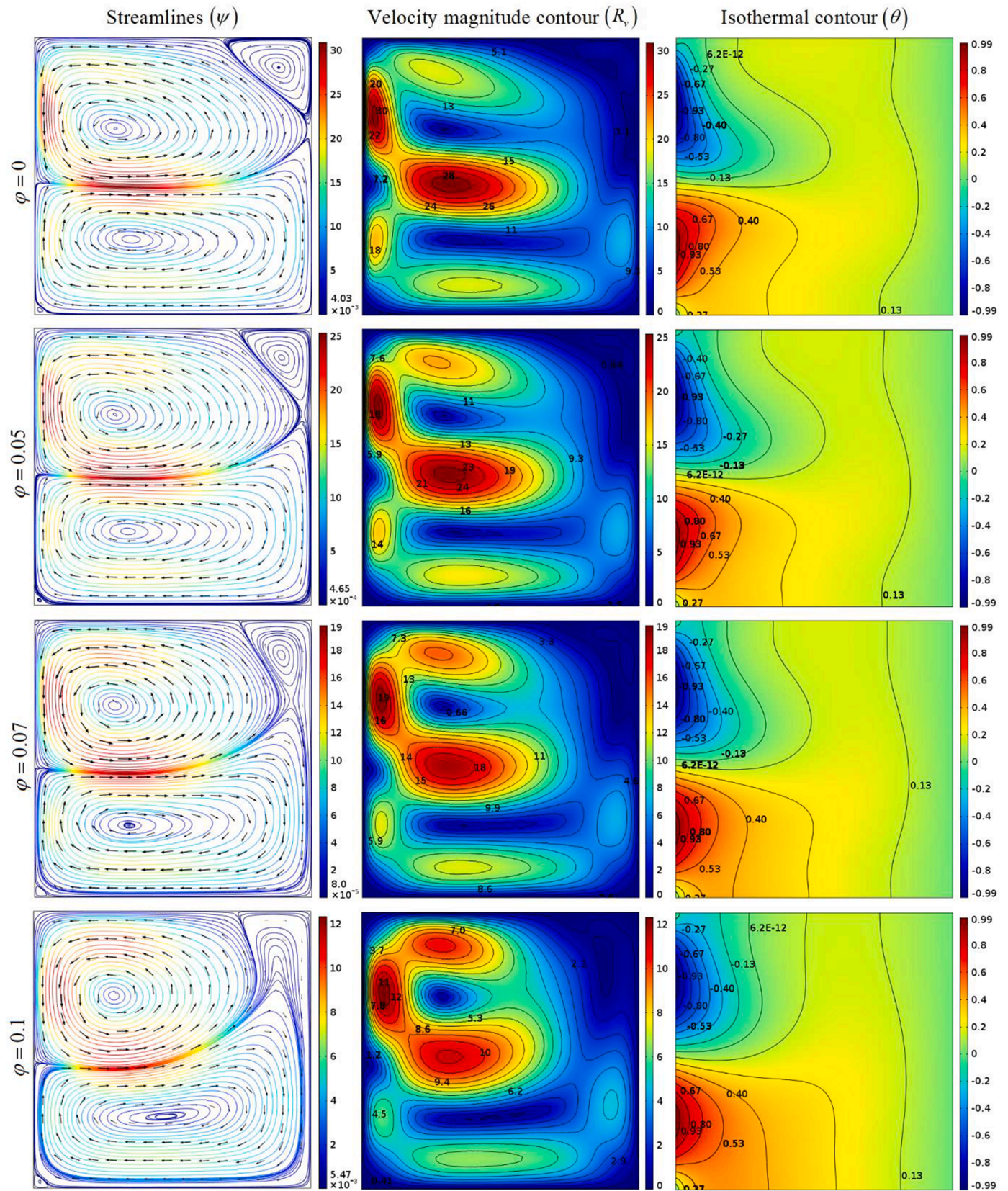


Fig. 6. Streamlines, velocity magnitude contour, and isothermal contour for several values of nanosolid volume fraction with $Ra = 10^5$, $Ha = 40$, $F_k = 2$, $\gamma = 0$, $T_0 = 25$ and $\tau = 5$.

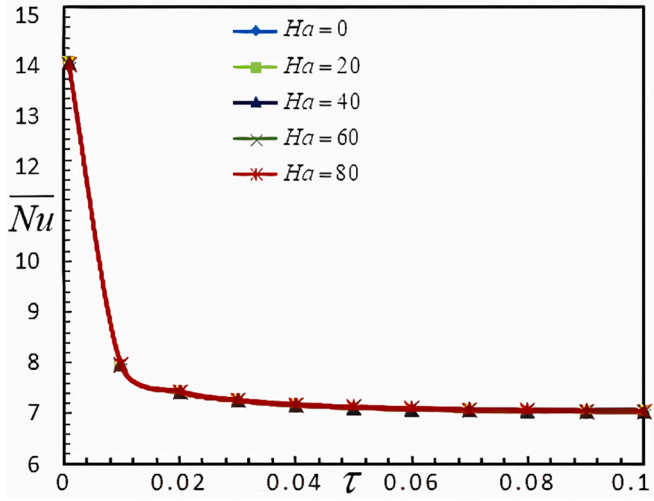


Fig. 7. Mean Nusselt number concerning time and Hartmann numbers for $\phi = 0.05$, $Ra = 10^4$, $F_k = 2$, and $\gamma = 0$

viscosity of nanofluid. The suffixes *bf* mean the base fluid, *pre* represent the nanoparticles, and *nf* mean the nanofluids. Different properties of ferric oxide (Fe_3O_4)-water nanofluids like effective density, mass expansion coefficient, thermal diffusivity, volumetric thermal, and heat capacities according to Islam and Nasrin [45] have been given as follows:

The effective density,

$$\rho_{nf} = \phi \rho_p + \rho_{bf} - \phi \rho_{bf} \quad (5)$$

The effective heat capacitance,

$$(\rho c_p)_{nf} = \phi (\rho c_p)_p + (\rho c_p)_{bf} - \phi (\rho c_p)_{bf} \quad (6)$$

The effective thermal diffusivity,

$$\alpha_{nf} = \frac{\kappa_{nf}}{(\rho c_p)_{nf}} \quad (7)$$

The volumetric thermal expansion,

$$(\rho \beta)_{nf} = \phi (\rho \beta)_p + (\rho \beta)_{bf} - \phi (\rho \beta)_{bf} \quad (8)$$

The volumetric mass expansion,

$$(\rho \beta^*)_{nf} = (1 - \phi)(\rho \beta^*)_{bf} + \phi(\rho \beta^*)_p \quad (9)$$

In this research, the equation of Chandrashekar et al. [46] has been utilized to estimate the effective dynamic viscosity of the nanofluid, which is defined as the following equation

$$\frac{\mu_{nf}}{\mu_{bf}} = 1 + 5300 \left(\frac{\phi}{1 - \phi} \right)^{2.8} \quad (10)$$

The thermal conductivity of the nanofluid is modelled by Bruggeman et al. [47] as

$$\kappa_{nf} = \frac{1}{4} [(3\phi - 1)\kappa_p + (2 - 3\phi)\kappa_{bf}] + \frac{\kappa_{bf}}{4} \sqrt{\Delta} \quad (11)$$

Here,

$$\Delta = (3\phi - 1)^2 \left(\frac{\kappa_p}{\kappa_{bf}} \right)^2 + (2 - 3\phi)^2 + 2(2 + 9\phi - 9\phi^2) \left(\frac{\kappa_p}{\kappa_{bf}} \right) \quad (11a)$$

The following are the boundary conditions for the current problem:
At bottom wall:

$$\frac{\partial T}{\partial y} = 0, \quad v = u = 0, \quad \forall \quad 0 \leq x \leq L, \quad y = L \quad (12a)$$

At top wall:

$$\frac{\partial T}{\partial y} = 0, \quad v = u = 0, \quad \forall \quad 0 \leq x \leq L, \quad y = 0 \quad (12b)$$

At left vertical wall:

$$T = T_c + (T_h - T_c) \sin 2\pi y, \quad v = u = 0, \quad \forall \quad x = 0, \quad 0 \leq y \leq L \quad (12c)$$

At right vertical wall:

$$T = T_c, \quad v = u = 0, \quad \forall \quad x = L, \quad 0 \leq y \leq L \quad (12d)$$

The velocity components are represented by *v* and *u* in the *y* and *x* - direction, while the vertical and horizontal distances are indicated by *y* and *x* respectively. *L* symbolizes the length of the square cavity, *T_c* and *T_h* are denoted low and high temperatures, respectively.

Dimensional analysis is one of the most effective mathematical approaches in the study of fluid mechanics. It is worthwhile to create the conservation equations dimensionless to characterize numerous transport processes in nanofluids. Uddin and Rahman [48] briefly illustrated the advantages of converting the nanofluid governing equation in non-dimensional form. The following non-dimensional variables in terms of Arrhenius kinetics are used to turn the Eqs. (1) to (4) and (12) into dimensionless versions for this purpose:

$$y = LY, \quad x = XL, \quad v = \frac{V \alpha_{bf}}{L}, \quad u = \frac{U \alpha_{bf}}{L}, \quad t = \frac{\tau L^2}{\alpha_{bf}}, \quad p = \frac{P \alpha_{bf}^2 \rho_{bf}}{L^2}, \quad \theta = \frac{(T - T_0)E}{RT_0^2} \quad (13)$$

The nondimensional core equations and boundary conditions are obtained by plugging Eq. (13) into Eqs. (1) to (4):

$$\frac{\partial V}{\partial Y} + \frac{\partial U}{\partial X} = 0 \quad (14)$$

$$V \frac{\partial U}{\partial Y} + U \frac{\partial U}{\partial X} = \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial X^2} \right) \frac{\text{Pr}}{\nu_{bf}} - \left(\frac{\rho_{bf}}{\rho_{nf}} \frac{\partial P}{\partial X} + \frac{\partial U}{\partial \tau} \right) - \{ U \sin^2(\gamma) - V \sin(\gamma) \cos(\gamma) \} \text{Pr} \cdot Ha \frac{\rho_{bf} \sigma_{nf}}{\rho_{nf} \sigma_{bf}} \quad (15)$$

$$V \frac{\partial V}{\partial Y} + U \frac{\partial V}{\partial X} = \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial X^2} \right) \frac{\text{Pr}}{\nu_{bf}} + \frac{(\rho \beta)_{nf}}{\beta_{bf} \rho_{nf}} \theta \cdot \text{Pr} \cdot Ra - \left(\frac{\rho_{bf}}{\rho_{nf}} \frac{\partial P}{\partial Y} + \frac{\partial V}{\partial \tau} \right) - \text{Pr} \cdot Ha \frac{\rho_{bf} \sigma_{nf}}{\rho_{nf} \sigma_{bf}} \{ V \cos^2(\gamma) - U \sin(\gamma) \cos(\gamma) \} \quad (16)$$

$$V \frac{\partial \theta}{\partial Y} + U \frac{\partial \theta}{\partial X} = \frac{\alpha_{nf}}{\alpha_{bf}} \left(\frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial^2 \theta}{\partial X^2} \right) + \frac{(\rho c_p)_{bf}}{(\rho c_p)_{nf}} F_k e^\theta - \frac{\partial \theta}{\partial \tau} \quad (17)$$

It is taken into account several dimensionless parameters, such as the Frank-Kamenetskii number $F_k = \left(\frac{E}{RT_0} \right) \frac{L^2 Q_0 a}{T_0 \kappa_{bf}} e^{-\frac{E}{RT_0}}$, the Hartmann number $Ha = B_0 L \sqrt{\frac{\sigma_{bf}}{\mu_{bf}}}$, the Rayleigh number $Ra = \frac{L^3 g \rho_{bf} \beta_{bf} RT_0^2}{E \alpha_{bf} \nu_{bf}}$, and the Prandtl number $\text{Pr} = \frac{\nu_{bf}}{\alpha_{bf}}$.

The non-dimensional boundary constraints are spelled out as follows:

At bottom wall:

$$\frac{\partial \theta}{\partial Y} = 0, \quad V = U = 0, \quad \forall \quad 0 \leq X \leq 1, \quad Y = 1 \quad (18a)$$

At top wall:

$$\frac{\partial \theta}{\partial Y} = 0, \quad V = U = 0, \quad \forall \quad 0 \leq X \leq 1, \quad Y = 0 \quad (18b)$$

At left vertical wall:

$$\theta = \sin(2\pi Y), \quad V = U = 0, \quad \forall \quad 0 \leq Y \leq 1, \quad X = 0 \quad (18c)$$

At right vertical wall:

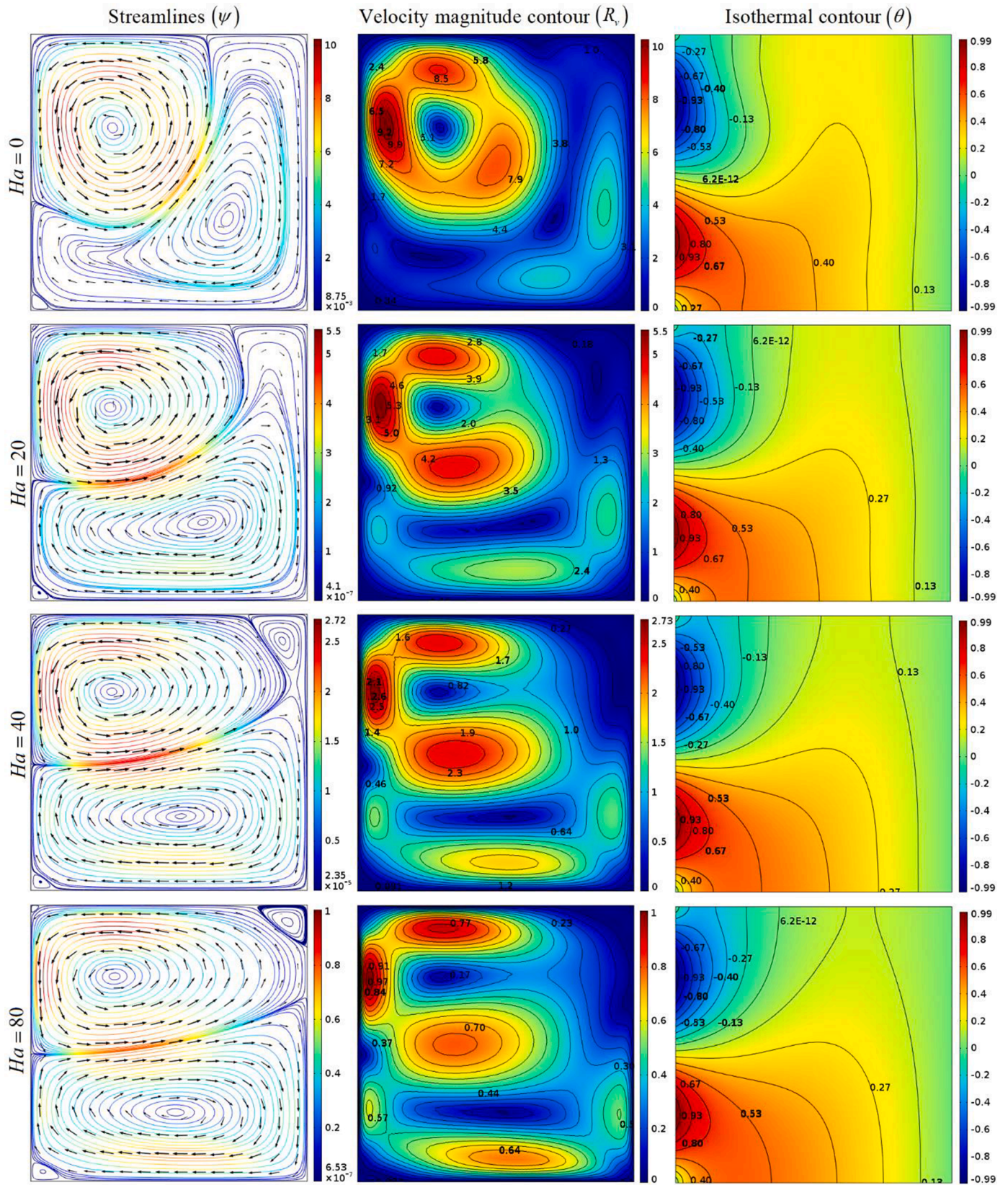


Fig. 8. Streamlines, velocity magnitude contour, and isothermal contour for various values of Hartmann numbers when $\phi = 0.05$, $Ra = 10^4$, $F_k = 2$, $\gamma = 0$, $T_0 = 25$ and $\tau = 5$

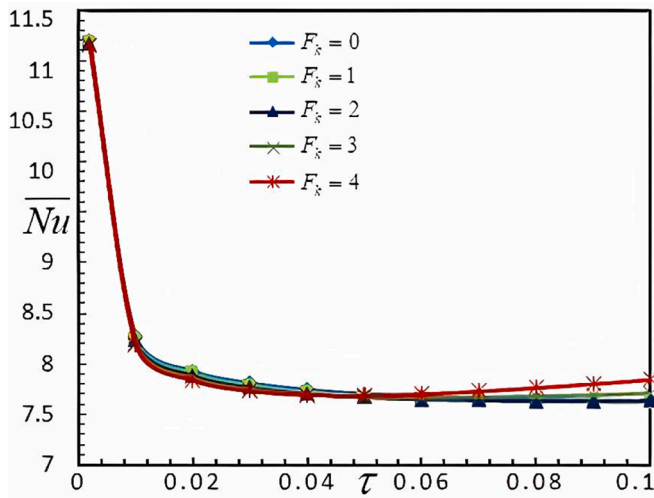


Fig. 9. Mean Nusselt number concerning time and Frank-Kamenetskii numbers for $\phi = 0.05$, $Ha = 40$, $Ra = 10^5$, and $\gamma = 0$

$$\theta = 0, \quad V = U = 0, \quad \forall 0 \leq Y \leq 1, \quad X = 1 \quad (18d)$$

The Nusselt number (local and average), and heat transmission coefficient are used to study heat transmission:

The chemical reaction's Nusselt number (local) near the heated wall can be expressed as,

$$Nu_L = \frac{Lq}{\kappa_{bf}T_0} \quad (19)$$

The heat flow is represented as due to the Arrhenius chemical process.

$$q = -\frac{\kappa_{nf}E}{RT_0} \left(\frac{\partial T}{\partial x} \right)_{x=0} \quad (20)$$

Applying the non-dimensional variables of Eq. (13), Eq. (20) transformed to

$$Nu_L = -\frac{\partial \theta}{\partial X} \frac{\kappa_{nf}}{\kappa_{bf}} \quad (21)$$

On the heated wall, the Nusselt number (average) becomes

$$Nu_{av} = -\frac{\kappa_{nf}}{\kappa_{bf}} \frac{1}{L} \int_0^L \frac{\partial \theta}{\partial X} dY \quad (22)$$

The resultant velocity of the flow of nanofluid can be defined as

$$R_v = \sqrt{v^2 + u^2} \quad (23)$$

Nanofluid velocities is related with the gradient of stream function (ψ) as

$$U = \frac{\partial \psi}{\partial y}, \quad V = -\frac{\partial \psi}{\partial x} \quad (24)$$

4. Computational technique

The finite element method (FEM) in the case of Galerkin's extension is used in the solution of the governing equations. The boundary conditions are incorporated with the final element equations for defining the domain boundaries. We followed many steps to convert the governing equations to FEM equations. In the finite element computing technique, first, we divide the domain into many subdomains called meshes. Nonuniform triangular mesh configuration is considered in the present studies. The triangular element in the meshes contains six nodal points which are related to the variables of the governing equations. The

pressure variable is associated with the corner nodes. The pressure term of the momentum equation is evaluated through a lower ordered polynomial such that it meets the continuity requirements of the set of equations. It is assumed that the pressure is discontinuous in the mesh elements, so it maintains different pressures in the domain. The detailed descriptions and the step-by-step calculations of FEM over the set of partial differential equations along with boundary conditions are available in the study of Rahman et al. [40], Zienkiewicz and Taylor [49], Uddin and Rahman [50], Khatun and Nasrin [51], Nasrin et al. [52], Priam and Nasrin [53]. In the method, the second task is to convert the nonlinear FEM equations into a system of integral equations. The exact integration formula is used in each term to calculate the integration and the thermal boundary conditions are used in the evaluation. After the evaluation of each term, the set of the global nonlinear algebraic equations are constructed into a global matrix form. The artificial compressibility scheme is used to remove the singularity problem in the matrix. the Newton-Raphson iteration technique and the Newton backward scheme have been adapted to find the final solution. The convergence criterion of 10^{-5} has been chosen to obtain a desired numerical solution.

5. Code validation

The comparison of the numerical results is a very imperative practice in computational dynamics. Data from Rahman et al. [40], who explored two-dimensional natural convection flow in a square domain occupied with copper oxide-water nanofluid in the existence of a gradient magnetic field generated by an exothermic chemical process, back up the numerical results of this work. In the comparison, the momentum equations are combined with the hydromagnetic field parameter ($Ha = 1.0$) and its inclination angle ($\gamma = \pi/6$). The geometric inclination is set to zero ($\delta = 0$) to be compatible with to work of Rahman et al. [40]. Fig. 2 displays a graphical depiction of the Nusselt number (average) due to the Frank-Kamenetskii number between Rahman et al. [40] and the current work. The current numerical code achieves excellent agreement for a particular situation with the results of Rahman et al. [40] which encourages the current study's numerical results to be expanded.

6. Grid check

A finite element strategy, also known as grid generation, divides a domain into a limited number of subdomains, which is a crucial step before solving the problem. In the current study, a widely utilized mesh testing technique has been implemented to assure a grid-independent solution for water- Fe_3O_4 nanofluid with the fixed value $\phi = 0.05$, $Fk = 2$, $Ha = 15$, $Ra = 10^6$, $\gamma = 0$, $T_0 = 25$, and $\tau = 1$ (steady-state case). Table 2 demonstrates the average Nusselt number ($\bar{Nu}$) for numerous values of the triangular shape of grids along the left upright wall.

Due to the upsurge in the number of elements in grid generation, the Nusselt number (average) has climbed to a certain level. After that, even if the elements grow larger, the rate of the Nusselt number (average) remains almost unchanged, as a result graph is almost parallel to $x -$ axis. The average Nusselt number grew by 7.13% as the number of elements enlarged from 202 to 17,038 whereas the $\bar{Nu}$ amplified by 0.0011% when the numeral of elements improved from 17,038 to 26,300. This is indicating that the findings are independent of the number of elements in the domain. It has been utilized 17,038 elements as a grid-independent outcome in this study, and the results are true across all domains.

7. Results and discussions

The overall purpose of this study is to use the finite element method to investigate the numerical evaluation of nanofluid flow kinetics in a square cavity driven by an exothermic chemical reaction. We have

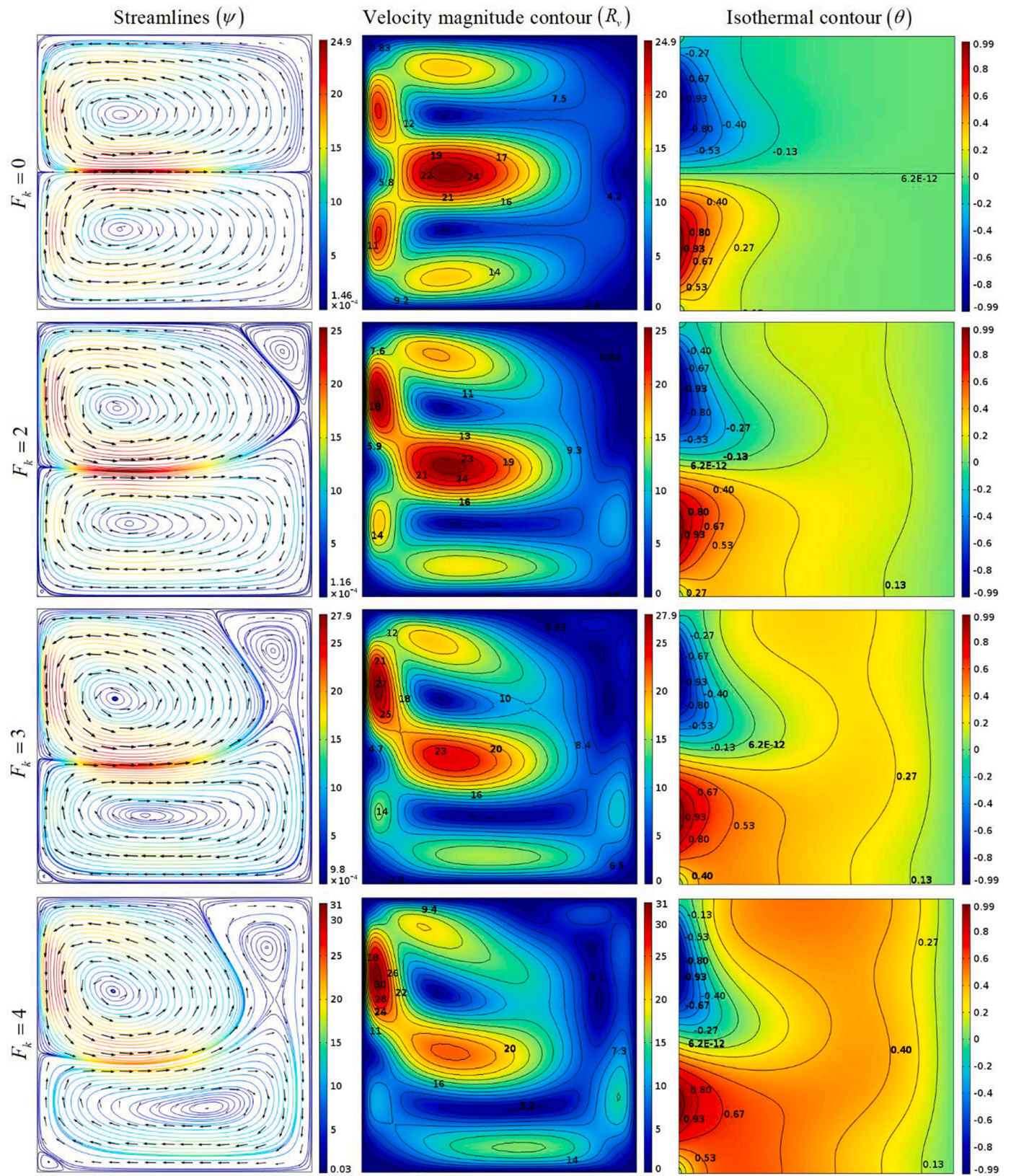


Fig. 10. Streamlines, velocity magnitude contour, and isothermal contour for various values of Frank-Kamenetskii numbers when $\phi = 0.05$, $Ha = 40$, $Ra = 10^5$, $\gamma = 0$, and $\tau = 5$

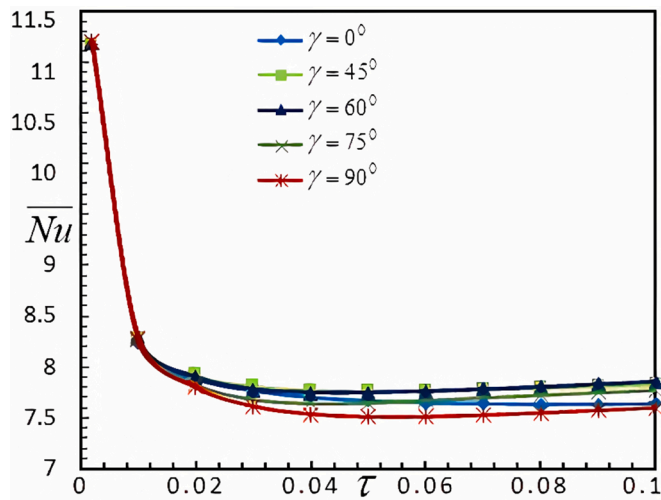


Fig. 11. Mean Nusselt number concerning time and inclined angles of the magnetic field for $\phi = 0.05$, $Ha = 40$, $F_k = 2$, and $Ra = 10^5$

considered Fe_3O_4 -water nanofluid flow inside the square vessel. The sinusoidal temperature dissemination is a source of the heating at the left vertical wall. The present study has been performed for several values of nanosolid volume fraction ($0 \leq \phi \leq 0.1$), Rayleigh number ($10^4 \leq Ra \leq 10^6$), Frank-Kamenetskii number ($0 \leq F_k \leq 4$), Hartmann number ($0 \leq Ha \leq 80$), and the magnetic field inclination ($0^\circ \leq \gamma \leq 90^\circ$). For the suggested boundary conditions, the numerical results are reported in terms of velocity magnitude contour streamlines, and isothermal contour inside the square cavity. For various parameters of the problem, the effects of several time steps are depicted concerning the Nusselt number (mean).

For varying values of the Rayleigh number, Fig. 3 depicts the impact of time on the average Nusselt number. For all values of Rayleigh number, $\overline{Nu}$ is uniformly greater at the primary stage of time. The heat transfer rate sharply declines up to a certain number and after that, it is plateaued for all considered Rayleigh numbers. This occurs because of the chaotic movement of solid particles of fluid at the early stage of the enclosure and as time goes on, the motion of the particles steadies down. It is importantly noted that at $Ra = 1 \times 10^6$, the Nusselt number (average) decreased by 13.41% and 0.04% for $0.001 \leq \tau \leq 0.07$, and $0.07 \leq \tau \leq 0.1$ respectively, whereas $\overline{Nu}$ reaches a steady-state situation at $\tau = 0.095$. For the Rayleigh number $1 \times 10^4 \leq Ra \leq 5 \times 10^5$, the value of $\overline{Nu}$ decreases from 49.39% to 24.89% for the time range $0.001 \leq \tau \leq 0.1$ while $\overline{Nu}$ declines by 0.1 to 0.05% for $0.1 \leq \tau \leq 0.15$. Henceforth for $1 \times 10^4 \leq Ra \leq 5 \times 10^5$ and $0.12 \leq \tau \leq 0.15$, Nusselt number (average) approaches a steady-state condition. Hence, most of the heat transmission happens at $Ra = 1 \times 10^6$.

Fig. 4 represents the streamlines, resultant velocity contour, and isothermal contour for four different thermal Rayleigh numbers. The significant change can be noticed in the pattern of streamlines, velocity magnitude, and isotherm for lower to higher Rayleigh numbers. The general trend towards iron oxide-based nanofluids is very intriguing. We have observed that the fluid rotates and flows in a horizontal direction in the cavity. All over the domain, two large vortices and a smaller vortex are revolving in the opposing directions. They are horizontally growing longer. In all instances, the velocity magnitude profile in various loops has a similar pattern but varies in intensity. The higher velocity intensity occurs near the left vertical wall of the cavity. The pattern of the isothermal lines is making two blooming semicircular loops on the hotter left vertical walls and spreads to the right directions. Due to the sinusoidal temperature source, which was captured on the cavity's left wall, these intriguing patterns of vortices and loops developed inside the vessel. Two big horizontal vortices are strongly visible for all values of

Ra where the direction of the upper whirlpool is anticlockwise and the lower is clockwise which is due to the natural behavior of convection. Besides this, a velocity layer forms between the vortices for all-Rayleigh numbers, and an additional tiny vortex can be seen in the top right corner of the cavity. In addition, the size of the tiny vortex in the geometry's top right corner has shrunk, and the velocity layer is practically parallel to the bottom wall, resulting in a larger buoyancy force. For higher Rayleigh numbers ($5 \times 10^5 \leq Ra \leq 1 \times 10^6$) the core of the vortices tends to come in the amid of the cavity and elongate horizontally to the colder wall. The density and the potency of the streamlines rose noticeably on account of enhancement of the temperature otherness within the entire domain. For higher buoyancy force ($Ra = 1 \times 10^6$) the size of the vortices is almost equal and covered most of the region of the cavity, which indicates the convection is superior. Velocity magnitude is the overview of the fluid flow within the cavity. In each step of the Rayleigh numbers, there are eight loops with different sizes and shapes in the duct whereas the bigger one is in the mid-point of the hole. For the lower buoyancy force ($1 \times 10^4 \leq Ra \leq 1 \times 10^5$), the highest velocity magnitude lies in the core neighboring to the warm wall whereas for the supreme buoyancy force ($5 \times 10^5 \leq Ra \leq 1 \times 10^6$) it shifted to the nearer of the center of the hole that indicating the fluid movement is dominant in the whole region. In addition, all loops are flattened and extended to the cooler wall to increase the thermal Rayleigh number, indicating that the fluid flows are more active in this scenario. The resulting velocity of increasing the Rayleigh number from $1 \times 10^4 \leq Ra \leq 1 \times 10^6$ is raised by 815%, 3307%, and 5211% respectively, indicating that the fluid motion has risen due to the higher buoyancy force. Isotherm reflects the heat distribution inside geometry in a normal sense, and the influence of isotherm for four distinct Rayleigh numbers has reflected in the last column of Fig. 4. For lower thermal Rayleigh number ($1 \times 10^4 \leq Ra \leq 1 \times 10^5$), heat lines are clustered adjacent to the hotter wall, which point out the conduction mode is active for this case. In the case of higher buoyancy force ($5 \times 10^5 \leq Ra \leq 1 \times 10^6$), the density of the distorted thermal lines decreases and uniformly elongates to the colder wall due to the convection. Consequently, convection heat transference is superior in the domain for the higher Rayleigh number. As the thermal Rayleigh numbers grew, the heat transmission method shifted from conduction to convection.

Fig. 5 demonstrates the influence of time on the rate of heat transmission when different amounts of nanosolids are mixed with the base fluid. As nanosolids are added to water, the heat energy is significantly increased as a result, heat is swiftly transported across the fluid by nanoparticles from the left and lower walls to the cooler walls. Due to the extreme phonon nature of the nanofluid, the velocity of the fluid flow is reduced for fluxing a higher quantity of nanosolids inside the normal fluid, while the rate of heat transmission increases at a slower pace than for the others. The heat transmission rate for the base fluid has decreased by 42.38% for $0.001 \leq \tau \leq 0.085$ while it declines by 0.016% for $0.085 \leq \tau \leq 0.1$ and the heat transfer rate achieves to steady-state for $\tau = 0.093$. Henceforth, for increasing the number of nanoparticles (2% to 7%) with the base fluid, the decline rate of heat transfer varies from 41.66% to 43.33% for the time range 0.001 to 0.085 whereas for the time range 0.085 to 0.09 decline rate of heat transfer varies from 0.003% to 0.002%.

In this case, the heat transfer rate achieves a steady-state condition from $0.085 \leq \tau \leq 0.09$. Heat transfer decrease by 42.66% for $0.001 \leq \tau \leq 0.06$ and 0.001% for $0.06 \leq \tau \leq 0.075$ in terms of 10% nanoparticle mixing with the base fluid and in this condition steady-state cases arise at $\tau = 0.07$. Finally, it can be stated that the presence of a higher amount of nanosolid in the base fluid aids in the faster attainment of the steady-state case.

The effect of streamline, velocity magnitude contour, and isothermal contour for the various nanosolid volumetric ratios are exhibited in Fig. 6. Generally, because of the physical model set up and the adoption of the sinusoidal boundary condition, streamlines are divided into two big opposite rotational vortices. Higher intensity velocity amplitudes

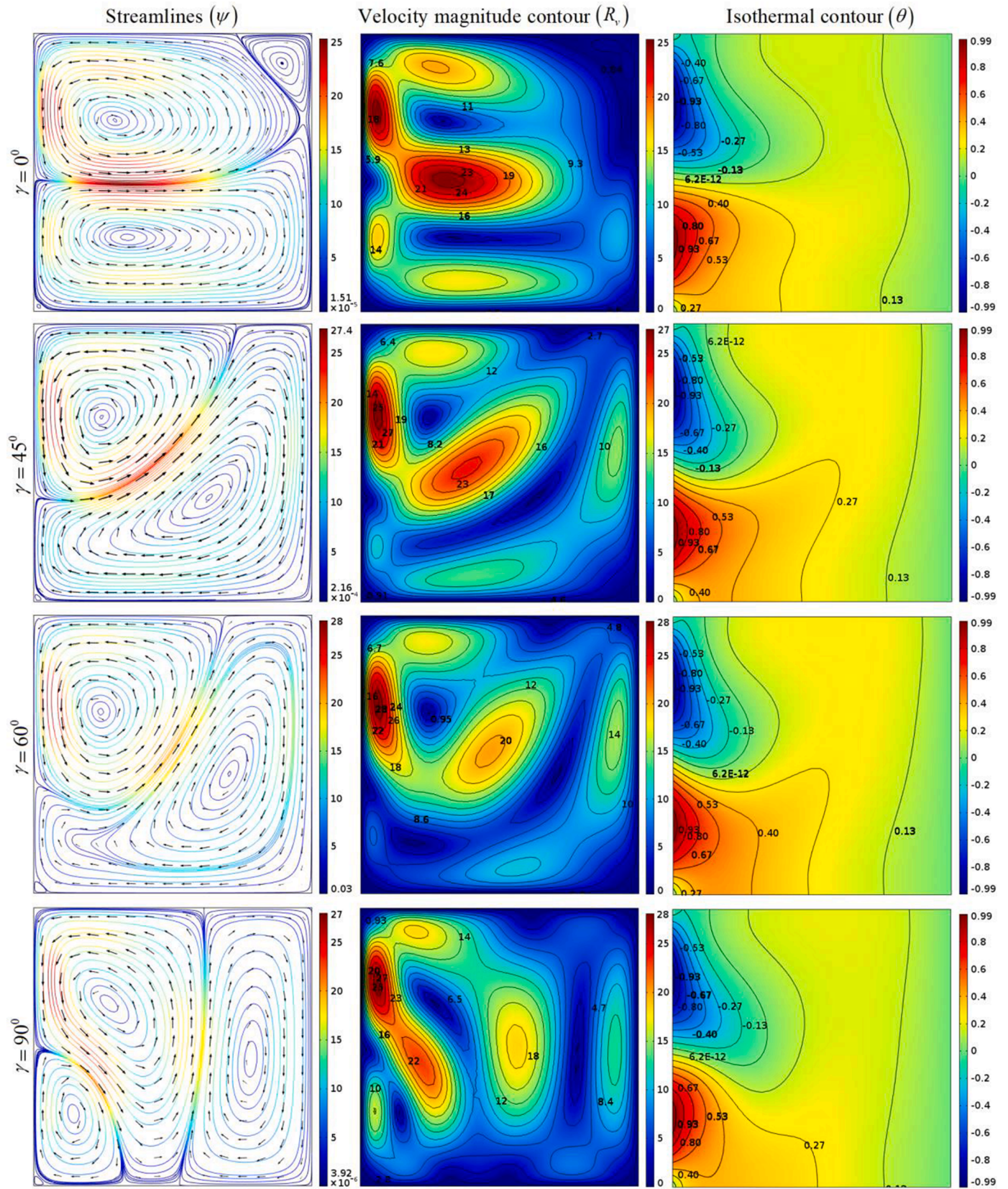


Fig. 12. Streamlines, velocity magnitude contour, and isothermal contour for inclined angles of the magnetic field with $\phi = 0.05$, $Ha = 40$, $F_k = 2$, $Ra = 10^5$, $T_0 = 25$ and $\tau = 5$

remain in the middle of the vessel. The isotherm is blooming, falling into two oppositely rotating rings from right to left. Interestingly, for both base fluid and nanofluid, two massive vortices along with two tiny vortices are observed, one in the left lower nook and the other in the right upper nook of the domain. The direction of the streamlines of the upper large vortex is counterclockwise, whereas the other two vortices are clockwise. Two clockwise-rotated vortices are combined in the event of a greater nanosolid ratio. The velocity layer is remarked among the vortices, but it is perceptible only in the tiny vortex in the uppermost right nook of the hollow for the base fluid. For nanofluid flow, the velocity layer is more active in the vicinity of the wall.

The core of the vortices tends to move to the center, indicating that the dominant fluid flows throughout the cavity, for a larger amount of nanoparticle volumetric ratio. The most noticeable difference between nanofluid and base fluid is that the streamlines become more clogged and tighter. The presence of nanoparticles is critical for increasing temperature differences and increasing nanofluid density due to collisions between nanoparticles. As a result of the increment of nanoparticle volumetric ratio from 5% to 10%, the streamline becomes a bit too relaxed and loosely connected to the boundary layer. This is because the viscosity of nanofluid increases as the nanosolid volume fraction increases, necessitating a larger buoyancy force to agitate the stream. In the case of velocity magnitude, it is noticed for the base fluid, all loops are flattened and elongated to the colder wall where the maximum resultant velocity magnitude is neighboring to the hotter wall. As a result of mixing the solid particle with the normal fluid, it is spectacularly realized that flattened vortices are becoming oval and the maximum velocity point is moving away from the hotter wall. The resultant velocity is decreased by 16.67%, 36.67%, and 60% after mixing the nanoparticle with base fluid 5%, 7%, and 10% respectively in an account for higher viscosity and the phonon nature of nanofluid. Hence, the velocity of the fluid flow in the normal fluid is superior to that of the nanofluid. It is remarkably noticed that the isotherms that revolve two counterclockwise and clockwise in the enclosure show the dominant effects of the sinusoidal thermal wall conditions on the heat distribution. Without nanoparticles in the solution, the isotherms are loosely connected with contorted lines indicating the convection nature of the heat transfer whereas the density of the nanoparticles and water mixer increased and indicated closely connected isotherm lines yield the significant heat transfer from species to species. For increasing the nanoparticle volumetric ratio 5% to 10%, the heat transmission rate (average) increased by 9.63% to 22.65%. In conclusion, when the velocity of the liquid flow decreases and the nanoparticle volumetric ratio increases, the average heat transfer rate increases.

Fig. 7 demonstrates the impact of the Hartmann number, a hydro-magnetic field parameter, on the average Nusselt number at the left vertical walls over time. In terms of the time range 0.001 to 0.9, the decreasing rate of heat transfer is 49.01%, 49.30%, 49.33%, 49.23%, 49.20% for the hydro-magnetic field parameter, $Ha = 0, 20, 40, 60, 80$ respectively, while the decreasing rate varies from 0.1% to 0.001% for the time range 0.9 to 0.1. It is striking to note that heat transmission takes approximately the same length of time to achieve a steady state for all values of the Hartmann number, which is around 0.95. The trend of the Hartmann number is shown in Fig. 8, and this graph shows that it has a substantial impact on the temperature application. In the non-appearance of any impact of the hydro-magnetic field, the streamlines, resultant velocity, and isotherms are stiff, but they are feeble with a nonzero value of the parameter. If the Hartmann number isn't available, the cavity contains two rotating clockwise and anticlockwise vortices that try to occupy the whole cavity. The fluid movement is hampered in the appearance of the Hartmann number because the Lorentz force, which acts right angles to the direction of velocity, is produced by a magnetic field. As a result, when $Ha = 0$ is compared to the greater number of Hartmann number parameters, heat transmission is more active. The velocity is relatively high in the non-existence of a magnetic field since the shape of the main loop is almost circular and the core of

the loop lies in its center. The structure of the loops is broken down, and the density of the flow line drop-off shows a lower velocity inside the cavity when larger Hartmann numbers are present. The velocity magnitude is lessened 45%, 72.7%, and 90% for the Hartmann numbers 0 to 20, 0 to 40, and 0 to 80 respectively.

The isothermal lines inside the cavity become increasingly deformed and denser at the left vertical warmed fence with the absenteeism of Hartmann number. Furthermore, in the center portion of the heated wall, the isothermal lines travel upward more, indicating a greater heat transmission zone. Due to $Ha = 0$, the convection is superior but $Ha > 0$ it turns to conduction which indicates that the heat transportation decrement with the enhancement of the intensity of Hartmann number. These characteristics are due to the multiferoic multipart structure of Fe_3O_4 nanoparticles in water, where natural magnetic dipoles alter the electric polarity. Fe_3O_4 nanoparticles' magnetically generated ferro-electricity is constant within water molecules.

The influence of the exothermic chemical reaction parameter named Frank-Kamenetskii number (F_k) on the Nusselt number (average) at the left vertical walls over time is exhibited in Fig. 9. It should be stated that the values of F_k depend on the parametric arrangement, as a result in the present research, the Frank-Kamenetskii number varies from 1 to 4. Initially, the average heat transfer rate declines rapidly (decline rate of heat transfer varies from 45% to 44% for $0 \leq F_k \leq 4$ respectively within $0.001 \leq \tau \leq 0.06$) due to the unsettled nanoparticles in the early time steps, subsequently the thermal transmission decline rate for all studied values of F_k varies from 0.6% to 1.8% over the time range 0.06 to 0.1 which specifies that the average rate of heat handover reaches a plateau state in that time.

Fig. 10 exemplify the effect of Frank-Kamenetskii numbers ($F_k = 0, 2, 3, 4$) over streamlines (1st column), velocity magnitude contour (2nd column), and isothermal contour (3rd column) with constant parameters $\phi = 0.05$, $Ha = 40$, $Ra = 10^5$, $\gamma = 0$, and $\tau = 5$. The exothermic process produces more heat as the Frank-Kamenetskii number rises, speeding up the convection processes within the cavity. The fluid flow is regulated by buoyant force in the absence of an exothermic chemical reaction, and two anti-directionally spinning and horizontally symmetrical vortices can be seen in the cavity. In this condition, the thickness of the velocity tier is significant among the two whirlpools. The flow pattern of the fluid as well as the shape of the vortices is tremendously changed and another vortex appeared in the upper right corner of the square vessel, due to the Frank-Kamenetskii number ($F_k = 2$). For the higher Frank-Kamenetskii number ($F_k = 3, 4$), the bottom and upper right corner vortices join, and the upper left vortex comes to the round shape, and a puny vortex is formed in the lower-left nook of the domain. It is also worth noting that the velocity layer gets thinner as the Frank-Kamenetskii number gets higher.

The fluid flow is more active in this scenario, and it covers the whole domain; consequently, the streamlines are highly connected, and the core of the hole extended further to the cooler wall. Finally, it is to say that the higher Frank-Kamenetskii number controls the fluid movement of nanofluid. In the absence of chemical reaction, the vortices and streamlines in the resulting velocity tend to extend to the cooler wall, and the greatest velocity is found in the middle of the vessel. The flow patterns are drastically altered when F_k is increased, and the vortices are pressed to the hotter wall, indicating that fluid flow is strong in the cavity and the resultant velocity increases by 0.4%, 12.05%, and 24.5% for $F_k = 2, 3$, and 4, respectively. The isotherm is only limited by the thermal Rayleigh number when a chemical reaction is an absenteeism, but when it is present, a massive, coiled isotherm arises over the geometry, driving a dominant convective heat transmission. The mean Nusselt number reduces by 0.09% for the little quantity of F_k ($F_k = 1$) due to the exothermic properties of the chemical process, but increases by 10.61% for the bigger value of F_k ($F_k = 4$).

The diversity in mean Nusselt number ($\overline{Nu}$) concerning time for five inclined angles of the magnetic field are depicted in Fig. 11. At the

earliest stages, the mean Nusselt number appears to be about similar for the varied inclined angles. For each inclination, $\overline{Nu}$ decreases significantly initially before stabilizing. For the all consider values of inclined angle, initially ($0.001 \leq \tau \leq 0.08$) the decreasing rate of $\overline{Nu}$ is near about 44.6% (average) while time progresses ($0.08 \leq \tau \leq 0.1$), the decreasing rate of $\overline{Nu}$ is near approximately 0.48%. The falling rate of $\overline{Nu}$ is considerably larger (45.09%) for $\gamma = 0^\circ$, although it achieves a steady-state sooner ($\tau = 0.085$) than the others whereas at $\gamma = 60^\circ$, occurs lower average heat transfer declination rate, but a significantly longer steady-state duration.

Fig. 12 depicts the effect of an inclined magnetic field's angle on heat transmission and fluid movement, with the nanofluid flow channel perfectly aligned with the magnetic field's direction. The streamlines are split into two huge vortices that rotate in opposing directions from the magnetic field while flowing in the same direction. A tiny vortex core emerges adjacent to a big vortex in this scenario, and the maximum velocities are achieved. As γ rises, the thickness of this velocity layer diminishes. Depending on the inclination angle, the magnetic flow splits and lengthens the vortices. These findings suggest that the magnetic field is directly controlling the flow of nanofluids. The cores and velocity lines in the resultant velocity lengthened to the colder wall in the absence of an inclined angle, and the highest velocity is found around the midpoint of the geometry. In the event of higher angle values, the flow shapes spoiled their previous shape, and the maximum velocity is found the nearer the hotter wall, suggesting that fluid flow is strong in these cases. The resultant velocity of the fluid flow increased by 8% for $\gamma = 45^\circ$ in terms of $\gamma = 0^\circ$ while it increased by 12% for $\gamma = 60^\circ$, and the increasing rate remains unchanged for $\gamma = 90^\circ$. When $\gamma = 0^\circ$, the magnetic field effect operates only on the momentum equation of the y – axis, but when $\gamma = 90^\circ$, it acts only on the momentum equation of the x – axis, and in the rest of the instances, it acts on the full domain of the geometry. In the case of $0^\circ < \gamma < 90^\circ$, the isotherms are substantially deformed compared to $\gamma = 0^\circ$ and maximum heat transfer is found at $\gamma = 60^\circ$. The conduction mode is highly dominant in the absenteeism of inclined magnetic field but includes the inclination angle conduction transformed to the convection.

8. Conclusions

This research looked at the thermal transmission and fluid movement characteristics of water- Fe_3O_4 nanofluid in a square domain in the existence of a hydromagnetic field and an exothermic chemical reaction governed by Arrhenius kinetics. The geometry has been designed with engineering in mind, with insulated top and bottom walls, as well as cooled and warmer right and left walls, respectively. Applying the finite element method (FEM) numerically solved the mathematical model which is dictated by the nondimensional temperature and stream function. Comparisons with earlier published articles have been made, and the findings show a high level of agreement. The solution's development from unstable to steady state has been studied throughout time. Over a wide range of model parameters, streamline distributions, resultant velocity, mean Nusselt number, and isotherms, the impacts of diverse model parameters for instance nanosolids volume percentage, and thermal Rayleigh number have been created and analyzed in depth. The major goals are to investigate the flow and thermal transmission properties of water- Fe_3O_4 nanofluid so that the outcomes may be used in manufacturing applications. The following key conclusions may be drawn from the simulation findings.

- It is obvious that fluid movement and thermal transmission are proportionally related to the thermal buoyancy forces. The resulting velocity of increasing the Rayleigh number from 1×10^4 to 1×10^6 is raised by 815%, 3307%, and 5211% respectively, indicating that the fluid motion rises due to the higher buoyancy force.

- Conduction heat transmission is dominant at lower buoyancy forces, whereas convection thermal transmission is stronger in the domain with larger Rayleigh numbers.
- The average Nusselt number increased by 75.92% for $Ra = 1 \times 10^6$ comparing with $Ra = 1 \times 10^4$. The maximum temperature gradient (11.10%) in the x – axis is located at $Ra = 5 \times 10^5$.
- Compared with the base fluid water, for increasing the nanoparticle volumetric ratio, although the resultant velocity of the fluid flow is decreased (maximum 60% for 10% nanoparticle) but the average heat transfer rate increased (maximum 22.65% for 10% nanoparticle). To increase the number of nanoparticle volumetric ratios, the temperature gradient in the x – axis is reduced by 40.01%.
- To get the best nanofluid flow, the volume percentage of nanoparticles and the buoyancy force must be adjusted. A larger thermal Rayleigh number is necessary for a higher nanoparticle volume fraction in the solution.
- The thermal transmission and flow of nanofluid may be regulated by the magnetic field. Heat transmission and the resultant velocity of the fluid flow typically decrease as the Hartmann number increases, however, in this case, the temperature gradient in the x – axis is somewhat enhanced.
- The Frank-Kamenetskii number has a significant impact on convective fluxes, and as the Frank-Kamenetskii number rises, so does the mean Nusselt number. The restricted solution can be blasted apart due to the heat created by the strongly exothermic reaction (higher Frank-Kamenetskii number).
- For the Frank-Kamenetskii number is enlarged from 0 to 4, the heat transfer in the midpoint of the domain is enhanced by 10.61%, the temperature gradient is more active, and the consequent velocity rises by about 24.5%.
- The magnetic field and its inclination angle govern the thermal transmission rate and flow direction. The convective cell core elongates in the direction of the applied magnetic field. At $\gamma = 60^\circ$, the maximum heat transmission is discovered in this study.
- The mean Nusselt number increases with increasing nanosolid volume percentage, thermal Rayleigh number, and Frank-Kamenetskii number, while it decreases with rising Hartmann number.

In a nutshell, this research has focused on the role played by the thermal Rayleigh number, the inclination of the magnetic field, and the Frank-Kamenetskii number in the heat transfer and fluid flow of nanofluid. According to the authors, the numerical results of this work will help scientists, engineers, and researchers better understand the use of different heat transfer and fluid flow characteristics as well as various engineering and industrial applications of this type of work, such as in chemical industry reactors, oil recovery, electronic chips, moisture migration in food storage, etc. The current study might offer future research that should consider other physics, such as radiation effects, viscous dissipations, changing nanofluid, and varied forms of geometry. The concentration conservation equation may be utilized to explore double-diffusive mixed convection, for example.

Declaration of Competing Interest

The authors have declared that there are no competing interests

Data availability

Data will be made available on request.

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References

- [1] S.U. Choi, A.E. Jeffrey, Enhancing thermal conductivity of fluids with nanoparticles, Argonne National Lab., ILUnited States, 1995. No. ANL/MSD/CP-84938; CONF-951135-29.
- [2] M.J. Uddin, S.K. Rasel, Numerical analysis of natural convective heat transport of copper oxide-water nanofluid flow inside a quadrilateral vessel, *Heliyon* 5 (2019) e01757.
- [3] Y. Yang, Z.G. Zhang, E.A. Grulke, W.B. Anderson, G. Wu, Heat transfer properties of nanoparticle-in-fluid dispersions (nanofluids) in laminar flow, *Int. J. Heat Mass Transf.* 48 (2005) 1107–1116.
- [4] M. Lomascolo, G. Colangelo, M. Milanese, A. De Risi, Review of heat transfer in nanofluids: conductive, convective and radiative experimental results, *Renew. Sustain. Energy Rev.* 43 (2015) 1182–1198.
- [5] M.U. Sajid, H.M. Ali, Recent advances in application of nanofluids in heat transfer devices: a critical review, *Renew. Sustain. Energy Rev.* 103 (2019) 556–592.
- [6] A.A.A. Arani, M. Kazemi, Analysis of fluid flow and heat transfer of nanofluid inside triangular enclosure equipped with rotational obstacle, *J. Mech. Sci. Technol.* 33 (2019) 4917–4929.
- [7] M.A. Mansour, S.E. Ahmed, A numerical study on natural convection in porous media-filled an inclined triangular enclosure with heat sources using nanofluid in the presence of heat generation effect, *Eng. Sci. Technol. Int. J.* 18 (2015) 485–495.
- [8] R. Nasrin, M.A. Alim, A.J. Chamkha, Combined convection flow in triangular wavy chamber filled with water–CuO nanofluid: effect of viscosity models, *Int. Commun. Heat Mass Transf.* 39 (2012) 1226–1236.
- [9] I. Zahan, R. Nasrin, S. Khatun, Thermal performance of ternary-hybrid nanofluids through a convergent-divergent nozzle using distilled water - ethylene glycol mixtures, *Int. Commun. Heat Mass Transf.* 137 (2022), 106254, <https://doi.org/10.1016/j.icheatmasstransfer.2022.106254>.
- [10] L.M. Al-Balushi, M.J. Uddin, M.M. Rahman, Natural convective heat transfer in a square enclosure utilizing magnetic nanoparticles, *Propuls. Power Res.* 8 (2019) 194–209.
- [11] R. Nasrin, S. Parvin, Investigation of buoyancy-driven flow and heat transfer in a trapezoidal cavity filled with water–Cu nanofluid, *Int. Commun. Heat Mass Transf.* 39 (2012) 270–274.
- [12] A.K. Kareem, H.A. Mohammed, A.K. Hussein, S. Gao, Numerical investigation of mixed convection heat transfer of nanofluids in a lid-driven trapezoidal cavity, *Int. Commun. Heat Mass Transf.* 77 (2016) 195–205.
- [13] N. Makulati, A. Kasaeipoor, M.M. Rashidi, Numerical study of natural convection of a water–alumina nanofluid in inclined C-shaped enclosures under the effect of magnetic field, *Adv. Powder Technol.* 27 (2016) 661–672.
- [14] S. Soleimani, M. Sheikholeslami, D.D. Ganji, M. Gorji-Bandpay, Natural convection heat transfer in a nanofluid filled semi-annulus enclosure, *Int. Commun. Heat Mass Transf.* 39 (2012) 565–574.
- [15] N.C. Roy, Natural convection in the annulus bounded by two wavy wall cylinders having a chemically reacting fluid, *Int. J. Heat Mass Transf.* 138 (2019) 1082–1095.
- [16] A.J. Chamkha, Hydromagnetic combined convection flow in a vertical lid-driven cavity with internal heat generation or absorption, *Numer. Heat Transf. A Appl.* 41 (2002) 529–546.
- [17] M.J. Uddin, J. Al-Balushi, S. Mahatabuddin, M.M. Rahman, Convective heat transport for copper oxide-water nanofluid in an isosceles triangular cavity with a rippled base wall in the presence of magnetic field, *Int. J. Thermofluids* 16 (2022), 100195.
- [18] T. Islam, N. Parveen, R. Nasrin, Mathematical modeling of unsteady flow with uniform/non-uniform temperature and magnetic intensity in a half-moon shaped domain, *Heliyon* 8 (2022) e09015.
- [19] S.S. Priam, R. Nasrin, Oriented magneto-conjugate heat transfer and entropy generation in an inclined domain having wavy partition, *Int. Commun. Heat Mass Transf.* 126 (2021), 105430.
- [20] M. Rahman, J. Uddin, Magnetohydrodynamic boundary layer flow of nanofluid with variable chemical reaction in a radiative vertical plate, *Heat Transf.* 50 (2021) 7879–7897.
- [21] A. Malleswaran, S. Sivasankaran, A numerical simulation on MHD mixed convection in a lid-driven cavity with corner heaters, *J. Appl. Fluid Mech.* 9 (2015) 311–319.
- [22] M.J. Uddin, S.K. Rasel, J.K. Adewole, K.S. Al Kalbani, Finite element simulation on the convective double diffusive water-based copper oxide nanofluid flow in a square cavity having vertical wavy surfaces in presence of hydro-magnetic field, *Results Eng.* 13 (2022), 100364.
- [23] D. Chatterjee, S.K. Gupta, Hydromagnetic mixed convective transport in a nonisothermally heated lid-driven square enclosure including a heat-conducting circular cylinder, *Ind. Eng. Chem. Res.* 53 (2014) 19775–19787.
- [24] M.J. Uddin, M. Arifuzzaman, M.M. Rahman, Thermohydromagnetic performance of water–alumina nanofluid in a trapezoidal vessel, *J. Thermophys. Heat Transf.* 36 (2022) 28–39.
- [25] M. Habibishandiz, Z. Saghir, MHD mixed convection heat transfer of nanofluid containing oxytactic microorganisms inside a vertical annular porous cylinder, *Int. J. Thermofluids* 14 (2022), 100151.
- [26] S. Yeasmin, M.M. Billah, M.Z. Molla, K.E. Hoque, Numerical analysis of unsteady mixed convection heat transfer characteristics of nanofluids confined within a porous lid-driven L-shaped cavity, *Int. J. Thermofluids* (2022), 100218.
- [27] F. Selimefendigil, H.F. Öztop, N. Abu-Hamdeh, Mixed convection due to rotating cylinder in an internally heated and flexible walled cavity filled with SiO₂–water nanofluids: effect of nanoparticle shape, *Int. Commun. Heat Mass Transf.* 71 (2016) 9–19.
- [28] N.C. Roy, Natural convection of nanofluids in a square enclosure with different shapes of inner geometry, *Phys. Fluids* 30 (2018), 113605.
- [29] S. Jani, M. Mahmoodi, M. Amini, Magnetohydrodynamic free convection in a square cavity heated from below and cooled from other, *World Acad. Sci. Eng. Technol.* 7 (2013) 329–334.
- [30] M.M. Ali, M.A. Alim, M.A. Maleque, S.S. Ahmed, Numerical simulation of MHD free convection flow in a differentially heated square enclosure with tilted obstacle, *AIP Conf. Proc.* 1851 (2017) 1–8.
- [31] M. Hasanuzzaman, R. Saidur, N.A. Rahim, Effectiveness enhancement of heat exchanger by using nanofluids, in: *Proceedings of the IEEE First Conference on Clean Energy and Technology, IEEE*, 2011, pp. 98–103.
- [32] D. Guerrieri, F. Viana, S.C. Frago, M.M. Aeliano, Shell and tube heat exchangers using nanofluids, in: *14th Brazilian Congress of Thermal Science and Engineering*, 2012.
- [33] M.C. Ece, E. Büyük, Natural-convection flow under a magnetic field in an inclined rectangular enclosure heated and cooled on adjacent walls, *Fluid Dyn. Res.* 38 (2006) 564–590.
- [34] K. Kahveci, S. Öztuna, MHD natural convection flow and heat transfer in a laterally heated partitioned enclosure, *Eur. J. Mech. B. Fluids* 28 (2009) 744–752.
- [35] O.S.K. Hamama, F.S. Sharif, M.N. Hidouri, Irreversibility investigation on MHD natural convection in a square cavity for different Prandtl numbers, *World J. Eng. Phys. Sci.* 2 (2014) 060–075.
- [36] M. Ashouri, M.B. Shafii, H.R. Kokande, MHD natural convection flow in cavities filled with square solid blocks, *Int. J. Numer. Methods Heat Fluid Flow* 24 (2014) 1813–1830.
- [37] A. Lazarovici, V. Volpert, J.H. Merkin, Steady states, oscillations and heat explosion in a combustion problem with convection, *Eur. J. Mech. B. Fluids* 2 (2005) 189–203.
- [38] J.H. Merkin, I. Pop, Stagnation point flow past a stretching/shrinking sheet driven by Arrhenius kinetics, *Appl. Math. Comput.* 337 (2018) 583–590.
- [39] C. Zhang, L. Zheng, X. Zhang, G. Chen, MHD flow and radiation heat transfer of nanofluids in porous media with variable surface heat flux and chemical reaction, *J. Appl. Math.* 39 (2015) 165–181.
- [40] M.M. Rahman, I. Pop, M.Z. Saghir, Steady free convection flow within a titled nanofluid saturated porous cavity in the presence of a sloping magnetic field energized by an exothermic chemical reaction administered by Arrhenius kinetics, *Int. J. Heat Mass Transf.* 129 (2019) 198–211.
- [41] M. Raees-ul-Haq, Analysis of heat transfer in a triangular enclosure filled with a porous medium saturated with magnetized nanofluid charged by an exothermic chemical reaction, *Math. Probl. Eng.* 2019 (2019) 1–19.
- [42] S.K. Scott, *Chemical Chaos*, Clarendon Press, 1991.
- [43] M.J. Uddin, M.M. Rahman, Heat transportation in copper oxide-water nanofluid filled triangular cavities, *Int. J. Heat Technol.* 38 (2020) 106–124.
- [44] A. Lazarovici, V. Volpert, J.H. Merkin, Steady states, oscillations and heat explosion in a combustion problem with convection, *Eur. J. Mech. B. Fluids* 24 (2005) 189–203.
- [45] T. Islam, R. Nasrin, Thermal operation by nanofluids with various aspects: a comprehensive numerical appraisal, *Waves Random Complex Media* 32 (2022), <https://doi.org/10.1080/17455030.2022.2117430>.
- [46] J. Chandrashekar, C. Kuhn, Y. Oka, D.A. Yarmolinsky, E. Hummler, N.J. Ryba, C. S. Zuker, The cells and peripheral representation of sodium taste in mice, *Nature* 464 (2010) 297–301.
- [47] V.D. Bruggeman, Calculation of different physical constants of heterogeneous substances I. Dielectric constants and conductivities of the mixed media of isotropic substances, *Ann. Phys.* 416 (1935) 636–664.
- [48] M.J. Uddin, M.M. Rahman, Numerical computation of natural convective heat transport within nanofluids filled semi-circular shaped enclosure using nonhomogeneous dynamic model, *Therm. Sci. Eng. Prog.* 1 (2017) 25–38.
- [49] O.C. Zienkiewicz, R.L. Taylor, *The finite element method*, McGraw-Hill 2 (1989) 1–648.
- [50] M.J. Uddin, M.M. Rahman, Finite element computational procedure for convective flow of nanofluids in an annulus, *Therm. Sci. Eng. Prog.* 6 (2018) 251–267.
- [51] S.S. Priam, R. Nasrin, Numerical appraisal of time-dependent peristaltic duct flow using Casson fluid, *Int. J. Mech. Sci.* 233 (2022), 107676, <https://doi.org/10.1016/j.jimecs.2022.107676>.
- [52] S. Khatun, R. Nasrin, Numerical modeling of Buongiorno's nanofluid on free convection: Thermophoresis and Brownian effects, *J. Naval Archit. Mar. Eng.* 18 (2) (2021) 217–239.
- [53] R. Nasrin, S.A. Sweetey, I. Zahan, Turbulent nanofluid flow analysis passing a shell and tube thermal exchanger with Kays-Crawford model, *J. Nanofluids* 10 (4) (2021) 518–537.