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Robust SOCP Model for Business Production Mix Problems

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***Abstract*—In this work, a robust optimization model for business production mix problem is introduced to minimize the production cost. We apply conic programming technique to the original production cost minimization problem and formulate the model in the form of second-order cone programming (SOCP) problem which is an emerging branch in current research. For decision taking, optimization plays the crucial role. In robust optimization, we have to optimize a problem where some data are not provided directly but it is declared that the data are contained in a predefined set. Minimizing the production cost for production business is a big interest to maximize the profit. In this research, we propose a robust optimization model for production in business which minimizes the production cost maintaining the quality of product. To formulate the propose model, we use conic programming technique considering the worst-case in the ratio of ingredients in product materials and introduce the model in the form of second-order cone programming (SOCP) problem. So far we studied, this is the first time application of conic programming in minimization of production cost problems. The proposed robust model is presented as an alternative option to minimize the production cost maintaining the quality of product rather than the optimal value of the problem. Of course, the proposed model can minimize the production cost compared to the original problem since it is more robust.**

***Keywords*: Robust optimization, production, constraints, ingredients, second-order cone.**

I. Introduction

Minimizing production or manufacturing cost is a major concern for any business or industry to maximize the profit. Production cost can be defined in many ways and it varies in point to point. Direct labor cost, materials cost, manufacturing overheads, and related others cost commonly define the total production cost. But materials or ingredients cost takes the major portion of the total cost. But business company should maintain the quality of the product to survive in the competitive market for their long term business. They should not compromise the quality of the product and quality of the product also depends on the use of appropriate ratio of ingredients required in many goods or product in production. To minimize the production cost there are lots of model have been introduced in the research history. Moolio and Islam [1] presented a model to apply necessary and sufficient conditions in the objective to minimize the production cost. They considered production function as an output constrain and applied Lagrange multiplier method to a company's cost minimization problem to formulate the model.

Galindo et al. [2] introduced a mathematical formulation to minimize the total production cost including transportation costs and raw material purchase costs. They used integer linear programming (ILP) to formulate their model and the considered problems was solved by using MATLAB programming. The proposed ILP model reduced 7.09% production cost.

Nobil et al. [3] developed a convex non-linear programming problem for economic production quantity. In this work, the authors lengthen a multi product single machine inventory model under production capacity with imperfect production.

To minimize the production cost, Tsyganov [4] integrated a training of pattern recognition model using machine learning technique. The author proposed a mechanism applying traditional machine supervised learning algorithms to maximize own incentives by minimizing the undesirable activity of the personnel which is an active element of the total cost function.

Zhang et al. [5] designed a model to minimize the manufacturing and material cost by evaluating the lamination stacks cost which is usually used in electric traction motors of electric or hybrid vehicles. They used stamping process by following design of the lamination which has an influence on the manufacturing cost and raw material utilization. The presented approach was developed to lamination cost estimation based on an optimum production decision which is determined by proper mold design and selecting suitable stamping machine.

A. Problem Statement

There are a lot of work has been introduced in the history to minimize the production cost using various techniques and algorithms or introducing various parameters related to production in business. And most of them have a lot of contribution/achievement compared to the previous studies, but so far we studied, there is no work in the history which applied conic programming (second-order cone programming) technique to formulate their models for minimization of production cost. In conic programming, semidefinite programming (SDP) or second-order cone programming (SOCP) is a branch which has a wide range of applications in research. Nowadays, a lots of work in engineering [6], communication [7], [8], control [9], finance [10], etc are done using the conic programming techniques. The aim of this study is to design a model that can minimizes the production cost using second-order cone programming technique as a first addition.

B. Contribution

In this paper, a robust optimization model is presented to minimize the production cost maintaining the quality of product. We apply second-order cone technique in production materials and formulate the model in the form of second-order cone programming (SOCP) optimization problem. Although there are a lots of work have been done in the past to minimize the production cost, our contribution is that we apply second-order cone programming technique firstly in the production process history to formulate the model. We consider the worst-case in the ratio of ingredients, the production parameters related to the price which transforms the model into robust form. The presented model can be run using modern optimization tools and shows well prediction to the cost minimization problem.

C. Preliminaries [11]

1) Cone: Any set $S \subseteq \mathbb{R}^n$ is said to be a cone iff for any $\mathbf{x} \in S$ and for any positive scalar α it satisfies that

$$\alpha \mathbf{x} \in S.$$

2) Convex: A set $S \subseteq \mathbb{R}^n$ is said to be convex if for any two points $\mathbf{x}_1, \mathbf{x}_2 \in S$ and for all $0 \leq \lambda \leq 1$ it holds that

$$\lambda \mathbf{x}_1 + (1-\lambda)\mathbf{x}_2 \in S.$$

3) Convex Cone: A cone S is said to be a convex cone iff $\forall \mathbf{x}_1, \mathbf{x}_2 \in S$ and $\forall \alpha, \beta > 0$ it holds that

$$\alpha \mathbf{x}_1 + \beta \mathbf{x}_2 \in S.$$

4) Convex Function: Let S be a convex set in $\mathbb{R}^n$. A function $f(x)$ is said to be a convex function if $\forall\ \mathbf{x}_1, \mathbf{x}_2 \in S$ and $\forall\ 0 \leq \lambda \leq 1$ it satisfies that

$$f(\lambda \mathbf{x}_1 + (1-\lambda)\mathbf{x}_2) \leq \lambda f(\mathbf{x}_1) + (1-\lambda) f(\mathbf{x}_2).$$

5) Affine Set: Let $C \subseteq \mathbb{R}^n$, then C is affine if $\forall\ \mathbf{x}_1, \mathbf{x}_2 \in C$ and $\forall\ l \in \mathbb{R}$

$$l\mathbf{x}_1 + (1-l)\mathbf{x}_2 \in C.$$

6) Second-order Cone: The $n+1$ dimensional second-order cone (SOC) is represented by

$$SOC(n+1) = \left\{ \begin{pmatrix} \mathbf{u} \\ t \end{pmatrix} : \mathbf{u} \in \mathbb{R}^n, t \in \mathbb{R}, ||\mathbf{u}|| \leq t \right\}, \quad (1)$$

where $||\cdot||$ represents the Euclidean norm.

The one dimensional SOC is a unit second-order cone defined by

$$SOC(1) = \{t : t \in \mathbb{R}, 0 \leq t\}. \quad (2)$$

The SOC is also known as Lorentz cone, ice-cream cone or quadratic cone.

7) Second-order Cone Programming (SOCP): The general form of SOCP in equality standard can be stated as follows-

$$\text{minimize } \mathbf{c}^T \mathbf{x} \quad (3a)$$

$$\text{s.t. } A\mathbf{x} = \mathbf{b}, \quad (3b)$$

$$\mathbf{x} \in K. \quad (3c)$$

Here, $\mathbf{c} \in \mathbb{R}^n$, $\mathbf{b} \in \mathbb{R}^m$, $A \in \mathbb{R}^{m \times n}$ are problems data, $K \subseteq \mathbb{R}^n$ is the direct product of second-order cones, $\mathbf{x} \in \mathbb{R}^n$ is decision variable. In SOCP, usually linear objective function is minimized over the intersection of a direct product of second-order cones and an affine set. SOCP is known as a convex optimization problem which is used in engineering [6], communication [7], [8], control [9], finance [10], transport [12] etc to conduct advance research.

II. Robust Optimization

Robust optimization problem is an essential subbranch of optimization problem. In robust case, the data are not given exactly and it is declared that the data is contained in a set which is know as uncertainty set. If the problem is optimized taking consideration in the worst-case of the objective function, then such kind of optimization problem is called a robust optimization problem. The general form of robust LP problem can be defined as

$$\text{minimize } \mathbf{c}^T \mathbf{x} \quad (4a)$$

$$\text{s.t. } \mathbf{p}_i^T \mathbf{x} \leq \mathbf{b}_i, \forall \mathbf{p}_i \in S_{pi}, \forall \mathbf{b}_i \in S_{bi}, \quad i = 1, 2, \cdots, n. \quad (4b)$$

Here $S_{bi} \in \mathbb{R}$ and $S_{pi} \in \mathbb{R}^n$ are given uncertainty sets. For more details on robust optimization, the readers are requested to consult with [13] [14]. There are lots of research work have been conducted applying the concept of robust optimization. For example, mechanics [15], finance [16], management [17], control [18], [19], informatics [20], [21] etc. Our goal is to propose a model where we can minimize the production cost using robust optimization technique. Of course, we use second-order cone programming technique to formulate the proposed model in the form of SOCP.

The remaining part of this article is designed as follows. In Section III, first we formulate a business production mix problem. Then we formulate the proposed robust model for production mix problem considering an error in the ingredients in the same section. Finally, future work directions and concluding remarks are presented in Section IV.

III. Model Formulation

A. Production Mix Problem Formulation

To formulate the robust optimization problem using second-order cone constraints, first we consider a general business production problem as follows:

A production factory produces a product by combining n kinds of materials. Let N be the set of materials. Each material consists of m ingredients and let M be the set of ingredients. To produce a product and also to ensure the quality, it is

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required no less than $l_j\%$ of ingredient j, $\forall j \in M$. We consider that the price of the material x_i per unit weight is p_i, $\forall i \in N$. The problem is to find out the lowest cost to produce one unit weight of the product and the ratio of the materials to be bought.

The above optimization problem can be stated as follows:

$$\text{minimize} \sum_{i \in N} p_i x_i \tag{5a}$$

$$\text{s.t.} \sum_{i \in N} a_{ij} x_i \geq l_j, \qquad \forall j \in M \tag{5b}$$

$$x_i \geq 0, \qquad \forall i \in N \tag{5c}$$

here, a_{ij} represents the ratio of ingredient j in material x_i. The objective function compute the production cost in terms of ingredient and ratio of materials. The constraint (5b) represents the quality constraints of the product and the constraint (5c) represents the non negativity restriction of materials. We label the name of the problem (5) as general production mix (GPM) model since it minimizes the production cost considering the usual technique of optimization. Our aim is to propose a new production mix problem using SOCP based on the problem (5) that can minimize production cost. We will also use robust optimization technique to allow some errors in ratio of ingredients in materials.

B. Robust SOCP Model for Production Mix Problem

To formulate the robust optimization model using second-order cone programming, we assume that the ratio of ingredient j in material x_i has some error e_{ij}, $\forall i \in N$ and $\forall j \in M$. Our assumption is that the error is very small, could be positive or negative, and satisfies the following condition

$$\sum_{i \in N} e_{ij}^2 \leq 1, \qquad \forall j \in M. \tag{6}$$

The equation (6) implies that the absolute value of each error is less than or equal to 1%.

It is important that the quality constraints (5b) should be satisfied even though we have error in ratio of the ingredient, i.e. the constraints (5b) under the error should be

$$\sum_{i \in N} (a_{ij} + e_{ij})\, x_i \geq l_j, \qquad \forall j \in M. \tag{7}$$

Since the error e_{ij} takes any value satisfying (6), for all $j \in M$, we can express this constraint as

$$\min \left\{ \sum_{i \in N} (a_{ij} + e_{ij})\, x_i : \sum_{i \in N} e_{ij}^2 \leq 1 \right\} \geq l_j. \tag{8}$$

For all j, the equation (8) can be written as

$$\min \left\{ \sum_{i \in N} e_{ij} x_i : \sum_{i \in N} e_{ij}^2 \leq 1 \right\} \geq l_j - \sum_{i \in N} a_{ij} x_i. \tag{9}$$

Our next goal is to compute the minimum value of left hand side of the equation (9). To compute the minimum value of left hand side of the equation (9), we use the Lemma 1

Lemma 1: For given $\mathbf{a} \in \mathbb{R}^n$ and $\theta > 0$, the minimum value of $\mathbf{a}^T\mathbf{x}$ i.e.

$$\min_{\mathbf{x} \in \Omega_\theta} \mathbf{a}^T \mathbf{x} = -\theta ||\mathbf{a}||,$$

where, $\Omega_\theta = \{\mathbf{x} \in \mathbb{R}^n : ||\mathbf{x}|| \leq \theta\}$ is a closed ball with radius θ.

For detail proof of the Lemma 1 the readers are reffered to go through the article by Das et al. [22].

Using the Lemma 1, it is easy to observe that the linear function is minimized at

$$e_{ij} = -\frac{\mathbf{x}_i}{||\mathbf{x}||} \tag{10}$$

for every $j \in M$, where

$$||\mathbf{x}|| = \sqrt{\sum_{i \in N} x_i^2}. \tag{11}$$

Therefore, the constraints (9) becomes

$$-\sum_{i \in N} \frac{x_i^2}{||\mathbf{x}||} = -||\mathbf{x}|| \geq l_j - \sum_{i \in N} a_{ij} x_i \tag{12}$$

which implies that

$$||\mathbf{x}|| \leq \sum_{i \in N} a_{ij} x_i - l_j, \qquad \forall j \in M. \tag{13}$$

The equation (14b) represents the second-order cone constraints considering the error in ratio of ingredients in production materials.

Finally, replacing the constraint (5b) with the constraint (14b) in the problem (5), we represent the proposed robust optimization model as follow:

$$\text{minimize} \sum_{i \in N} p_i x_i \tag{14a}$$

$$\text{s.t.}\ ||\mathbf{x}|| \leq \sum_{i \in N} a_{ij} x_i - l_j, \qquad \forall j \in M \tag{14b}$$

$$x_i \geq 0, \qquad \forall i \in N. \tag{14c}$$

The proposed optimization problem (14) is a robust optimization model in the form of second-order cone programming (SOCP) problem since the constraint (14b) represents the second-order cone (SOC) constraint. The model is formulated using the worst-case of the ratio of ingredients in production materials which confirm the robustness of the model. The proposed optimization model minimizes the production cost and the model is tractable within the reasonable time using software tools. We label the name of the model (14) as SOCP production mix (SOCP-PM) model since it minimizes the production cost considering the SOCP concepts of conic programming.

IV. Concluding remarks and Future work Directions

Minimizing production cost is a common goal for any commercial production industry, but ensuring the quality of product is also an important fact for any production business. In competitive market, no one can survive in the long run without ensuring the quality of product but the business company also should maximizes the profit as well by minimizing the associate cost related to the product. This work propose a new model that can minimizes the production cost by ensuring the quality of product. To minimize the production cost, there are lots of work have been done considering different concepts. This paper presents a new model applying second-order cone programming (SOCP) technique as a first addition in business production sector. We also consider the worst-case of the ratio of ingredients in production materials to confirm the robustness and formulate the proposed model in the form of SOCP. We propose the model as an alternative option of techniques to minimize the production cost ensuring the product quality rather than the optimal value of problem. We did not conduct any numerical experiment in the present work to claim the optimal value compare to the original production problem, but the proposed model is robust in ratio of ingredients in production materials. It should minimizes the production cost compare to the original problem since SOCP model is more robust. Our future work is to conduct some numerical experiments to prove the effectiveness of the robust SOCP model compare the original model considering the different directions and different values of the parameters.

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