# UNRELIABLE SERVER MARKOVIAN QUEUE WITH HETEROGENEOUS ARRIVAL UNDER N-POLICY

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## 1. Model Description and Notations

Present investigation deals with optimal management policy for Markovian queue with server breakdowns and vacations in which arrival occurs according to the state of the server. The customers arrive in Poisson fashion to get the service. The server may breakdown during the service and goes for repair immediately. By applying probability generating function technique queue length distribution is obtain for different states of the server. Further we determine the probability of empty system, expected number of units in the system.

Following notations and probabilities are used through out the paper for formulating the model mathematically:

| | |
|---|---|
| N | Threshold level of queue length when server turns on |
| $\lambda_i$(i=0,1,2,3) | Arrival rate of customers in various status |
| $\mu$ | Mean service rate of the server |
| $\alpha$ | Mean break down rate of the server |
| $\beta$ | Mean repair rate of the server |
| $P_0(n)$ | The probability of being n customers in the system and server is on vacation. |
| $P_1(n)$ | The probability of being n customers in system when server is working |
| $P_2(n)$ | The probability of being n customers in the system when server is found to be broken down. |
| $P_3(n)$ | The probability of being n customers in the system when server is under repair. |

## 2. Governing Equations

Steady state equations governing the model are given as follows:

$$\lambda_0 P_0(0) = \mu P_0(1) \tag{1}$$

$$\lambda_0 P_0(n) = \lambda_0 P_0(n-1), \qquad 1 \le n \le N-1 \tag{2}$$

$$\lambda_0 P_0(n) = \lambda_0 P_0(n-1), \qquad n \ge N \tag{3}$$

$$(\lambda_1 + \mu + \alpha) P_1(1) = \mu P_1(2) + \beta P_3(1) \tag{4}$$

$$(\lambda_1 + \mu + \alpha) P_1(n) = \mu P_1(n+1) + \beta P_3(n) + \lambda_1 P_1(n-1), \quad 2 \le n \le N-1 \tag{5}$$

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$$(\lambda_1+\mu+\alpha)P_1(n)=\mu P_1(n+1)+\beta P_3(n)+\lambda_1 P_1(n-1), \qquad n\ge N \tag{6}$$

$$\lambda_2 P_2(1)=\alpha P_1(1) \tag{7}$$

$$\lambda_2 P_2(n)=\alpha P_1(n)+\lambda_2 P_2(n-1), \qquad n\ge 2 \tag{8}$$

$$(\lambda_3+\beta)P_3(1)=P_2(1) \tag{9}$$

$$(\lambda_3+\beta)P_3(n)=P_2(n)+\lambda_3 P_3(n-1), \qquad n\ge 2 \tag{10}$$

## 3. Probability Generating Functions

We define the generating function corresponding the probabilities $P_i(n)$, i=0,1,2,3 as follows:

$$H_0(z)=\sum_{n=0}^{\infty} z^n P_0(n)\ ; \quad H_i(z)=\sum_{n=1}^{\infty} z^n P_i(n), \quad i=1,2,3$$

Solving the equations (1)- (10) for $H_0(z)$, $H_1(z)$, $H_2(z)$ and $H_3(z)$, we find

$$H_0(z)=\left[\frac{1-z^n}{1-z}+\frac{\lambda_0 z^n}{\lambda_0+\lambda_0 z}\right]P_0(0) \tag{11}$$

$$H_1(z)=\frac{\lambda_0(\lambda_2 z-\lambda_2)(\lambda_3 z-\lambda_3-\beta)(z\lambda_0+z-\lambda_0 z^2-z^{N+1})P_0(0)}{[(\lambda_1 z^2-(\lambda_1+\mu+\alpha)z+\mu)(\lambda_3 z-\lambda_{3-}\beta)(\lambda_2 z-\lambda_2)](\lambda_0+\lambda_0 z)} \tag{12}$$

$$H_2(z)=\frac{(-\alpha)(\lambda_3 z-\lambda_3-\beta)(z\lambda_0+-\lambda_0 z^2-z^{N+1})\lambda_0 P_0(0)}{[(\lambda_1 z^2-(\lambda_1+\mu+\alpha)z+\mu)(\lambda_3 z-\lambda_3\beta)(\lambda_2 z-\lambda_2)+\alpha\beta z](\lambda_0+\lambda_0 z)} \tag{13}$$

$$H_3(z)=\frac{\lambda_0\alpha(z\lambda_0-\lambda_0 z^2-z^{N+1})P_0(0)}{[(\lambda_1 z^2-(\lambda_1+\mu+\alpha)z+\mu)(\lambda_3 z-\lambda_{3-}\beta)(\lambda_2 z-\lambda_2)+\alpha\beta z](\lambda_0+\lambda_0 z)} \tag{14}$$

Now H(z) which represents the p.g.f. of the total number of customers in the system, is obtained as

$$H(z)=\sum_{i=0}^{3} H_i(z) \tag{15}$$

We evaluate $H_0(1)$, $H_1(1)$, $H_2(1)$ & $H_3(1)$ by applying L'Hospital's rule in equations (11- 14) and obtain $H_0(1)=[N+\lambda_0]P_0(0)$ (16)

$$H_1(1)=\frac{\beta\lambda_0(\lambda_0+N)P_0(0)}{(\mu\beta-\lambda_1\beta-\alpha\lambda_3-\alpha\beta\lambda_2)} \tag{17}$$

$$H_2(1)=\frac{\alpha\beta\lambda_0(\lambda_0+N)P_0(0)}{(\mu\beta-\lambda_1\beta-\alpha\lambda_3-\alpha\beta\lambda_2)} \tag{18}$$

$$H_3(1) = \frac{\lambda_0\alpha(\lambda_0 + N)P_0(0)}{(\mu\beta - \lambda_1\beta - \alpha\lambda_3 - \alpha\beta\lambda_2)} \tag{19}$$

To establish $P_0(0)$, we use the normalizing condition and obtain

$$P_0(0) = \left[N + \lambda_0 + \frac{\lambda_0(\lambda_0 + N)(\beta + \alpha\beta + \alpha)}{(\mu\beta - \lambda_1\beta - \alpha\lambda_3 - \alpha\beta\lambda_2)}\right]^{-1} \tag{20}$$

## 4. The Expected Queue Length

To determine $E(N_i)$, i=0,1,2,3 we use the probability generating function $H_i(Z)$ given in equations (11)-(14) and obtain

$$E(N_0) = \left\{\frac{N(N-1)}{2} + \lambda_0(\lambda_0 + N)\right\}P_0(0) \tag{21}$$

$$E(N_1) = \left[\frac{\lambda_0\beta(2\lambda_0 + N(N+1)) - 2(\lambda_0 + N)}{2(\mu\beta - \lambda_1\beta - \alpha\lambda_3 - \alpha\beta\lambda_2)} + \frac{\beta\lambda_0^2(\lambda_0 + N)}{(\mu\beta - \lambda_1\beta - \alpha\lambda_3 - \alpha\beta\lambda_2)} - \frac{\lambda_0\beta(\lambda_0 + N)\{(\lambda_1 - \mu - \alpha)(\lambda_3 + \beta\lambda_2) + \alpha\lambda_2\lambda_3 - \lambda_1\beta\}}{(\mu\beta - \lambda_1\beta - \alpha\lambda_3 - \alpha\beta\lambda_2)^2}\right]P_0(0) \tag{22}$$

$$E(N_2) = \left[\frac{\alpha\beta\lambda_0(2\lambda_0 + N(N+1)) - 2\alpha\lambda_0\lambda_3(\lambda_0 + N)}{2(\mu\beta - \lambda_1\beta - \alpha\lambda_3 - \alpha\beta\lambda_2)} + \frac{\alpha\beta\lambda_0^2(\lambda_0 + N)}{(\mu\beta - \lambda_1\beta - \alpha\lambda_3 - \alpha\beta\lambda_2)} - \frac{\lambda_0\alpha\beta(\lambda_0 + N)\{(\lambda_1 - \mu - \alpha)(\lambda_3 + \beta\lambda_2) + \alpha\lambda_2\lambda_3 - \lambda_1\beta\}}{(\mu\beta - \lambda_1\beta - \alpha\lambda_3 - \alpha\beta\lambda_2)^2}\right]P_0(0) \tag{23}$$

$$E(N_3) = \left[\frac{\alpha\lambda_0(2\lambda_0 + N(N+1))}{2(\mu\beta - \lambda_1\beta - \alpha\lambda_3 - \alpha\beta\lambda_2)} + \frac{\alpha\lambda_0^2(\lambda_0 + N)}{(\mu\beta - \lambda_1\beta - \alpha\lambda_3 - \alpha\beta\lambda_2)} - \frac{\lambda_0\alpha(\lambda_0 + N)\{(\lambda_1 - \mu - \alpha)(\lambda_3 + \beta\lambda_2) + \alpha\lambda_2\lambda_3 - \lambda_1\beta\}}{(\mu\beta - \lambda_1\beta - \alpha\lambda_3 - \alpha\beta\lambda_2)^2}\right]P_0(0) \tag{24}$$

The expected number of customers is given by

$$E(N) = \sum_{i=0}^{3} E(N_i)$$

$$
\begin{aligned}
&=\left[\frac{N(N-1)}{2}+\lambda_0(\lambda_0+N)\right. \\
&+\frac{\lambda_0(2\lambda_0+N(N+1))(\beta+\alpha\beta+\alpha)-2\lambda_0(\lambda_0+N)(\lambda_3+\lambda_2\beta+\lambda_3\alpha)}{2(\mu\beta-\lambda_1\beta-\alpha\lambda_3-\alpha\beta\lambda_2)} \\
&+\frac{\lambda_0^2(\lambda_0+N)(\alpha\beta+\alpha+\beta)}{(\mu\beta-\lambda_1\beta-\alpha\lambda_3-\alpha\beta\lambda_2)} \\
&\left.-\frac{\lambda_0(\lambda_0+N)(\alpha\beta+\alpha+\beta)\{(\lambda_1-\mu-\alpha)(\lambda_3+\beta\lambda_2)+\alpha\lambda_2\lambda_3-\lambda_1\beta\}}{(\mu\beta-\lambda_1\beta-\alpha\lambda_3-\alpha\beta\lambda_2)^2}\right]P_0(0)
\end{aligned}
\tag{25}
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