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CHARACTERIZATION OF QUALITY OF SERVICE IN MOBILE COMMUNICATION MODEL

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Abstract: In this paper, mathematical model of mobile communication systems with traffic congestion and interference was considered in an attempt to study/ determine the quality of service rendered by the mobile networks service provider. The model was mathematically characterized; existence and uniqueness of solution was investigated, interference free steady and persistence steady states equilibria was established. Stability analysis was also carried out. For effective understanding, numerical simulation was carried out with classical fourth order Runge-kutta method using Maple-18 software for different interference rate. Finally, the results were analyzed appropriately and relative conclusions were drawn.

Keywords: *Mathematical model, quality of service, traffic congestion, interference, call dropout, throughput, mobile communication.*

I. INTRODUCTION

The use of mathematical concepts, laws and principles to represent the basic characteristics of the practical situations which usually pose serious challenges to human existence and well-being is known as model. Mobile communication is very essential in today's world because of its benefits to the government agencies, ministries, industries, organizations and institutions. And to achieve these, quality of service must be given maximum priority. Quality of service (QoS) is the level of satisfaction a customer or an end-user obtained from a service. The specific requirements provided by a network provider to the users which are necessary in order to achieve the required functionality of an application, [1], [2], [3], [4].

Challenging situation that cause quality of service to degrade can be congestion which is caused by traffic overflow. Delays caused by networking equipment, shared communication channels (where collision and large delays become common) and limited bandwidth networks with poor capacity management. The challenging nature of services can be measured by calls drop, fading, setup, success rate and cell data throughput, [1], [3], [4], [5], [6], [7]. The use of mathematical model as an alternative to costly or sometimes dangerous experiments in mobile communication industry has been very impressive.

Many researchers that have worked on mobile communications and its Quality of Services includes but not limited to the following; Aja et al. in [1] applied second order necessary condition to establish a free flow of connection between Mobile Telephone Network (MTN) mobile phone users at different locations in Michael Okpara university of Agriculture, Nigeria. The technique of minimum spanning tree to optimize the design for mobile communication network in some areas in Adamawa state of Nigeria was explored in [2]. In [3] analysis of global system for mobile communication network in Port Harcourt, Nigeria was carried out and the results shows that voice channels are underutilized if traffic loads are too small compare to the installed voice channels in a base station. Assessments and comparism of the quality of service provided by mobile network operators in Akure, Nigeria was studied in [4]. The quality of outgoing call service in Epe, Lagos state of Nigeria was investigated by [8] using key performance index.

In addition, [9] analyzed and identified the areas on the Global System of Mobile (GSM) network where congestion occurred and the causes of the congestion. A study on Quality of Service of GSM operators using a Real Time Methodology was studied in [10]. The evaluation and optimization of Quality of Service of Mobile Cellular Networks in Nigeria using drive test was carried out in [6]. [11] Investigated the real-time assessment QoS of mobile network in Nigeria using some Network Key Performance Indicators. The measurement and analysis of quality of service of mobile networks in Afghanistan and Ghana was studied in [12] and [13] respectively. Quality of service of the global system for mobile communication using empirical method was evaluated by [14] in Owerri environments and established that there are degradation in the level of debasement in the network. In [15] the effects of customer relationship marketing existing between complaint handling and customer loyalty was investigated. In [16] it was discovered that everyday increment in the population of mobile subscribers sky-high the demand for high-speed connectivity.

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Studies in [17] established the need for controls on quality of services of mobile communication and so on. Queuing is one of the vital mechanism in traffic management system,[18].

However, the problems of quality of service still remain the major problem to the end users. And since little or less has been done on the mathematical model of mobile communication networks in relation to key performance indicators and quality of service using drive test. Hence, this paper is focused on a mathematical model and simulations that will investigate the effect of traffic congestion and interference on the quality of service provided by mobile telecommunication operators using drive test. This assessment improves some of the existing results in the study of quality of service in moble communication.

II. MODEL FORMULATION

Let M, G and N be the number of subscribers to MTN, Glo(Global communications limited) and 9Mobile respectively. The traffic congestion (k_i) on a network is independent while interference (β) can affect cell through put. Call drop (μ) in either of the network is assumed varies but call fading (α) is the same because of combined interference. The following assumptions were imposed: All parameters unit are assumed in percentage (%).

i) Call blocked in MTN is independent on Glo or 9Mobile

ii) Call drop varies depending on location and the number of subscribers attempting to make call at a particular time *t*. It can also be affected by the old age of radio/facilities at the base station.

iii) Interference is assumed the same for all network at time *t* but congestion is not except for single network and same SIM.

iv) Only calls/network, which is less affected with traffic congestion and interference have cell data through put (γ). Cell data throughput is also less affected by interference.

v) Call setup success rate (CSSR) is homogeneous for the three networks and not all call setup success enjoys cell data throughput. Removal rate due to interference is (μ + α) and total time expired before removal is $\frac{1}{(\mu+\alpha)}$.

vi) Some mobile devices pull network than another, thus, the weak devices may experience much of call drop and fading.

vii) Call setup success rate is $\beta(k_1M + k_2G + k_3N)I$, the product of number of subscribers attempting and proportion of interference.

viii) Subscribers assumed either voluntarily exit cell data throughput or unwilling terminate the call before cell data throughput if poor connectivity is experienced.

TABLE 1: DESCRIPTION OF PARAMETER AND VARIABLES (The Values are Hypothetical)

Parameter	Description	Value
k_1	Traffic congestion (success rate) on MTN	0.9
k_2	Traffic congestion (success rate) on Glo	0.4
k_3	Traffic congestion (success rate) on 9 Mobile	0.2
B	Interference rate on the network	[0.1, 1]
μ_1	Proportion of call drop as a result of congestion on MTN	0.1
μ_2	Proportion of call drop as a result of congestion on Glo	0.01
μ_3	Proportion of call drop as a result of congestion on 9 Mobile	0.05
Λ_1	New subscribers to MTN	12
Λ_2	New subscribers to Glo	10
Λ_3	New subscribers to 9 Mobile	4
A	Call fading proportion due to combination of interference and congestion on call setup	0.15
α_2	Voluntary removal	0.8
Γ	Cell data throughput rate	0.8
μ	Call drop due to poor service	0.2
M_0	Initial subscribers to MTN	12
G_0	Initial subscribers to Glo	11
N_0	Initial subscribers to 9 Mobile	5
I_0	Initial subscribers susceptible to interference and congestion	20
S_0	Initial successful subscribers	13

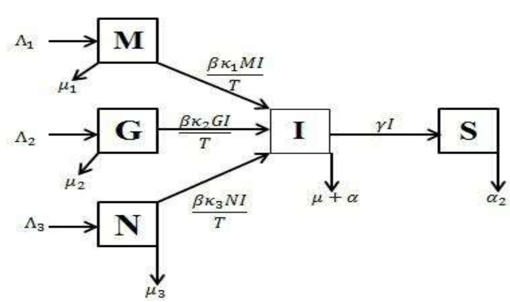

Figure1: Model Flow Diagram

In view of the above assumptions, the model equation is obtained as:

$$\frac{dM}{dt} = \Lambda_1 - \mu_1 M - \frac{\beta k_1 MI}{T} \quad (1)$$

$$\frac{dG}{dt} = \Lambda_2 - \mu_2 G - \frac{\beta k_2 GI}{T} \quad (2)$$

$$\frac{dN}{dt} = \Lambda_3 - \mu_3 N - \frac{\beta k_3 NI}{T} \quad (3)$$

$$\frac{dI}{dt} = \beta(k_1 M + k_2 G + k_3 N)\frac{I}{T} - (\mu + \alpha + \gamma)I \quad (4)$$

$$\frac{dS}{dt} = \gamma I - \alpha_2 S \quad (5)$$

$$T = M + G + N + I + S \quad (6)$$

T is the Total number of subscribers attempted calls).

III. CHARACTERIZATION OF MODEL

The model formulated is hereby characterized

(i) Existence and Uniqueness of Solution to the Model

The existence and uniqueness of solution to equations (1-6) shall be investigated using Lemma 1

Lemma 1: [19]

A function, F (x, y) is said to satisfy a Lipschitz condition in a region D on x-y plane, if there exist a positive constant K such that
$|F(x, y_2) - F(x,y_1)| \leq K|y_2 - y_1|$ whenever the points (x, y_2) and (x, y_1) lies in D. The constant K is called a Lipschitz constant for the function F (x, y).

Theorem 1

Suppose $F_i(t, M, G, N, I, S)$, $i(1)5$ satisfies the Lipschtz condition in the region then the solution to 1-6 exists and unique in the same region.

Proof: Using Lemma 1 to have

$$F_1(t, M, G, N, I, S) = \Lambda_1 - \mu_1 M - \frac{\beta k_1 MI}{T} \quad (7)$$

$$F_2(t, M, G, N, I, S) = \Lambda_2 - \mu_2 G - \frac{\beta k_2 GI}{T} \quad (8)$$

$$F_3(t, M, G, N, I, S) = \Lambda_3 - \mu_3 N - \frac{\beta k_3 NI}{T} \quad (9)$$

$$F_4(t, M, G, N, I, S) = \beta(k_1 M + k_2 G + k_3 N)\frac{I}{T} - (\mu + \alpha + \gamma)I \quad (10)$$

$$F_5(t, M, G, N, I, S) = \gamma I - \alpha_2 S \quad (11)$$

From (7–11) the following were obtained respectively

$$F_1(t, M_2, G, N, I, S) - F_1(t, M_1, G, N, I, S) = -\left(\mu_1 + \frac{\beta k_1 I}{T}\right)(M_2 - M_1)$$

$$|F_1(t, M_2, G, N, I, S) - F_1(t, M_1, G, N, I, S)| \leq \left|\mu_1 + \frac{\beta k_1 I}{T}\right| |M_2 - M_1|$$

i.e.

$$|F_1(t, M, G, N, I_2, S) - F_1(t, M, G, N, I_1, S)| \leq \left|\frac{\beta k_1 M}{T}\right| |I_2 - I_1| \quad (12)$$

$$|F_2(t, M, G_2, N, I, S) - F_2(t, M, G_1, N, I, S)| \leq \left|\mu_2 + \frac{\beta k_2 I}{T}\right| |G_2 - G_1|$$

$$|F_2(t, M, G, N, I_2, S) - F_2(t, M, G, N, I_1, S)| \leq \left|\frac{\beta k_2 G}{T}\right| |I_2 - I_1| \quad (13)$$

$$|F_3(t, M, G, N_2, I, S) - F_3(t, M, G, N_1, I, S)| \leq \left|\mu_3 + \frac{\beta k_3 I}{T}\right| |N_2 - N_1|$$

$$|F_3(t, M, G, N, I_2, S) - F_3(t, M, G, N, I_1, S)| \leq \left|\frac{\beta k_3 N}{T}\right| |I_2 - I_1| \quad (14)$$

$$|F_4(t, M_2, G, N, I, S) - F_4(t, M_1, G, N, I, S)| \leq \left|\frac{\beta k_1 I}{T}\right| |M_2 - M_1|$$

$$|F_4(t, M, G_2, N, I, S) - F_4(t, M, G_1, N, I, S)| \leq \left|\frac{\beta k_2 I}{T}\right| |G_2 - G_1|$$

$$|F_4(t, M, G, N_2, I, S) - F_4(t, M, G, N_1, I, S)| \leq \left|\frac{\beta k_3 I}{T}\right| |N_2 - N_1|$$

$$|F_4(t, M, G, N, I_2, S) - F_4(t, M, G, N, I_1, S)| \leq |l||I_2 - I_1| \quad (15)$$

Where

$$l = \frac{\beta}{T}(K_1 M + K_2 G + K_3 N) - (\mu + \alpha + \gamma)$$

$$|F_5(t, M, G, N, I_2, S) - F_5(t, M, G, N, I_1, S)| \leq |\gamma||I_2 - I_1|$$

$$|F_5(t, M, G, N, I, S_2) - F_5(t, M, G, N, I, S_1)| \leq |\alpha_2||S_2 - S_1| \quad (16)$$

Since (12-16) satisfy the Lipschtz inequality, hence, the solution to (1-6) exists and unique.

Theorem 2

Let D denote the region bounded in $0 \leq R < \infty$. Then $\frac{\partial F_i(t,M,G,N,I,S)}{\partial t}$, i = 1,…, 5 is continuous in the region D, if $\left|\frac{\partial F_i(t,M,G,N,I,S)}{\partial t}\right| < \infty$.

Proof: Using Theorem 1

For F_1

$$\left|\frac{\partial F_1}{\partial M}\right| = \left|\mu_1 + \frac{Bk_1 I}{T}\right| < \infty, \left|\frac{\partial F_1}{\partial G}\right| = 0 < \infty,$$

$$\left|\frac{\partial F_1}{\partial N}\right| = 0 < \infty,$$

$$\left|\frac{\partial F_1}{\partial I}\right| = \left|\frac{Bk_1 M}{T}\right| < \infty, \quad \left|\frac{\partial F_1}{\partial S}\right| = 0 < \infty$$

For F_2

$$\left|\frac{\partial F_2}{\partial M}\right| = 0 < \infty, \left|\frac{\partial F_2}{\partial G}\right| = \left|\mu_2 + \frac{Bk_2 I}{T}\right| < \infty,$$

$$\left|\frac{\partial F_2}{\partial N}\right| = 0 < \infty, \left|\frac{\partial F_2}{\partial I}\right| = \left|\frac{Bk_2 G}{T}\right| < \infty,$$

$$\left|\frac{\partial F_2}{\partial S}\right| = 0 < \infty$$

For F_3

$$\left|\frac{\partial F_3}{\partial M}\right| = 0 < \infty, \quad \left|\frac{\partial F_3}{\partial G}\right| = 0 < \infty,$$

$$\left|\frac{\partial F_3}{\partial N}\right| = \left|\mu_3 + \frac{Bk_3 I}{T}\right| < \infty, \left|\frac{\partial F_3}{\partial I}\right| = \left|\frac{Bk_3 N}{T}\right| < \infty,$$

$$\left|\frac{\partial F_3}{\partial S}\right| = 0 < \infty$$

For F_4

$$\left|\frac{\partial F_4}{\partial M}\right| = \left|\frac{Bk_1 I}{T}\right| < \infty, \left|\frac{\partial F_4}{\partial G}\right| = \left|\frac{Bk_2 I}{T}\right| < \infty,$$

$$\left|\frac{\partial F_4}{\partial N}\right| = \left|\frac{Bk_3 I}{T}\right| < \infty, \left|\frac{\partial F_4}{\partial I}\right| = |\omega| < \infty,$$

$$\left|\frac{\partial F_4}{\partial S}\right| = 0 < \infty$$

Where

$$\omega = \frac{\beta}{T}(K_1 M + K_2 G + K_3 N) - (\mu + \alpha + \gamma)$$

For F_5

$$\left|\frac{\partial F_5}{\partial M}\right| = 0 < \infty, \left|\frac{\partial F_5}{\partial G}\right| = 0 < \infty, \left|\frac{\partial F_5}{\partial N}\right| = 0 < \infty,$$

$$\left|\frac{\partial F_5}{\partial I}\right| = |\gamma| < \infty, \left|\frac{\partial F_5}{\partial S}\right| = |\alpha_2|$$

Since $\left|\frac{\partial F_i(t,M,G,N,I,S)}{\partial t}\right| < \infty$ for i = 1,…,5, hence the result.

(ii) Equilibrium State and Stability Analysis

Theorem 3

Consider the function,

$$F_i(t, M, G, N, I, S) = \frac{dM}{dt},$$

where $i = 1, \ldots, 5$, then there exist $E = (t, M, G, N, I, S)$ and $E^* = (t^*, M^*, G^*, N^*, I^*, S^*)$ for all $t \geq 0$ for which the function is at equilibrium

Proof: Equations (1–6) can be written as:

$$\Lambda_1 - \mu_1 M - \frac{Bk_1 MI}{T} = 0$$

$$\Lambda_2 - \mu_2 G - \frac{Bk_2 GI}{T} = 0$$

$$\Lambda_3 - \mu_3 N - \frac{Bk_3 NI}{T} = 0$$

$$\beta(K_1 M + K_2 G + K_3 N)\frac{I}{T} - (\mu + \alpha + \gamma)I = 0$$

$$\gamma I - \alpha_2 S = 0$$

Two cases were considered

Case 1: For $i = 0$

$$E = \{(t, M, G, N, I, S): \frac{\Lambda_1}{\mu_1}, \frac{\Lambda_2}{\mu_2}, \frac{\Lambda_3}{\mu_3}, 0 \text{ for all } t \geq 0\}$$

Case 2: For $i \neq 0$

$$E^* = \{(t^*, M^*, G^*, N^*, I^*, S^*): \frac{T\Lambda_1}{T\mu_1 + \beta k_1 I},$$

$$\frac{T\Lambda_2}{T\mu_2 + \beta k_2 I}, \frac{T\Lambda_3}{T\mu_3 + \beta k_3 I}, \frac{\alpha_2 S}{\gamma}, \frac{\gamma I}{\alpha_2} \text{ for all } t \geq 0\}$$

Two equilibria states; Interference Free Steady State (E) and Persistence Steady State (E^*) were exhibited. To establish stability in the Traffic Congestion and Interference of mobile network, the eigenvalues of the Jacobian matrix on (1–6) will be discussed in two cases. That is:

$$J = \begin{pmatrix} -\mu_1 - \frac{Bk_1 I}{T} & 0 & 0 & \frac{-Bk_1 M}{T} & 0 \\ 0 & -\mu_2 - \frac{Bk_2 I}{T} & 0 & \frac{-Bk_2 G}{T} & 0 \\ 0 & 0 & -\mu_3 - \frac{Bk_3 I}{T} & \frac{-Bk_3 N}{T} & 0 \\ \frac{Bk_1 I}{T} & \frac{Bk_2 I}{T} & \frac{Bk_3 I}{T} & \omega & 0 \\ 0 & 0 & 0 & \gamma & -\alpha_2 \end{pmatrix} \quad (17)$$

Theorem 4

Let (17) be the Jacobian matrix for systems (1–6), then it will be locally asymptotically stable for all λ_i, where $i(1)5$, if eigenvalues have strictly negative roots.

Proof:

Case 1: Interference Free State; $|J - \lambda I| = 0$

From (17)

$$|J-\lambda I| = \begin{vmatrix} -(\mu_1+\lambda) & 0 & 0 & \frac{-Bk_1M}{T} & 0 \\ 0 & -(\mu_2+\lambda) & 0 & \frac{-Bk_2G}{T} & 0 \\ 0 & 0 & -(\mu_3+\lambda) & \frac{-Bk_3N}{T} & 0 \\ 0 & 0 & 0 & \omega-\lambda & 0 \\ 0 & 0 & 0 & \gamma & -(\mu_2+\lambda) \end{vmatrix} = 0$$

Which implies

$$|J-\lambda I| = -(\mu_1+\lambda)\{-(\mu_2+\lambda)[-(\mu_3+\lambda\ \ )(-(\omega-\lambda)(\alpha_2+\lambda)]\} = 0$$

$$= (\mu_1+\lambda)(\mu_2+\lambda)(\mu_3+\lambda)(\omega-\lambda)(\alpha_2+\lambda) = 0$$

Therefore the Eigenvalues are:

$$\lambda_1 = -\mu_1, \lambda_2 = -\mu_2\ \ \lambda_3 = -\mu_3\ \ \lambda_4 = -\alpha_2$$

and

$$\lambda_5 = \frac{\beta}{T}(K_1M\ + K_2G\ + K_3N) - (\mu\ +\ \alpha\ +\ \gamma) \qquad (18)$$

Case 2: Interference Persistence State; $|J^* - \lambda I^*| = 0$
From (17)

$$|J^*-\lambda I^*| = \begin{vmatrix} -\left(\mu_1+\frac{Bk_1I}{T}+\lambda^*\right) & 0 & 0 & \frac{-Bk_1M}{T} & 0 \\ 0 & -\left(\mu_2+\frac{Bk_2I}{T}+\lambda^*\right) & 0 & \frac{-Bk_2G}{T} & 0 \\ 0 & 0 & -\left(\mu_3+\frac{Bk_3I}{T}+\lambda^*\right) & \frac{-Bk_3N}{T} & 0 \\ \frac{Bk_1I}{T} & \frac{Bk_2I}{T} & \frac{Bk_3I}{T} & (\omega^*-\lambda^*) & 0 \\ 0 & 0 & 0 & \gamma & -(\alpha_2+\lambda^*) \end{vmatrix}$$

Where

$$\omega^* = \frac{\beta}{T}(K_1M^*\ + K_2G^*\ + K_3N^*) - (\mu\ +\ \alpha\ +\ \gamma)$$

$$|J^*-\lambda I^*| = -\left(\mu_1+\frac{Bk_1I}{T}+\lambda^*\right)\left\{-\left(\mu_2+\frac{Bk_2I}{T}+\lambda^*\right)\left[-\left(\mu_3+\frac{Bk_3I}{T}+\lambda^*\right)\left((\omega^*-\lambda^*)(\alpha_2+\lambda^*)\right)\right]\right\} = 0$$

$$= \left(\mu_1+\frac{Bk_1I}{T}+\lambda^*\right)\left(\mu_2+\frac{Bk_2I}{T}+\lambda^*\right)\left(\mu_3+\frac{Bk_3I}{T}+\lambda^*\right)(\omega^*-\lambda^*)(\alpha_2+\lambda^*) = 0$$

The eigenvalues are:

$$\lambda_1^* = -\left(\mu_1+\frac{Bk_1I}{T}\right), \lambda_2^* = -\left(\mu_2+\frac{Bk_2I}{T}\right), \lambda_3^* = -\left(\mu_3+\frac{Bk_3I}{T}\right), \lambda_4^* = -\alpha_2$$

and

$$\lambda_5^* = \frac{\beta}{T}(K_1M^*\ + K_2G^*\ + K_3N^*) - (\mu\ +\ \alpha\ +\ \gamma) \qquad (19)$$

Thus the eigenvalues for Cases 1 and 2, (i.e $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and $\lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_4^*$) respectively returned negative values but λ_5 and λ_5^* requires further analysis before the conclusion can be made on the stability analysis. That is the case where $\frac{\beta}{T}(K_1M\ + K_2G\ + K_3N) < (\mu\ +\ \alpha\ +\ \gamma)$ and $\frac{\beta}{T}(K_1M^*\ + K_2G^*\ + K_3N^*) < (\mu +\ \alpha +\ \gamma)$.

IV. NUMERICAL APPLICATION

In this section numerical solution was carried out with Runge-Kutta method of fourth order [20], [21]. The effect of traffic congestion and interference was solved using Classical Fourth order Runge-Kutta method which is more accurate and also takes weighted average of slopes at more number of points than lower orders, Maple-18 software was used for different interference rate [20], [21], [22], [23]. The numerical simulation and results were shown in Tables 2 and 3 and Figures 2-4.

TABLE 2: NUMERICAL VALUES FOR INTERFERENCE FREE STATE

β	μ_1	μ_2	μ_3	k_1	k_2	k_3	Λ_1	Λ_2	Λ_3	λ_5	Remark
0	0.05	0.08	0.07	0.5	0.8	0.8	12	10	4	– 1.1500	Stable
0.01	0.05	0.08	0.07	0.5	0.8	0.8	12	10	4	–1.1064	Stable
0.03	0.05	0.08	0.07	0.5	0.8	0.8	12	10	4	–1.0193	Stable
0.06	0.05	0.08	0.07	0.5	0.8	0.8	12	10	4	–0.8886	Stable
0.15	0.05	0.08	0.07	0.5	0.1	0.8	12	10	4	–0.7118	Stable
0.15	0.05	0.08	0.07	0.1	0.1	0.8	18	18	9	– 0.7679	Stable
0.15	0.05	0.08	0.07	0.1	0.1	0.1	18	18	9	– 0.9745	Stable
0.7	0.05	0.08	0.07	0.1	0.1	0.8	18	18	9	+0.6238	Unstable
0.7	0.05	0.08	0.07	0.5	0.5	0.5	18	18	9	+2:9443	Unstable

TABLE 3: NUMERICAL VALUES FOR INTERFERENCE PERSISTENCE STATE

β	μ_1	μ_2	μ_3	k_1	k_2	k_3	Λ_1	Λ_2	Λ_3	λ_5	Remark
0	0.05	0.08	0.07	0.5	0.8	0.8	12	10	4	-1.1500	Stable
0.01	0.05	0.08	0.07	0.5	0.8	0.8	12	10	4	-1.1113	Stable
0.03	0.05	0.08	0.07	0.5	0.8	0.8	12	10	4	-1.0047	Stable
0.06	0.05	0.08	0.07	0.5	0.8	0.8	12	10	4	-0.8226	Stable
0.15	0.05	0.08	0.07	0.5	0.1	0.8	12	10	4	-0.2757	Stable
0.15	0.05	0.08	0.07	0.1	0.1	0.8	18	18	9	-0.4153	Stable
0.15	0.05	0.08	0.07	0.1	0.1	0.1	18	18	9	-0.9589	Stable
0.7	0.05	0.08	0.07	0.1	0.1	0.8	18	18	9	-0.7516	Stable
0.7	0.05	0.08	0.07	0.5	0.5	0.5	18	18	9	-6.8704	Stable

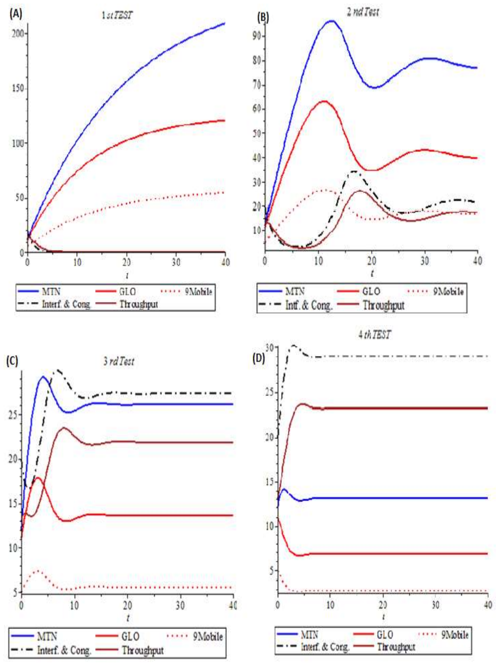

Figure 2: Investigations for; absence of interference rate ($\beta = 0$), 1% interference rate, 3% interference rate, and interference rate at 6%

V. DISCUSSION

The result is hereby discussed; Table 2 shows that the model is stable for high congestion ($\boldsymbol{k_1 = 0.5, k_2 = 0.8, k_3 = 0.8}$) and low interference rate but became unstable for high congestion and high interference rate, while Table 3 shows that the model is everywhere stable for low and high interference with independence of congestion, thus the model is locally asymptotically stable. In another word the simulation depends on interference and congestion. The behavior of the solution was first checked on 1st test, which has no interference leading to very low call drop, high call setup success rate, throughput is proportional to the product of interference and subscribers call attempted rate. In the 2nd test, subscribers fairly enjoyed communication as interference increases to 1% and congestion reduces CSSR. For 3rd test, it was 3% interference and reduction in number of people making calls which made active subscribers enjoyed the dividend as there are CSSR increases. 4th test investigated interference at 6% reveals subscribers reduced in call attempt, while the performance (throughput) increases. In 5th test, interference was at 15%, which shows significant difference between interference and throughput. At 6th test, 15% interference increase in number of subscribers considered in the analysis. And CSSR dropped due to congestion made little significant of throughput compared to the 5th test. 7th test maintained 15% interference and reduction in CSSR due to congestion but high throughput. Test 8 shows 70% interference rate with low CSSR whereby the behavior could best describe using concept of diminishing return. In order words, only few subscribers will enjoy quality communication as interference and congestion keep growing. Subscribers attempting making would be dropping. Finally in Test 9, the proportion of subscribers for the three networks was considered equal while interference is 70% with slight increment in CSSR for $\boldsymbol{\beta}$ from 0.01 to 0.7. Discouragement would drop call attempt rate because of poor or low throughput.

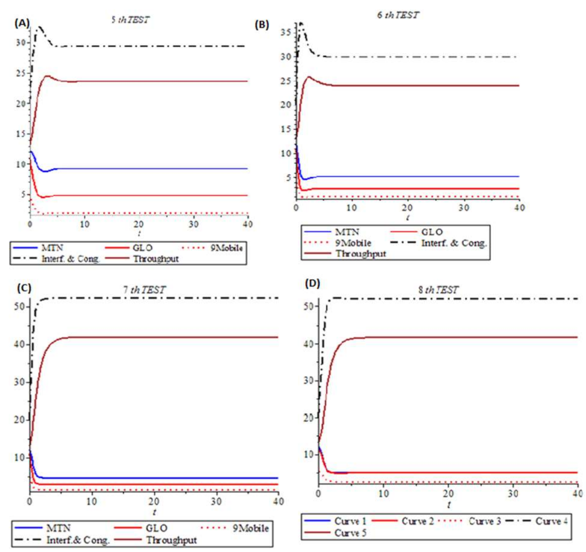

Figure 3: Investigations of ; interference rate at (15%), 15% interference rate but increase in call attempt, 15% Interference rate and reduction in call setup success rate(CSSR), interference rate at 70% with low CSSR.

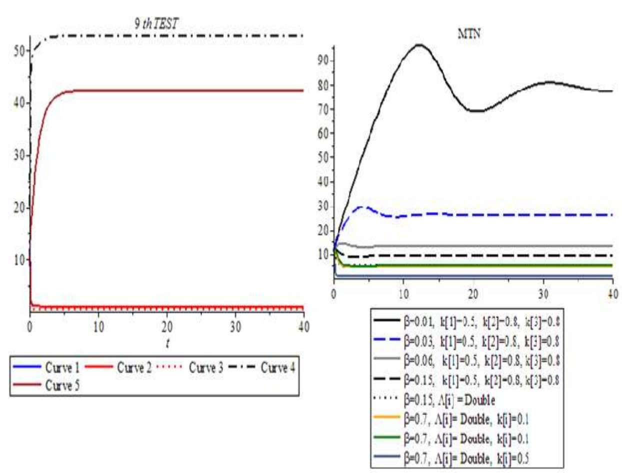

Figure 4: Investigation of interference rate at 70% with slight increment in CSSR

VI. CONCLUSION

In this paper, mathematical model for mobile communication was considered. The model was characterized mathematically; stability analysis and equilibrium state were investigated for both interference free state and interference persistence state of the system. Numerical application and simulation was finally carried out using Maple 18 software for better interpretation of the quality of service of the system and the result obtained shows that though interference aid throughput as call setup success rate will be high, the system is stable for high congestion $(\boldsymbol{k_1 = 0.5, k_2 = 0.8, k_3 = 0.8})$ and low interference rate but unstable for high congestion and high interference rate. In other words, in the absence of interference, the call attempt increases such that the capacity of facilities of the mobile operators exhausted.

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