NUMERICAL STUDY OF HEAT AND MASS TRANSFER IN A TRANSIENT THIRD GRADE FLUID FLOW IN THE PRESENCE OF HEAT SOURCE, CHEMICAL REACTION AND THERMAL RADIATION

Olajuwon, B.I. [1] **Baoku, I.G.** [2]

[1]Department of Mathematics, Federal University of Agriculture, P.M.B. 2240, Abeokuta, Ogun State, Nigeria.
[2]Department of Physical Sciences, Bells University of Technology, P.M.B. 1015, Ota, Ogun State, Nigeria.

e-mail: baokuig@bellsuniversity.edu.ng, olajuwonbi@funaab.edu.ng

Abstract: *This paper investigates the combined influence of internal heat generation, suction, thermal radiation and chemical reaction of n^{th} order on an unsteady free convective heat and mass transfer flow of a chemically reactive third grade fluid over an infinite vertical porous plate with an oscillating temperature applied to the plate. The dimensionless partial differential equations governing this investigation are solved numerically by employing Crank-Nicolson finite difference scheme with modified Newton's iterative technique. Graphical results obtained for velocity, temperature and species concentration profiles are presented and discussed. The results show that the profiles are appreciably influenced by the suction, heat generation/absorption, thermal radiation, order of chemical reaction and viscoelastic parameters. The findings depict that the suction parameter has the influence of reducing velocity, temperature and species concentration fields. The fluid velocity increases as the value of the second grade viscoelastic parameter increases. Also, the third grade viscoelastic parameter has a significant effect on the velocity profiles. Finally, an increase in the thermal radiation parameter decreases the velocity distribution of flow field but it increases the temperature distribution*

Keywords: *Heat source, suction, Chemical Reaction, Heat andMass Transfer, Thermal Radiation.*

1. Introduction

The study of natural free convection flow with heat and mass transfer under the influence of chemical reaction and heat source has practical applications in many scientific and industrial processes such as drying, evaporation at the surface of a water body, energy transfer in a wet tower flow in a desert cooler. Natural convection flow occurs frequently in nature. It occurs due to temperature differences, as well as due to concentration differences or the combination of these two. Many practical diffusive operations involve the molecular diffusion of a species in the presence of chemical reaction within or at the boundary. These are homogeneous and heterogeneous reactions in which reactions occur uniformly throughout a given phase and in a restricted region or within the boundary of a phase respectively. During a chemical reaction between two species, heat is also generated. Hence, the growing need for chemical reaction and heat generation/absorption in industries and engineering requires the study of heat and mass transfer in the presence of different physical geometries and thermophysical conditions.

Vajravelu [1] studied the exact solution for hydrodynamic boundary layer flow and heat transfer over a continuous, moving horizontal plate surface with uniform suction and internal heat generation/ absorption. Das et al. [2] investigated the effect of a homogeneous first-order chemical reaction on the flow past an impulsively started infinite vertical plate with constant heat flux and mass transfer. Muthucumaraswamy [3] examined a first-order chemical reaction on the flow past an impulsively started vertical plate with uniform heat and mass flux. He [4] further considered the effects of chemical reaction on a moving isothermal vertical infinitely long surface with suction. Chamka [5] presented an analytical

Date of acceptance : 29.04.2014

solution for heat and mass transfer by laminar flow of a Newtonian, viscous electrically conducting fluid with heat generation/absorption. Kandasamy et al. [6] reported the problem of nonlinear MHD flow with heat and mass transfer characteristics of an incompressible, viscous, electrically conducting Boussinesq fluid on a vertical stretching surface with chemical reaction and thermal stratification effects.

Muthucumaraswawy and Ganesan [7] investigated the effects of a chemical reaction on an unsteady flow past an impulsively started semi-infinite vertical plate which is subjected to uniform heat flux. They [8] also investigated the effect of chemical reaction and injection as well as flow characteristics in an unsteady upward motion of an isothermal plate. Pal et al. [9] reported the perturbation analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. Shateyi and Motsa [10] studied the unsteady magnetohydrodynamic convective heat and mass transfer past an infinite vertical plate in a porous medium with thermal radiation, heat generation/absorption and chemical reaction.

Kesavaiah et al. [11] considered the effects of chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction. Kandasamy et al. [12] obtained an approximate numerical solution for a steady laminar boundary layer flow over a wall of the wedge with suction or injection in the presence of species concentration and mass diffusion and their results showed that the flow is influenced appreciably by the chemical reaction, heat source and suction/injection at the wall of the wedge. Das et al. [13] estimated the mass transfer effects on an unsteady flow past an accelerated vertical porous plate with suction employing finite difference analysis. Das et al. [14] also investigated the hydromagnetic convective flow past a vertical porous plate through a porous medium with suction and heat source. Makinde et al. [15] reported the unsteady free convective flow with suction on an accelerating porous plate.

The aforementioned studies focused on heat and mass transfer of viscous Newtonian fluid. However, non-Newtonian fluids are prevalent in many transport processes that exist in nature and industrial processings. The problem of convective heat and mass transfer in a hydromagnetic flow of a second grade fluid in the presence of thermal radiation and thermal diffusion was investigated by Olajuwon [16]. He used numerical approach of Runge-Kutta shooting method to solve the problem and presented the velocity, temperature and concentration fields for different values of parameters entering into the problem. Ellahi et al. [17] presented the effects of heat and mass transfer with slip on Couette and generalized Couette flow in a homogeneous and thermodynamically compatible third grade non-Newtonian viscous fluid. They derived exact solutions of velocity and temperature in Couette flow problem and the nonlinear analysis for generalized Couette flow problem was performed by using spectral homotopy analysis method (SHAM).

Sonth et al. [18] examined the heat and mass transfer in a visco-elastic fluid flow over an accelerating surface in the presence of heat source/sink and viscous dissipation. Similarly, Seddeek [19] investigated the problem of heat and mass transfer with heat source or sink and magnetic field on a stretching sheet in a viscoelastic fluid flow through a porous medium. Hayat et al. [20] studied the unsteady flow over a stretching surface in the presence of chemical reaction with heat and mass transfer of a third grade fluid. Simultaneous effects of heat and mass transfer over a stretching surface on time-dependent flow have been analyzed by Hayat et al. [21]. Baoku et al. [22] investigated the magnetohydrodynamic partial slip flow, heat and mass transfer of a thermodynamically compatible, viscoelastic third grade fluid over an insulated porous plate embedded in a porous medium. They employed a numerical technique of midpoint scheme with Richardson's extrapolation to solve the emerging higher order, nonlinear coupled differential equations.

Heat source and chemical reaction effects are crucial in controlling the heat and mass transfer especially for non-Newtonian fluids. Hence, the present paper attempts to investigate the influence of heat generation and chemical reaction of order n in a viscoelastic third grade fluid flow, heat and mass transfer in the presence of thermal radiation and suction. There has been no work in scientific literature that has earlier considered the problem of combined effects of chemical reaction and heat source/sink over an infinite vertical porous plate set in motion with an oscillating temperature in an unsteady free convective flow, heat and mass transfer of an incompressible and chemically reactive third grade fluid. The coupled nonlinear partial differential equations governing the flow, heat and mass transfer have been solved numerically by discretizing the equations using an unconditionally stable Crank-Nicolson finite difference scheme. The effects of various physical parameters on velocity, temperature and species concentration profiles are studied.

2. Problem Formulation

Consider an unsteady free convective flow, heat and mass transfer of an incompressible and chemically reactive third grade fluid past an infinite vertical porous plate. The x'- axis is taken along the plate vertically upwards and y'-axis is normal to it. The plate is set in motion in its own plane with a velocity $U(t')$. An oscillating temperature is assumed to be applied on the plate in the presence of thermal radiation and heat source/sink. The plate surface temperature is T'_w and the temperature of the ambient medium is T'_∞. The concentration of diffusing species is very small in comparison to the other chemical species. The flow contains a species A slightly soluble in the fluid B, the concentration at the plate surface is C'_w and the solubility of A in B away from the plate, C'_∞ is infinitesimally small. The chemical reactions are taking place in the flow regime and there exists a homogeneous n^{th} order chemical reaction within the fluid and species concentration of A in B. All other physical properties are assumed to be constant. Since the plate is infinitely long, the physical variables are functions of y' and t' only. Hence, from the continuity equation, the velocity field is obtained as:

$$u' = u'(y', t'),\ v' = -V_0 \qquad (1)$$

where u' and v' are the velocities of the fluid along x' and y' axes respectively and $V_0 > 0$ indicates suction velocity.

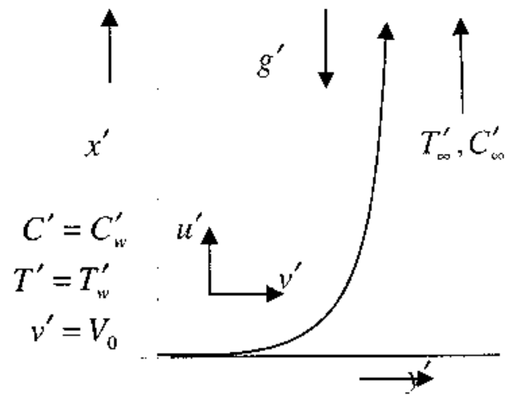

Figure 1: Physical sketch and coordinate system of the problem.

2.1 Flow Analysis

The constitutive equation of an incompressible third grade fluid as given by Coleman and Noll [23] is:

$$\begin{aligned}\tau' = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 \\ + \beta_2 (A_2 A_1 + A_1 A_2) + \beta_3 (trA_2) A_1\end{aligned} \qquad (2)$$

$\mu, \alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3$ being material constants, τ' the stress-tensor, p the scalar pressure, I the identity tensor and A_n represents the kinematical tensors defined by, $A_0 = I$, $A_1 = \nabla u' + (\nabla u')^T$,

$$A_{n+1} = \left(\frac{\partial}{\partial t} + u' \cdot \nabla\right) A_n + \nabla u' \cdot A_n + (\nabla u' \cdot A_n)^T,$$

$n = 1, 2$.

where u' is the velocity and t' is the time.

In a third grade fluid, the expression for the Cauchy stress tensor satisfying the thermodynamic constraints (Fosdick and Rajagopal [24]): $\mu \geq 0,\ \alpha_1 \geq 0$, $|\alpha_1 + \alpha_2| \leq \sqrt{24\mu\beta_3}$, $\beta_1 = \beta_2 = 0, \beta_3 \geq 0$.

is given by Truesdell and Noll [25]:

$$\tau' = -pI + \mu_1 A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_3 \left(trA_1^2\right) A_1 \quad (3)$$

The stress components (2) by virtue of equation (1) are:

$$\tau_{x'x'} = -p + \alpha_2 \left(\frac{\partial u'}{\partial y'}\right)^2 + 2\beta_2 \frac{\partial u'}{\partial y'} \frac{\partial^2 u'}{\partial y' \partial t'} \quad (4)$$

$$\tau_{y'y'} = -p + (2\alpha_1 + \alpha_2)\left(\frac{\partial u'}{\partial y'}\right)^2 + (6\beta_1 + 2\beta_2)\frac{\partial u'}{\partial y'} \frac{\partial^2 u'}{\partial y' \partial t'} \quad (5)$$

$$\tau_{z'z'} = -p, \quad (6)$$

$$\tau_{x'y'} = \mu \frac{\partial u'}{\partial y'} - \alpha_1 V_0 \frac{\partial^2 u'}{\partial y'^2} + \alpha_1 \frac{\partial^2 u'}{\partial y' \partial t'} + 2(\beta_2 + \beta_3)\left(\frac{\partial u'}{\partial y'}\right)^3 + \beta_1 \left(\frac{\partial^3 u'}{\partial y' \partial t'^2}\right) \quad (7)$$

$$\tau_{x'z'} = \tau_{z'y'} = 0 \quad (8)$$

where $\tau_{x'y'} = \tau_{y'x'}$, $\tau_{x'z'} = \tau_{z'x'}$, $\tau_{y'z'} = \tau_{z'y'}$

Inserting the stress components, using equation (3) and velocity field given by (1) in the equation of motion:

$$\rho \frac{Dv_i}{Dt} = -\tau'_{,i} + \rho X_i + \tau'_{ij,j} \quad (9)$$

where $\frac{D}{Dt}$ denotes the material derivative and X_i is the external force per unit mass in i^{th} direction while neglecting body force and introducing the similarity transformations used for scaling respective heat transport and species concentration equations in heat and mass transfer analysis, the equation of motion (9) becomes:

$$\frac{\partial u'}{\partial t'} = V_0 \frac{\partial u'}{\partial y'} + \frac{\mu}{\rho}\frac{\partial^2 u'}{\partial y'^2} + \frac{\alpha_1}{\rho}\left(\frac{\partial^3 u'}{\partial y'^2 \partial t'}\right) - \frac{\alpha_1 V_0}{\rho}\frac{\partial^3 u'}{\partial y'^3} + \frac{6\beta_3}{\rho}\left(\frac{\partial u'}{\partial y'}\right)^2 \frac{\partial^2 u'}{\partial y'^2} + g'\beta_T (T'_w - T'_\infty)\theta + g'\beta_C (C'_w - C'_\infty)\phi \quad (10)$$

2.2 Heat Transfer Analysis

The heat transport equation for the problem under consideration without viscous dissipation and work done to deformation is:

$$\rho C_p \frac{DT'}{Dt'} = K \nabla^2 T' - \nabla q_r + Q(T' - T'_\infty) \quad (11)$$

In the equation of energy (11), the term representing viscous and joule dissipation are assumed to be neglected as they are really very small in slow motion free convection flows. So, Q is the volumetric rate of internal heat generation/absorption (heat source/sink) and q_r is the radiative heat flux.

Making use of the Rosseland approximation for radiation of the optically thick layer (see Pop et al. [26]) to obtain:

$$q_r = -\frac{4\sigma^*}{3\xi}\frac{\partial T^4}{\partial y} \quad (12)$$

where σ^* is the Stefan-Boltzmann constant and ξ is the mean absorption coefficient. One can express the term T^4 as a linear function of temperature. It is recognized by expanding in a

Taylor series about T_∞ and neglecting higher terms, one can write:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \qquad (13)$$

Hence, the governing equation of temperature flow field, with equations (12) and (13), is obtained as:

$$\frac{\partial T'}{\partial t'} - V_0 \frac{\partial T'}{\partial y'} = \frac{K}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma^* T_\infty^3}{3\xi} \frac{1}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{Q}{\rho C_p}(T' - T'_\infty) \qquad (14)$$

2.3 Mass Transfer Analysis

The species concentration equation for the thermodynamically compatible third grade fluid with nth order homogeneous chemical reactions:

$$\frac{DC'}{Dt} = D_{AB}\nabla^2 C' - k_1 (C' - C'_\infty)^n \qquad (15)$$

Using (1), the governing equation of species concentration flow field is:

$$\frac{\partial C'}{\partial t'} - V_0 \frac{\partial C'}{\partial y'} = D_{AB}\frac{\partial^2 C'}{\partial y'^2} - k_1 (C' - C'_\infty)^n \qquad (16)$$

where D_{AB} , n and k_1 are mass diffusivity, order and rate of chemical reaction respectively.

The initial and boundary conditions are:

$$t' \le 0 : y = 0, u' = 0, T' = 0, \qquad (17)$$

$$t' \succ 0 : u' = U(t') = \frac{U^{2m+1}}{v^m}\ell^{a't'}t'^m, \quad when\ y' = 0 \qquad (18)$$

$$T' = T'_\infty + (T'_w - T'_\infty)\cos b't', \quad when\ \ y' = 0 \qquad (19)$$

$$C' = C'_\infty + (C'_w - C'_\infty) = 1, \quad when\ \ y' = 0 \qquad (20)$$

$$u' = 0, \frac{\partial u'}{\partial y'} = 0, when\ y \to \infty \qquad (21)$$

$$T' = T'_\infty, when\ \ y' \to \infty \qquad (22)$$

$$C' = C'_\infty, when\ \ y' \to \infty \qquad (23)$$

Introducing the following dimensionless variables:

$$u = \frac{u'}{U}, y = \frac{y'U}{v},\ a = \frac{a'v}{U^2},\ t = \frac{t'U^2}{v} \qquad (24)$$

Using equation (24) alongside the similarity transformations used for scaling respective heat transport and species concentration equations in heat and mass transfer analysis, equations (10), (14) and (16) become:

$$\frac{\partial u}{\partial t} - \omega\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial y^2 \partial t} - \omega\alpha\frac{\partial^3 u}{\partial y^3} + \beta\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} + Gr\,\theta + Gc\phi \qquad (25)$$

$$\frac{\partial \theta}{\partial t} = \omega\frac{\partial \theta}{\partial y} + \left(\frac{1}{\Pr} + \frac{4}{3}R_d\right)\frac{\partial^2 \theta}{\partial y^2} + S\theta \qquad (26)$$

$$\frac{\partial \phi}{\partial t} = \omega\frac{\partial \phi}{\partial y} + \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} - \gamma\phi^n \qquad (27)$$

where $\omega = \dfrac{V_0}{U}$, $\alpha = \dfrac{\alpha_1 U^2}{\rho v^2}$, $\beta = \dfrac{6\beta_3 U^4}{\rho v^3}$, $Gr = \dfrac{\beta g' v (T'_w - T'_\infty)}{U^3}$, $\Pr = \dfrac{v\rho C_p}{K}$, $Gr = \dfrac{\beta g' v (C'_w - C'_\infty)}{U^3}$, $R_d = \dfrac{4\sigma T_\infty^3}{\xi\rho C_p v}$, $S = \dfrac{Qv}{\rho C_p U^2}$, $Sc = \dfrac{D}{v}$, $\gamma = \dfrac{k_1 v}{U^2}$

3. Method of Solution

The governing nonlinear coupled partial differential equations (22) - (24) with the initial and boundary conditions (25) – (27) are solved using Crank-Nicolson finite difference scheme which has been discussed by Conte and De Boor [27], Jain [28], Ganesan and Palani [29] and Baoku et al. [30]. One therefore discretizes the governing equations based on the transient state conditions. The numerical method of Crank-Nicolson type does not restrict the value of r to be chosen. The finite difference equations corresponding to these governing equations are given as:

$$
\begin{aligned}
u_{i,j+1}-u_{i,j} &= \left(\frac{\omega h r}{4}+\frac{r}{2}+\frac{\alpha}{h^2}-\frac{\omega\alpha r}{h}\right)u_{i+1,j+1} \\
&+\left(\frac{r}{2}-\frac{\omega h r}{4}+\frac{\alpha}{h^2}-\frac{\omega\alpha r}{h}\right)u_{i-1,j+1} \\
&+\left(\frac{\omega h r}{4}+\frac{r}{2}\right)u_{i+1,j}+\left(\frac{r}{2}-\frac{\omega h r}{4}+\frac{\alpha}{h^2}\right)u_{i-1,j} \\
&-2\left(\frac{r}{2}+\frac{\alpha}{h^2}\right)u_{i+1,j}-2\left(\frac{r}{2}+\frac{\alpha}{h^2}\right)u_{i,j} \\
&+\frac{\alpha}{h^2}u_{i+1,j}-\frac{\omega\alpha r}{2h}\left[\begin{array}{l}\left(-u_{i-2,j+1}+u_{i+2,j+1}\right) \\ -\left(-u_{i-2,j}+2u_{i-1,j}-2u_{i+1,j}+u_{i+2,j}\right)\end{array}\right] \\
&+\frac{\beta r}{3h^2}\left[\begin{array}{l}\left(u_{i+1,j+1}\right)^2+\left(u_{i-1,j-1}\right)^2+\left(u_{i+1,j}\right)^2+\left(u_{i-1,j}\right)^2- \\ 2u_{i+1,j+1}u_{i-1,j-1}+2u_{i+1,j+1}u_{i+1,j}-2u_{i+1,j+1}u_{i-1,j} \\ -2u_{i+1,j}u_{i-1,j-1}+2u_{i-1,j}u_{i-1,j-1}-2u_{i+1,j}u_{i-1,j}\end{array}\right] \\
&\left(u_{i+1,j+1}-2u_{i,j+1}+u_{i-1,j+1}+u_{i+1,j}-2u_{i,j}+u_{i-1,j}\right) \\
&+2rh^2\left[G\left(\theta_{i,j+1}-\theta_{i,j}\right)+G\left(\phi_{i,j+1}+\phi_{i,j}\right)\right] \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
\theta_{i,j+1} &= \left(\frac{3\Pr\omega h r+6r+8R_d}{6\Pr}\right)\theta_{i+1,j+1} \\
&-\left(\frac{3\Pr\omega h r-6r-8rR_d}{6\Pr}\right)\theta_{i-1,j+1} \\
&-\left(\frac{12r-16rR_d-3\Pr S h^2 r}{6\Pr}\right)\theta_{i,j+1} \qquad (29) \\
&+\left(\frac{Sh^2 r+2}{2}\right)\theta_{i,j}
\end{aligned}
$$

$$
\begin{aligned}
\phi_{i,j+1} &= \left(\frac{\omega Sc\, h\, r+2r}{2Sc}\right)\phi_{i+1,j+1} \\
&+\left(\frac{2r-\omega Sc\, h\, r}{2Sc}\right)\phi_{i-1,j+1} \\
&-\left(\frac{4r+2\gamma Sc h^2 r}{2Sc}\right)\phi_{i,j+1} \qquad (30) \\
&+\phi_{i,j}-\frac{\gamma h^2 r}{2}\left(\phi_{i,j}\right)^n
\end{aligned}
$$

where i dessignates the grip point along y-direction, j along t-direction and $r=\Delta t/h^2$. Hence, the equations of motion, heat transport and species concentration are reduced to system of algebraic nonlinear coupled-equations. The mesh size h is 0.05 with time step $t=0.1$. The values of $u(y,t)$, $\theta(y,t)$ and $\phi(y,t)$ are known at all grip points when $t=0$ from the initial conditions. Modified Newton's iterative technique is employed to solve the system of nonlinear algebraic equations. Computations are carried out by moving along y-direction. After computing values corresponding to each i at a time level, the values at the next time level are determined in similar manner.

The implicit nature of Crank-Nicolson method is unconditionally stable and has local truncation error $O\left[(\Delta t)^2, h^2\right]$ which tends to zero as Δt and h^2 tend to zero. There is no drawback of

conditionally stability from one level to the next. The implicit method gives stable solutions and requires iterative procedure which we did at each step forward in time because this problem is an initial-boundary value problem with a finite number of spatial grip points. Though, the corresponding difference equations do not automatically guarantee the convergence of the mesh $h \to 0$. To achieve maximum numerical efficiency, we used the tridiagonal procedure to solve the two point conditions for (20) and four point conditions for (19). Transforming the above procedure into Maple code as described by Heck [31], the convergence of the process was quite satisfactory and the numerical stability of the method was guaranteed by the implicit nature of the scheme. Hence, the scheme is consistent; stability and consistency ensure convergence.

4. Discussion of the Results

The study examines the reactive flow fields when a vertically upward plate suddenly starts moving with a velocity U in its own plane and temperature field assumed to be oscillating is applied to the plate in the presence of suction, chemical reaction and thermal radiation. The governing equations of the flow, temperature and species concentration fields are solved using Crank-Nicolson implicit finite difference scheme with modified Newton method and approximate solutions are obtained for the velocity, tempearture and species concentration profiles. The effects of the pertinent parameters on the flow, tempearture and species concentration fields are analyzed and discussed with the help of velocity profiles (Figures 2 - 6), temperature profiles (Figures 7 - 9) and species concentration profiles (Figures 10 - 11).

4.1 Velocity Profiles

The effects of various parameters on the velocity field are investigated through simulations using the method described above and results are produced as graphs for $n = 0.8$, i.e. when the plate starts moving with variable acceleration. Figure 2 analyzes the influence of suction parameter ω. It is observed that an increase in the suction parameter ω increases the fluid velocity at any point of the fluid. Figure 3 depicts the effect of viscoelastic parameter α on the velocity field. It is observed that as the viscoelastic parameter α increases, the velocity field increases. It is important to note from Figure 4 that β has a decreasing effect on the velocity field thereby displaying the shear thickening property of a third grade fluid. Also, Figure 5 shows that an increase in S decreases the fluid velocity for the variable accelerations. Figure 6 analyzes the influence of the thermal radiation parameter Rd on the velocity profile. It is observed that an increase in Rd decreases the fluid velocity at any point of the fluid.

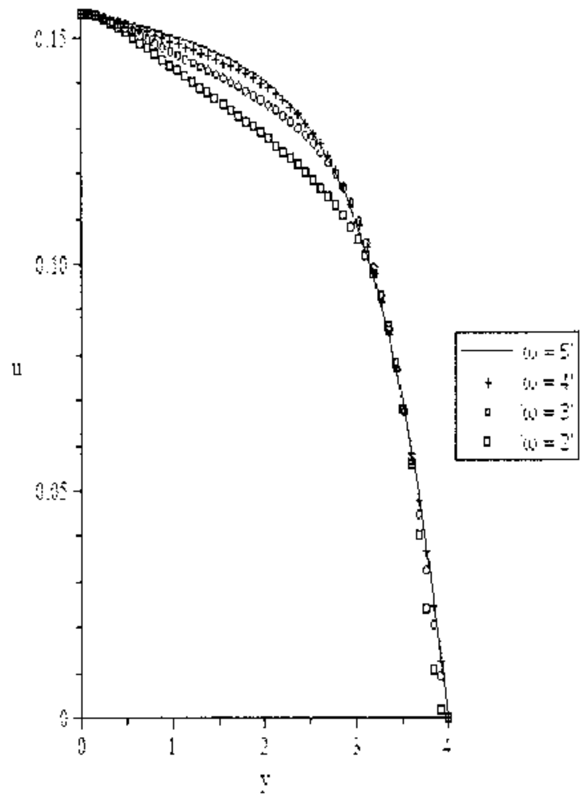

Figure 2: Velocity profiles for values of ω when $\alpha = 2.0$, $\beta = 5.0$, $Gc = 10$, $Gr = 10$.

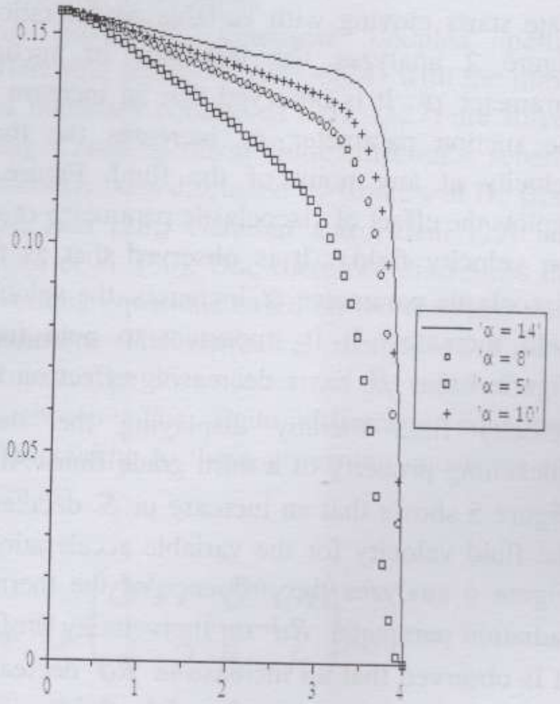

Figure 3: Velocity profiles for values of α when $\omega = 3.0$, $\beta = 5.0$, $Gc = 10$, $Gr = 10$.

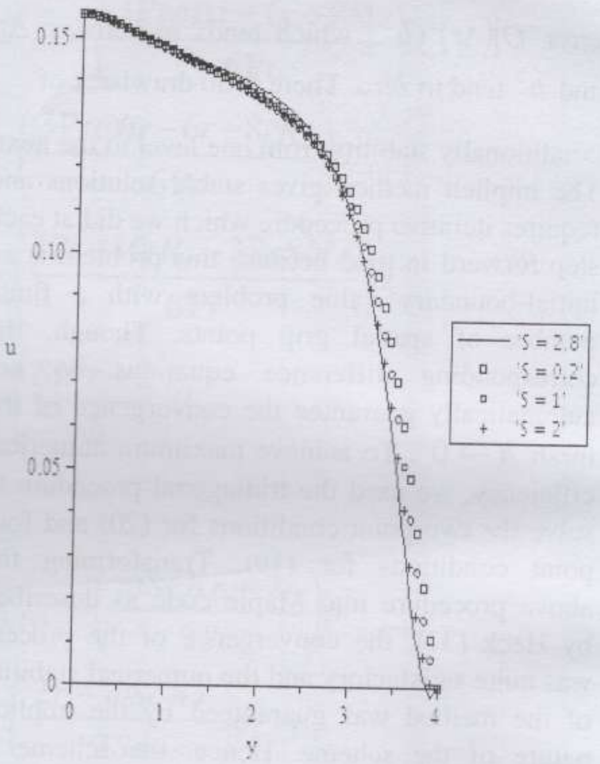

Figure 5: Velocity profiles for values S when $\alpha = 2.0$, $\beta = 5.0$, $Gc = 10$, $Gr = 10$.

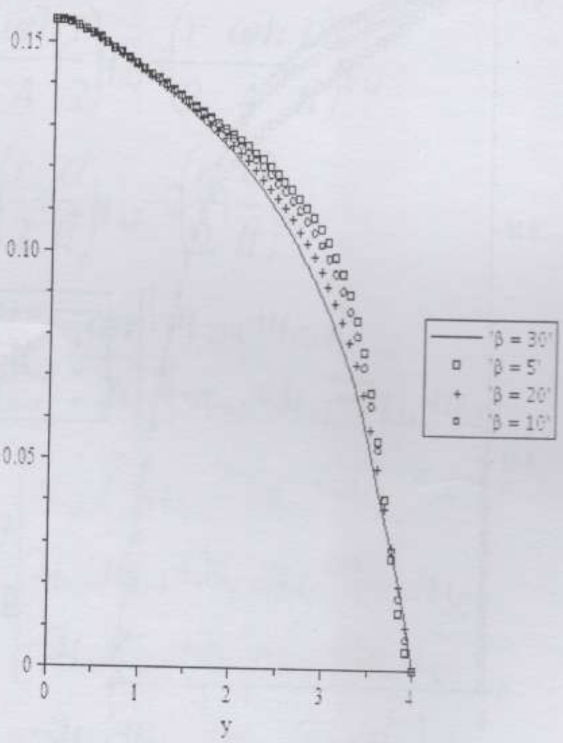

Figure 4: Velocity profiles for values of β when $\alpha = 2.0$, $\omega = 2.0$, $Gc = 10$, $Gr = 10$.

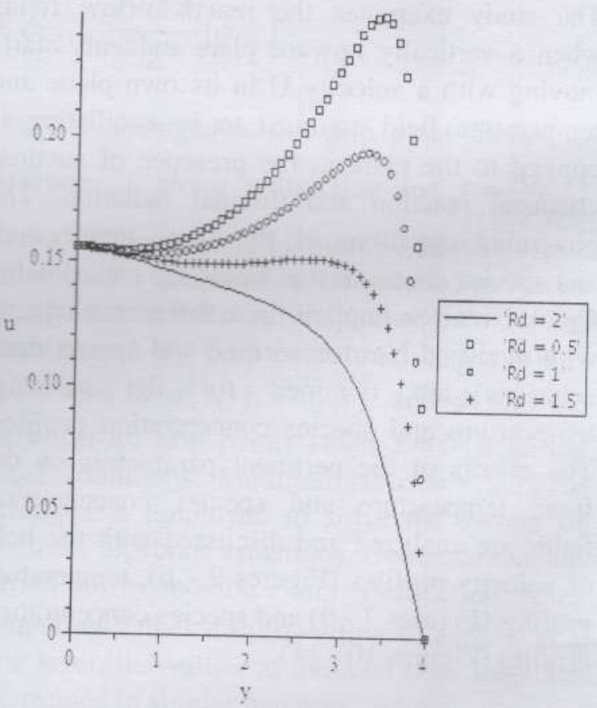

Figure 6: Velocity profiles for values of Rd when $\alpha = 2.0$, $\beta = 5.0$, $Gc = 10$, $Gr = 10$.

4.2 Temperature Profiles

The temperature of the flow field suffers a substantial change with the variation of the flow parameters such as suction parameter ω, heat generation/absorption parameter S and thermal radiation parameter Rd. It is pertinent to note that the effects of these thermophysical parameters on the temperature field remain constant within the specified range in the interval before satisfying the boundary conditions at the free stream. These variations are shown in Figures 7 – 9. Figure 7 depicts the influence of suction parameter ω on the temperature profile. A growing ω is found to decrease the temperature of the flow field at all points in the domain. Figure 8 analyzes the influence of S on the temperature field. It is observed that an increase in the S increases the fluid velocity at any point of the fluid. Lastly, it is evident from Figure 9 that Rd has an appreciable influence on the temperature profile. The effect of Rd on temperature distribution is noticeable such that a rise in Rd increases the temperature field.

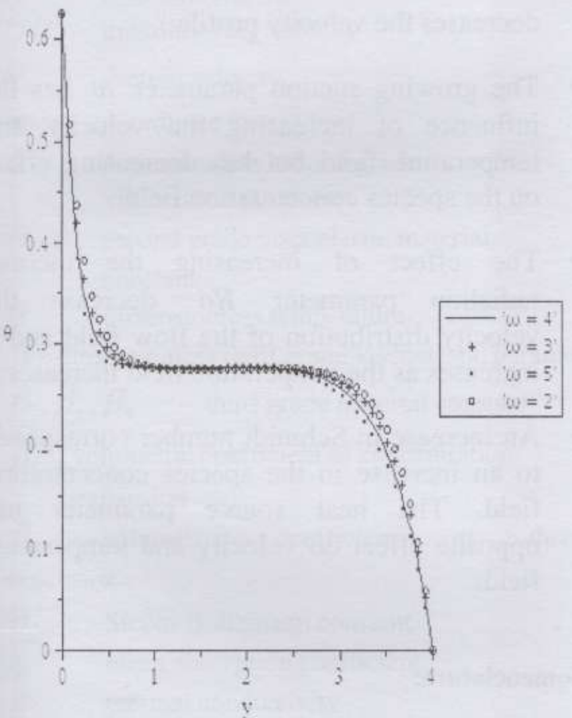

Figure 7: Temperature profiles for values of ω when $Rd = 2.0$, $S = 0.5$, $\Pr = 1$.

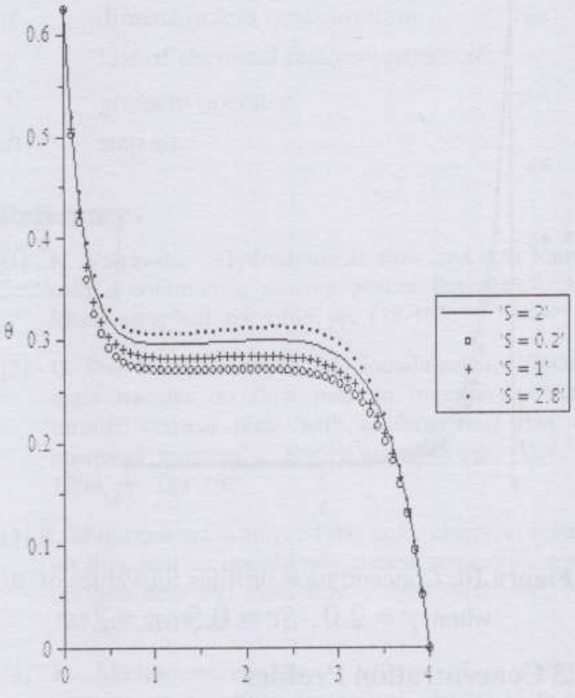

Figure 8: Temperature profiles for values of S when $Rd = 2.0$, $\omega = 3.0$, $\Pr = 1$.

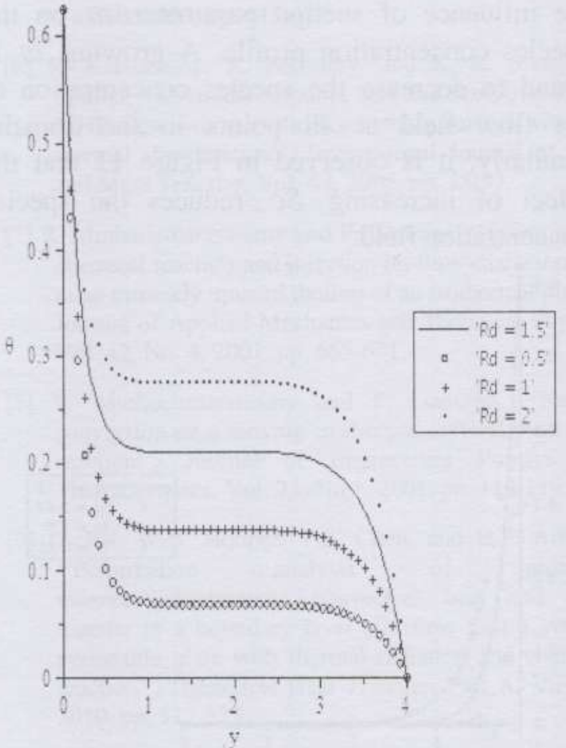

Figure 9: Temperature profiles for values of Rd when $\omega = 3.0$, $S = 0.5$, $\Pr = 1$.

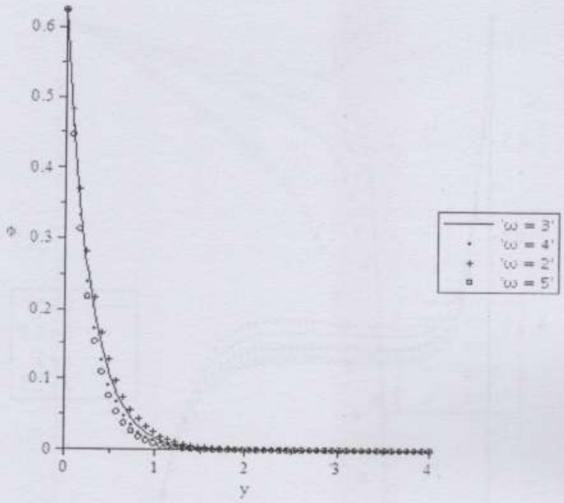

Figure 10: Concentration profiles for values of ω when $\gamma = 2.0$, $Sc = 0.5$, $n = 2$.

4.3 Concentration Profiles

The species concentration of the flow field also suffers a substantial change with the variation of the flow parameters such as suction parameter ω and Schmidt number Sc. These variations are shown in Figures 10 and 11. Figure 10 depicts the influence of suction parameter ω on the species concentration profile. A growing ω is found to decrease the species concentration of the flow field at all points in the domain. Similarly, it is observed in Figure 11 that the effect of increasing Sc reduces the species concentration field.

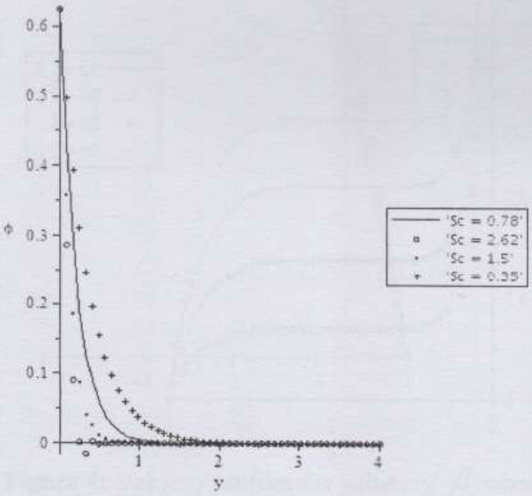

Figure 11: Concentration profiles for values of Sc when $\gamma = 2.0$, $\omega = 3$, $n = 2$.

5. Concluding Remarks

In this study, the influences of thermophysical parameters on the flow, heat and mass transfer of a third grade viscoelastic fluid through a vertical porous plate were investigated. An implicit finite difference numerical scheme of Crank-Nicolson type was employed to discretize the system of coupled partial diffential equations, and modified Newton's method was used to solve the system of algebraic nonlinear equations obtained by the discretization. The above scheme is transformed into the Maple code to simulate the solutions of the problem. This solution procedure is valid for all values of viscoelastic parameters unlike perturbation and power series methods that are only valid for small values of viscoelastic parameters.

Therefore, the following results of physical interest on the velocity, temperature and species concentration distribution of the flow field are summarized below:

- The fluid velocity increases whthe value of the viscoelastic second grade parameter α increases. However, as the viscoelastic third grade parameter β increases, it significantly decreases the velocity profile;
- The growing suction parameter ω has the influence of increasing the velocity and temperature field but has decreasing effect on the species concentration field;
- The effect of increasing the thermal radiation parameter Rd decrease the velocity distribution of the flow field and it increases as the temperature field increases;
- An increase in Schmidt number corresponds to an increase in the species concentration field. The heat source parameter has opposite effect on velocity and temperature fields.

Nomenclature:

A_1, A_2, A_3 Rivlin-Ericksen tensor
C' ambient concentration
C'_w concentration at the wall/plate

C'_∞ free stream concentration
C_p specific heat at constant pressure
D_{AB} mass diffusivity
Gc solutal Grashof number
Gr thermal Grashof number
g' acceleration due to gravity
I identity tensor
i, j grip points along y-direction and t-direction
k_1 rate of chemical reaction
m, a, b constants
n order of chemical reaction
$\Pr$ Prandtl number
p scalar pressure
r convergent term
S heat generation or absorption
Sc Schmidt number
T' ambient temperature
T'_w temperature at the wall/plate
T'_∞ free stream concentration
t', t local time, dimensionless time
u', v' velocity components in x and y directions
u fluid velocity
$U(t)$ initial moving velocity
V_0 suction velocity
x, y coordinate axes
α dimensionless second grade viscoelastic parameter
α_1, α_2 second grade viscoelastic material constant
θ dimensionless temperature
β dimensionless third grade viscoelastic parameter
$\beta_1, \beta_2, \beta_3$ third grade material constant
β_C volumetric coefficient of concentration expansion
β_T volumetric coefficient of thermal expansion
σ^* Stefan-Boltzmann constant
ξ mean absorption coefficient
κ thermal conductivity
μ dynamic viscosity
ν kinematic viscosity
ω suction parameter
ρ fluid density
τ' stress tensor
ϕ dimensionless concentration
γ rate of chemical reaction parameter
∇ gradient operator
h step size

References

[1] K. Vajravelu, ''Hydrodynamic flow and heat transfer over a continuous, moving porous flat plate'', Acta Mechanica, Vol. 64, 1986, pp. 179-185.

[2] U. Das, N. Deka, and V. M. Soundalgekar, ''ffects of mass transfer on flow past an impulsively started infinite vertical plate with uniform heat flux and chemical recation'', Forsch.ingenieurwes, Vol. 60, 1994, pp. 284-287.

[3] R. Muthucumaraswamy, ''First order chemical reaction on flow past an impulsively started vertical plate with uniform heat and mass flux'', Acta Mechanica, Vol. 147, pp. 2001, 45-57.

[4] R. Muthucumaraswamy, ''Effects of a chemical reaction on a moving isothermal vertical plate with suction'', Acta Mechanica, Vol. 155, 2002, pp. 65-70.

[5] A. Chamka, ''MHD flow of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction'', International Communication in Heat Mass Transfer, Vol. 30, 2003, pp. 413-422.

[6] R. Kandasamy, K. Perisamy, and K. K. Sivagnana Prabhu, ''Chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with and thermal stratification'', International Journal of Heat and Mass Transfer, Vol. 48, 2005, pp. 45-57.

[7] R. Muthucumaraswamy and P. Ganesan, ''Effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate'', Journal of Applied Mechanics and Technical Physics, Vol. 42, No. 4, 2001, pp. 665-671.

[8] R. Muthucumarswamy and P. Ganesan, ''Natural convection on a moving isothermal vertical plate with reaction'', Journal of Engineering Physics and Thermophysics, Vol. 73, No.1, 2001, pp. 113-119.

[9] D. Pal, W.B. Hooper, T.S. Chen, and B.F. Armaly, ''Perturbation analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction'', Numerical Heat Transfer, Part A, Vol. 25, 2010, pp. 317-329.

[10] S. Shateyi and S. Motsa, ''Unsteady MHD convective heat and mass transfer past an infinite vertical plate in a porous medium with thermal radiation, heat generation/absorption and chemical reaction'', Advanced Topics in Mass Transfer, 2011, pp. 145-162.

[11] D.C. Kesavaiah, P.V. Satyanarayana, and S. Venkataramana, "Effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction". Int. J. of Appl. Math. And Mech., Vol. 7, No. 1, 2011, pp. 52-69.

[12] R. Kandasamy, K. Periasamy, and P.K.K., Sivagnana, "Effects of chemical reaction, heat and mass transfer along a wedge with heat source and concentration in the presence of suction or injection", International Journal of Heat and Mass Transfer, Vol. 48, No. 7, 2005, pp. 1388-1394.

[13] S.S. Das, S.K. Sahoo, and G.C. Dash, "Numerical solution of mass transfer effects on unsteady flow past an accelerated vertical porous plate with suction", Bulletin of the Malaysian Mathematical Society, Vol. 29, No. 1, 2006, pp. 33-42.

[14] S.S. Das, U.K. Tripathy, and J.K. Das, "Hydromagnetic convective flow past a vertical porous plate through a porous medium with suction and heat source", International Journal of Energy and Environment, Vol. 1, No. 3, 2010, pp. 467-478.

[15] O.D. Makinde, P.O. Olanrewaju, and W.M. Charles, "Unsteady convection with chemical reaction and radiative heat transfer past a flate porous plate moving through, a binary mixture", Afrika Matematika, AM 21, 2011, pp. 1-17.

[16] B.I. Olajuwon, "Convection heat and mass transfer in a hydromagnetic flow of a second grade fluid in the presence of thermal radiation and thermal diffusion", International Communications in Heat and Mass Transfer, Vol. 38, No. 3, 2011, pp. 377-382.

[17] R. Ellahi, E. Shivanian, S. Abbasbandy, S.U. Rahman, and T. Hayat, "Analysis of steady flows in viscous fluid with heat/mass transfers and slip effects", International Journal of Heat and Mass Transfer, Vol. 55, 2012, pp. 6384-6390.

[18] R.M. Sonth, S.K. Khan, M.S. Abel, and K.V. Prasad, "Heat and Mass transfer in a visco–elastic fluid flow over an accelerating surface with heat source/sink and viscous dissipation", Heat mass Transfer, Vol. 38, 2002, pp. 213 – 220.

[19] M. A. Seddek, "Heat and mass transfer on a stretching sheet with magnetic field in a viscoelastic fluid flow through a porous medium with heat source or sink", Comput. Mater. Sci. Vol. 38, 2007, pp. 781-787.

[20] T. Hayat and M. Awais, "Simultaneous effects of heat and mass transfer on time-dependent flow over a stretching surface", International Journal for Numerical Methods in Fluids, Vol. 67, 2011, pp. 1341-1357.

[21] T. Hayat, M. Mustafa, and S. Asghar, "Unsteady flow with heat and mass transfer of a third grade fluid over a stretching surface in the presence of chemical reaction", Nonlinear Anal.: RWA, Vol. 11, 2010, pp. 3186-3197.

[22] I.G. Baoku, B.I. Olajuwon, and A.O. Mustapha, "Heat and mass transfer on a MHD third grade fluid with partial slip flow past an infinite vertical insulated porous plate in a porous medium", International Journal of Heat and Fluid Flow, Vol. 40, 2013, pp. 81 – 88.

[23] B.D. Coleman and W. Noll, "An approximation theorem for functional with application in continuum mechanics", Arch. Rat. Mech. Anal., Vol. 56, 1965, pp. 191 - 256.

[24] R.L. Fosdick and K.R. Rajagopal, "Thermodynamics and stability of fluids of third grade", Proc. R. Soc. London A339, 1980, pp. 351-377.

[25] C. Truesdell and W. Noll, "The Nonlinear Field Theories of Mechanics", 3rd Edn., Springer, Berlin, 2004.

[26] S.R. Pop, T. Grosan, and I. Pop, "Radiation effects on the flow near the stagnation point of a stretching sheet", Tech. Mech., 2004, pp. 100-106.

[27] S.D. Conte and C. De Boor, "Elementary Numerical Analysis an Algorithmic Approach", McGraw-Hill Inc., New York, 1980.

[28] M.K. Jain, "Numerical Solution of Differential equations", Wiley Eastern, New Delhi, 1984.

[29] P. Ganesan and G. Palani, "Natural convection effects on an impulsively started isothermal inclined plate", Acta Mechanica, Vol. 153, 2002, pp. 127 – 132.

[30] I.G. Baoku, C. Israel-Cookey, and B.I. Olajuwon, "Influence of thermal radiation on a transient MHD Couette flow through a porous medium", Journal of Applied Fluid Mechanics, Vol. 5, No. 1, 2012, pp. 81 – 87.

[31] A. Heck, "Introduction to Maple", Third edition. Springer-Verlag, Germany, 2003.

[32] B.I. Olajuwon is a researcher and academic staff at the Department of Mathematics, Federal University of Agriculture, Abeokuta, Ogun State, Nigeria.

[33] I.G. Baoku is an academic staff and researcher at Department of Physical Sciences, Bells University of Technology, Ota, Nigeria.