ON NYORRAUF TRANSFORMATION ALGORITHM TO FLIGHT CREW SCHEDULING PROBLEMS

Rauf Kamilu [1], Nyor Ngutor [2], Omolehin Joseph Olorunju [3], Owolabi Abiodun Adebayo [4]

[1] Department of Mathematics, University of Ilorin, Nigeria.
[2] Department of Mathematics, Federal University of Technology, Minna, Nigeria.
[3] Department of Mathematics, Federal University Lokoja, Nigeria.
[4] Department of Mathematics and Computer Sciences, Fountain University, Osogbo, Nigeria.

Email: krauf@unilorin.edu.ng, ngutornyor@yahoo.com, joseph.omolehin@fulokoja.edu.ng, abiowolab@yahoo.com

Abstract: *In this work, we applied NyorRauf Transformation Algorithm to a flight crew scheduling problem to assess its impact on crew scheduling models. The result yielded a solution that is implementable by airlines. It was concluded that, generally speaking, the transformation algorithm is efficient in the Karmarkar Interior Point scheme for solving Linear Programming Problems.*

Keywords: *Flight Crew, Flight Crew Scheduling Problem, NyorRauf Transformation Algorithm, Karmarkar Interior Point scheme*

1. Introduction

According to Balaji and Ellis [1], the airline industry is characterized by some of the largest scheduling problems of any industry. The problem of crew scheduling involves the optimal allocation of crews to flights. Balaji and Ellis [1] argued that, over the last two decades the magnitude and complexity of crew scheduling problems have grown enormously and airlines are depending more and more on automated mathematical procedures as a practical necessity. Michael [4] also reiterated that, One major problem for airlines is the scheduling of their flight crews.

The airline industry is severely unionized and there are stringent limitations on how to use a crew. For example, there are rules on how many hours a crew must be in the air in a day; and there are restrictions on the number of hours a crew can be away from their home base before they must stopover in a hotel. But crew Overheads are the second largest operating expense an airline has (after gasoline). Therefore, there is an opening to work with a hard problem influenced by enormous potential cost savings (Michael [4]).

According to Karla and Manfred [3], the air scheduling problem is one that has been studied almost continually for the past 40 years. It is obvious that, the problem is much more important today since costs for flying personnel of organizations or companies or major government parastatals have so much grown and are second largest cost (next to fuel) of the total operating costs for airlines. As a result of this, even small percentage savings amount to substantial amounts.

NyorRauf Transformation Algorithm is a transformation procedure that converts an (LP) problem from standard maximization form to a Karmarkar form. Karmarkar assumes that the LP is given as:

Minimize

$$Z=CX$$

Subject to

$$AX = 0$$

All the constraints are homogeneous equations except for the constraint

$$1X = \sum_{j=i}^{n} x_j = 1,$$

which defines an n-dimensional simplex. The validity of Karmarkar algorithm rests on satisfying two conditions:

1. $X = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})$ satisfies $AX = 0$
2. min $z = 0$ (Taha [6]).

Date of submission : 17.10.2016 Date of acceptance : 04.04.2017

1.1 NyorRauf Transformation Procedure.

Given the original LP problem:

Maximize $Z = CX$

Subject to $AX \le b$

$X \ge 0$

NyorRauf has the following steps for the transformation:

Step 1. Convert the constraints inequalities of the original LP problem into equations by augmenting slack or surplus variables appropriately as:

$$\sum AX_{ij} = b; \qquad i = 1,2,\cdots,m; \; j = 1,2,\cdots,n$$

where n is the number of X variables and m is the number of constraints in the original LP.

Step 2. Define the augmented equations in step 1 leaving out the coefficient of X_j but considering all the X_{js} as:

$$\sum_{j=1}^{n+1} X_j \le U_i; \qquad i = 1,2,\cdots,m$$

where m is the number of constraint equations,

U is the value that is sufficiently large so as not to eliminate any feasible point from the solution space.

Step 3. For each standardized constraint i, obtain

$$X_j = L.U.I.\left|\frac{b_i}{a_{ij}}\right|$$

if fractional part, by setting other n variables equal to zero.

Where L.U.I. is Least Upper Integers.

Step 4. Obtain $U_i = \sum_{j=1}^{n+1} X_j$

Step 5. Obtain U_{val} from

$$U_{val} = G.L.I.\left(\frac{\sum_{i=1}^{m} U_i}{m}\right)$$; whether fractional parts or not.

Where G.L.I. is Greatest Lower Integers.

Step 6. Thus, step 2 becomes

$$\sum_{j=1}^{n+1} X_j \le U_{val}$$

Step 7. Augment again the defined constraint in step 6 to have

$$\sum_{j=1}^{n+2} X_j = U_{val}$$

Step 8 Homogenize the Right Hand Side (RHS) of each augmented constraints in step 1 by

$$\frac{\sum_{j=1}^{n+2} X_j}{U_{val}}$$. Hence,

$$\left(\sum_{j=1}^{n+1} a_{ij} X_{ij}\right) U_{val} = b_i\left(\sum_{j=1}^{n+2} X_j\right).$$

Where a is the coefficient of X in the original LP problem

and b is the RHS constraint of the original LP. Thus,

$$\left(\sum_{j=1}^{n+1} a_{ij} X_{ij}\right) U_{val} - b_i\left(\sum_{j=1}^{n+2} X_j\right) = 0$$

Step 9. Ensure that the sum of the coefficient of the LHS equals zero by adding artificial variables where necessary.

Step 10. Penalize the artificial variables introduced in step 7 in the objective function.

Step 11. Define new variables for the objective function as:

$$Y_i = \frac{X_i}{U_{val}}$$

Step 12. Substitute the new variables as defined in step 11 into the constraints to maintain consistency; hence the transformed karmarkar algorithm (Omolehin et al., [5]).

2. Application

The aim of this paper is to apply NyorRauf Transformation Algorithm in order to assess its impact on flight crew scheduling models. The flight scheduling problem under consideration in section 3 is in a standard LP form. It was a case study of IRS [2] airline, which was formulated first as an integer program then its LP dual was standardized to this state.

2.1 Flight Crew Scheduling Mathematics

We have n flights and assign m crews. One possibility is to define decision variables $y_{ij}.1 \le i \le m, 1 \le j \le m;$ Where

$$y_{ij} = \begin{cases} 1 \ \textit{flight } j \textit{ has a crew } i \\ 0 \ \textit{otherwise} \end{cases}$$

To cover flight j, we introduce a constraint of the form:

$$\sum_{i=1}^{n} y_{ij} \ge 1$$

for each flight j. A crew pairing problem can be visualized as:

Given:

i. A set of scheduled flight;

ii. safety and working rules;

iii. Minimum or maximum credited hours per crew base.

Find least-cost feasible crew pairings

Subject to

i. each flight is covered by an active crew

ii. the maximum or minimum credited hours per crew base is represented (Tran, [7]).

3. The Flight Crew Scheduling Problem

The Flight Crew Scheduling problem given below is in its standard form which NyorRauf can be applied. Slack variables x_{32} - x_{67} are added to the left hand side of the inequalities to obtain the equality constraints.

Maximize **Z=**

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$
$$+ x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} + x_{21} + x_{22} + x_{23}$$
$$+ x_{24} + x_{25} + x_{26} + x_{27} + x_{28} + x_{29} + x_{30} +$$
$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36}$$

Subject to:

$$x_1 + x_{15} + x_{32} = 128 \qquad (3.1)$$

$$x_1 + x_7 + x_{16} + x_{33} = 181 \qquad (3.2)$$

$$x_1 + x_7 + x_8 + x_{17} + x_{34} = 245 \qquad (3.3)$$

$$x_1 + x_7 + x_8 + x_{11} + x_{17} + x_{35} = 280 \qquad (3.4)$$

$$x_1 + x_7 + x_8 + x_{11} + x_{13} + x_{19} + x_{36} = 384$$

$$x_1 + x_7 + x_8 + x_{11} + x_{13} + x_{14} + x_{37} = 445 \ (3.6)$$

$$x_2 + x_{16} + x_{20} + x_{38} = 146 \qquad (3.7)$$

$$x_2 + x_8 + x_{17} + x_{39} = 210 \qquad (3.8)$$

$$x_2 + x_8 + x_{11} + x_{18} + x_{40} = 245 \qquad (3.9)$$

$$x_2 + x_8 + x_{11} + x_{13} + x_{19} + x_{41} = 349 \qquad (3.10)$$

$$x_2 + x_8 + x_{11} + x_{13} + x_{14} + x_{20} + x_{42} = 410 \ (3.11)$$

$$x_3 + x_{17} + x_{43} = 168 \qquad (3.12)$$

$$x_3 + x_{11} + x_{18} + x_{44} = 203 \qquad (3.13)$$

$$x_3 + x_{11} + x_{13} + x_{19} + x_{45} = 307 \qquad (3.14)$$

$$x_3 + x_{11} + x_{13} + x_{14} + x_{20} + x_{46} = 368 \qquad (3.15)$$

$$x_4 + x_{18} + x_{47} = 82 \qquad (3.16)$$

$$x_4 + x_{13} + x_{20} + x_{48} = 186 \qquad (3.17)$$

$$x_4 + x_{13} + x_{14} + x_{20} + x_{49} = 247 \qquad (3.18)$$

$$x_{15} + x_{19} + x_{50} = 136 \qquad (3.19)$$

$$x_5 + x_{14} + x_{20} + x_{51} = 197 \qquad (3.20)$$

$$x_6 + x_{20} + x_{52} = 132 \qquad (3.21)$$

$$x_1 + x_7 + x_{15} + x_{21} + x_{53} = 216 \qquad (3.22)$$

$$x_2 + x_8 + x_{15} + x_{22} + x_{54} = 241 \qquad (3.23)$$

$$x_3 + x_{16} + x_{23} + x_{55} = 210 \qquad (3.24)$$

$$x_5 + x_{14} + x_{20} + x_{56} = 197 \qquad (3.25)$$

$$x_{14} + x_{31} + x_{57} = 126 \qquad (3.26)$$

$$x_{14} + x_{30} + x_{58} = 217 \qquad (3.27)$$

$$x_5 + x_{16} + x_{24} + x_{59} = 249 \qquad (3.28)$$

$$x_5 + x_{18} + x_{29} + x_{60} = 186 \qquad (3.29)$$

$$x_{10} + x_{23} + x_{27} + x_{61} = 281 \qquad (3.30)$$

$$x_4 + x_{13} + x_{19} + x_{62} = 186 \qquad (3.31)$$

$$x_8 + x_{12} + x_{14} + x_{25} + x_{63} = 326 \quad (3.32)$$

$$x_9 + x_{14} + x_{25} + x_{64} = 301 \quad (3.33)$$

$$x_{14} + x_{28} + x_{65} = 292 \quad (3.34)$$

$$x_4 + x_{15} + x_{21} + x_{23} + x_{26} + x_{66} = 280 \quad (3.35)$$

$$x_{13} + x_{29} + x_{67} = 154 \quad (3.36)$$

$$x_j \geq 0; (j = 1 - 31) \quad (3.37)$$

4. NyorRauf Transformation of the LP

Using the standardized problem above, we begin the transformation into NyorRauf form as:

Define

$$x_1 + x_2 + x_3 + \ldots + x_{67} \leq U$$

From equation (3.1),

$$x_1 + x_{15} + x_{32} = 128$$

If $x_1 = x_{15} = 0$, *then* $x_{32} = 128$.

If $x_1 = x_{32} = 0$, *then* $x_{15} = 128$.

If $x_{15} = x_{32} = 0$, *then* $x_1 = 128$.

Since there is no fractional part,

$$U = L.U.I.\left(\frac{\sum_{i=1}^{n} U_i}{m}\right)$$

will not be applied here.

$$\therefore U_1 = 128 + 128 + 128 = 384.$$

$$Or\ U_1 = 128 \times 3 = 384$$

Since there is no fractional part,

From equation (3.2),

$$x_1 + x_7 + x_{16} + x_{33} = 181$$

If $x_1 = x_7 = x_{16} = 0$, *then* $x_{33} = 181$

If $x_1 = x_7 = x_{33} = 0$, *then* $x_{16} = 181$

If $x_1 = x_{16} = x_{33} = 0$, *then* $x_7 = 181$

If $x_7 = x_{16} = x_{33} = 0$, *then* $x_1 = 181$

Again, since there is no fractional part,

$$U = L.U.I.\left(\frac{\sum_{i=1}^{n} U_i}{m}\right)$$

will not be applied here.

$$\therefore U_2 = 181 \times 4 = 724$$

Similarly, solving equations $(3.3) - (3.36)$ as above, the following $U_{i's}$ are obtained:

$U_3 = 1225$	$U_4 = 1680$
$U_5 = 2688$	$U_6 = 3115$
$U_7 = 584$	$U_8 = 840$
$U_9 = 1225$	$U_{10} = 2094$
$U_{11} = 2870$	$U_{12} = 504$
$U_{13} = 812$	$U_{14} = 1535$
$U_{15} = 2208$	$U_{16} = 246$
$U_{17} = 744$	$U_{18} = 1235$
$U_{19} = 408$	$U_{20} = 788$
$U_{21} = 396$	$U_{22} = 1080$
$U_{23} = 1205$	$U_{24} = 840$
$U_{25} = 788$	$U_{26} = 378$
$U_{27} = 651$	$U_{28} = 996$
$U_{29} = 744$	$U_{30} = 1124$
$U_{31} = 744$	$U_{32} = 1304$
$U_{33} = 1204$	$U_{34} = 876$
$U_{35} = 1680$	$U_{36} = 462$

$$But\ U_{val} = G.L.I.\left(\frac{\sum_{i=1}^{36} U_i}{m}\right) = G.L.I.\left(\frac{40382}{36}\right) = 1121$$

Equation (3.38) becomes,

$$x_1 + x_2 + x_3 + \cdots x_{67} \leq 1121 \quad (3.39)$$

Augmenting again,

$$x_1 + x_2 + x_3 + \cdots x_{67} + x_{68} = 1121 \quad (3.40)$$

By multiplying the right hand side of equations (3.1) to (3.36) respectively by $\frac{x_1 + x_2 + x_3 + \cdots x_{67} + x_{68}}{1121}$, we obtain the constraints of NyorRauf form as follows:

From equation (3.1)

$$x_1 + x_{15} + x_{32} = 128\left(\frac{x_1 + x_2 + x_3 + \cdots + x_{67} + x_{68}}{1121}\right)$$

$$= 128x_1 + 128x_2 + 128x_3 + \cdots + 128x_{67} + 128x_{68}$$

But

$$1121x_1 + 1121x_{15} + 1121x_{32}$$

$$993x_1 - 128x_2 - \cdots - 128x_{40} - \cdots - 128x_{68}$$

$$+ 993x_{15} - 128x_{16} - \cdots - 128x_{31} + 993x_{32} - 128x_{32} = 0$$

$$993-1664+993-2048+993-4608=-5341$$

Thus

$$993x_1-128x_2-\cdots-128x_{14}+993x_{15}-128x_{16}-\cdots-128x_{31}+993x_{32}-128x_{33}-\cdots-128x_{68}+5341x_{69}=0 \qquad (3.41)$$

From equation (3.2)

$$x_1+x_7+x_{16}+x_{33}=181\left(\frac{x_1+x_2+x_3+\ldots+x_{67}+x_{68}}{1121}\right)$$

$$1121x_1+1121x_7+1121x_{16}+1121x_{33}=181x_1+181x_2+181x_3+\ldots+181x_{67}+181x_{68}$$

$$940x_1-181x_2-\ldots-181x_6+940x_7-181x_8-\ldots-181x_{15}+940x_{16}-181x_{17}-\ldots-181x_{32}+940x_{33}-181x_{34}-\ldots-181x_{68}=0$$

But

$$940-905+940-1448+940-2896+940-6335=-7824$$

Thus

$$940x_1-181x_2-\ldots-181x_6+940x_7-181x_8-\ldots-181x_{15}+940x_{16}-181x_{17}-\ldots-181x_{32}+940x_{33}-181x_{34}-\ldots-181x_{68}+7824x_{70}=0 \qquad (3.42)$$

Similarly, solving equations $(3.3)-(3.36)$ as above, equations $(3.43)-(3.76)$ are obtained respectively as follows:

$$876x_1-245x_2-\cdots-245x_6+876x_7+876x_8-245x_9-\cdots-245x_{16}+876x_{17}-245x_{18}-\cdots-245x_{33}+876x_{34}-245x_{35}-\cdots-245x_{68}+11055x_{71}=0$$

$$841x_1-280x_2-\cdots-280x_6+841x_7+841x_8-280x_9-\cdots-280x_{10}+841x_{11}-280x_{12}-\cdots-280x_{16}+841x_{17}-280x_{18}-\cdots-280x_{34}+841x_{35}-280x_{36}-\cdots-280x_{68}+12311x_{72}=0$$

$$737x_1-384x_2-\cdots-384x_6+737x_7+737x_8-384x_9-\cdots-384x_{10}+737x_{11}-384x_{12}+737x_{13}-384x_{14}-\cdots-384x_{18}+737x_{19}-384x_{20}-\cdots-384x_{35}+737x_{36}-384x_{37}-\cdots-384x_{68}+18265x_{73}=0$$

$$676x_1-445x_2-\cdots-445x_6+676x_7+676x_8-445x_9-445x_{10}+676x_{11}-445x_{12}+676x_{13}+676x_{14}-445x_{15}-\cdots-445x_{36}+676x_{37}-445x_{38}-\cdots-445x_{68}+22412x_{74}=0$$

$$-146x_1+975x_2-146x_3-\cdots-146x_{15}+975x_{16}-146x_{17}-\cdots-146x_{19}+975x_{20}-146x_{21}-\cdots-146x_{37}+975x_{38}-146x_{39}-\cdots-146x_{68}+5444x_{75}=0$$

$$-210x_1+911x_2-210x_3-\cdots-210x_7+911x_8-210x_9-\cdots-210x_{16}+911x_{17}-210x_{18}-\cdots-210x_{38}+911x_{39}-210x_{40}-\cdots-210x_{68}+9796x_{76}=0$$

$$-245x_1+876x_2-245x_3-\cdots-245x_7+876x_8-245x_9-245x_{10}+876x_{11}-245x_{12}-\cdots-245x_{17}+876x_{18}-245x_{19}-\cdots-245x_{39}+876x_{40}-245x_{41}-\cdots-245x_{68}+11055x_{77}=0$$

$$-349x_1+772x_2-349x_3-\cdots-349x_7+772x_8-349x_9-349x_{10}+772x_{11}-349x_{12}+772x_{13}-349x_{14}-\cdots-349x_{18}+772x_{19}-349x_{20}-\cdots-349x_{40}+772x_{41}-349x_{42}-\cdots-349x_{68}+17006x_{78}=0$$

$$-410x_1+711x_2-410x_3-\cdots-410x_7+711x_8-410x_9-410x_{10}+711x_{11}-410x_{12}+711x_{13}+711x_{14}-410x_{15}-\cdots-410x_{19}+711x_{20}-410x_{21}-\cdots-410x_{41}+711x_{42}-410x_{43}-\cdots-410x_{68}+20033x_{79}=0$$

$$-168x_1-168x_2+953x_3-168x_4-\cdots-168x_{16}+953x_{17}-168x_{18}-\cdots-168x_{42}+953x_{43}-168x_{44}-\cdots-168x_{68}+8061x_{80}=0$$

$$-203x_1-203x_2+918x_3-203x_4-\cdots-203x_{10}+918x_{11}-203x_{12}-203x_{17}+918x_{18}-203x_{19}-\cdots-203x_{43}+918x_{44}-203x_{45}-\cdots-203x_{68}+9320x_{81}=0$$

$$-307x_1-307x_2+814x_3-307x_4-\cdots-307x_{10}+814x_{11}-307x_{12}+814x_{13}-+307x_{14}-\cdots-307x_{18}+814x_{19}-307x_{20}-\cdots-307x_{44}+814x_{45}-307x_{46}-\cdots-307x_{68}+15271x_{82}=0$$

$$\begin{aligned}
&-368x_1-368x_2+753x_3-368x_4-\cdots-349x_{10}\\
&+753x_{10}+753x_{11}-368x_{12}+753x_{13}+\\
&753x_{14}-368x_{15}-\cdots-368x_{19}+753x_{20}-368x_{21}\\
&-\cdots-368x_{45}+753x_{46}-368x_{47}-\cdots\\
&-368x_{68}+18298x_{83}=0
\end{aligned}$$

$$\begin{aligned}
&-82x_1-82x_2-82x_3+1039x_4-82x_5-\cdots-82x_{17}\\
&+1039x_{18}-82x_{19}-\cdots-82x_{46}+1039x_{47}\\
&-82x_{48}-\cdots-82x_{68}+2213x_{84}=0
\end{aligned}$$

$$\begin{aligned}
&-186x_1-186x_2-186x_3+935x_4-186x_5\\
&-\cdots-186x_{12}+935x_{13}-186x_{14}-\cdots-186x_{19}+\\
&935x_{20}-186x_{21}-\cdots-186x_{47}+935x_{48}\\
&-186x_{49}-\cdots-186x_{68}+8164x_{85}=0
\end{aligned}$$

$$\begin{aligned}
&-247x_1-247x_2-247x_3+874x_4-347x_5\\
&-\cdots-247x_{12}+874x_{13}+874x_{14}-247x_{15}-\cdots\\
&-247x_{19}+874x_{20}-247x_{21}-\cdots-247x_{48}\\
&+874x_{49}-247x_{50}-\cdots-247x_{68}+11438x_{86}=0
\end{aligned}$$

$$\begin{aligned}
&-136x_1-\cdots-136x_4+985x_5-136x_6-\cdots-136x_{18}\quad+985x_{19}-136x_{20}-\cdots-136x_{49}+\\
&985x_{50}-136x_{51}-\cdots-136x_{68}+5885x_{87}=0
\end{aligned}$$

$$\begin{aligned}
&-197x_1-\cdots-197x_4+924x_5-197x_6-\cdots-197x_{13}\\
&+924x_{14}-197x_{15}-\cdots-197x_{19}\\
&+924x_{20}-197x_{21}-\cdots-197x_{50}+924x_{51}-197x_{52}\\
&-\cdots-197x_{68}+8912x_{88}=0
\end{aligned}$$

$$\begin{aligned}
&-132x_1-\cdots-132x_5+989x_6-132x_7-\\
&\cdots-132x_{19}+989x_{20}-132x_{21}-\cdots-132x_{51}\\
&+989x_{52}-132x_{53}-\cdots-132x_{68}+5613x_{89}=0
\end{aligned}$$

$$\begin{aligned}
&905x_1-216x_2-\cdots-216x_6+905x_7\\
&-216x_8-\cdots-216x_{14}+905x_{15}-216x_{15}\\
&-216x_{16}-\cdots\\
&-216x_{20}+905x_{21}-216x_{22}-\cdots\\
&-2167x_{52}+905x_{53}-216x_{54}-\cdots\\
&-216x_{68}+9021x_{90}=0
\end{aligned}$$

$$\begin{aligned}
&-241x_1+880x_2-241x_3-\cdots-241x_7\\
&+880x_8-241x_9-\cdots-241x_{14}+880x_{15}\\
&-241x_{16}-\cdots\\
&-241x_{21}+880x_{22}-241x_{23}-\cdots-2417x_{53}\\
&+880x_{54}-241x_{55}-\cdots-241x_{68}+10783x_{91}=0
\end{aligned}$$

$$\begin{aligned}
&-210x_1-210x_2+911x_3-210x_4-\\
&\cdots-210x_{15}+911x_{16}-210x_{17}-\cdots\\
&-210x_{22}\\
&+911x_{23}-210x_{24}-\cdots-210x_{54}\\
&+911x_{55}-210x_{56}-\cdots-210x_{68}\\
&+9796x_{92}=0
\end{aligned}$$

$$\begin{aligned}
&-197x_1-\cdots-197x_4+924x_5-197x_6\\
&-\cdots-197x_{13}+924x_{14}-197x_{15}-\cdots\\
&-197x_{19}\\
&+924x_{20}-197x_{21}-\cdots-197x_{55}+924x_{56}\\
&-197x_{57}-\cdots-197x_{68}+8912x_{93}=0
\end{aligned}$$

$$\begin{aligned}
&-126x_1-\cdots-126x_{13}+995x_{14}-126x_{15}\\
&-\cdots-126x_{30}+995x_{31}-126x_{32}-\cdots\\
&-126x_{56}+995x_{57}-126x_{58}-\cdots\\
&-126x_{68}+5205x_{94}=0
\end{aligned}$$

$$\begin{aligned}
&-217x_1-\cdots-217x_{13}+905x_{14}-217x_{15}\\
&-\cdots-217x_{29}+905x_{30}-217x_{31}-\cdots\\
&-217x_{57}+905x_{58}-217x_{59}-\cdots-217x_{68}\\
&+11393x_{95}=0
\end{aligned}$$

$$\begin{aligned}
&-249x_1-\cdots-249x_4+872x_5-249x_6\\
&-\cdots-249x_{15}+872x_{16}-249x_{17}\\
&-\cdots-249x_{23}\\
&+872x_{24}-249x_{25}-\cdots-249x_{58}+872x_{59}\\
&-249x_{60}-\cdots-249x_{68}+12448x_{96}=0
\end{aligned}$$

$$\begin{aligned}
&-186x_1-\cdots-186x_4+935x_5-186x_6\\
&-\cdots-186x_{17}+935x_{18}-186x_{19}\\
&-\cdots-186x_{28}\\
&+935x_{29}-186x_{30}-\cdots-186x_{59}+935x_{60}\\
&-186x_{61}-\cdots-186x_{68}+8164x_{97}=0
\end{aligned}$$

$$-281x_1 - \cdots - 281x_9 + 840x_{10} - 281x_{11} - \cdots - 281x_{22} + 840x_{23} - 281x_{24} - \cdots - 281x_{26} + 840x_{27} - 281x_{28} - \cdots - 281x_{60} + 840x_{61} - 281x_{62} - \cdots - 281x_{68} + 14624x_{98} = 0$$

$$-186x_1 - \cdots - 186x_3 + 935x_4 - 186x_5 - \cdots - 186x_{12} + 935x_{13} - 186x_{14} - \cdots - 186x_{18} + 935x_{19} - 186x_{20} - \cdots - 186x_{61} + 935x_{62} - 186x_{63} - \cdots - 186x_{68} + 8164x_{99} = 0$$

$$-326x_1 - \cdots - 326x_7 + 795x_8 - 326x_9 - \cdots - 326x_{11} + 795x_{12} - 326x_{13} + 795x_{14} - 326x_{15} - \cdots - 326x_{24} + 795x_{25} - 326x_{26} - \cdots - 326x_{62} + 795x_{63} - 326x_{64} - \cdots - 326x_{68} + 16563x_{100} = 0$$

$$-301x_1 - \cdots - 301x_8 + 820x_9 - 301x_{10} - \cdots - 301x_{13} + 820x_{14} - 301x_{15} - \cdots - 301x_{24} + 820x_{25} - 301x_{26} - \cdots - 301x_{63} + 820x_{64} - 301x_{65} - \cdots - 281x_{68} + 15984x_{101} = 0$$

$$-292x_1 - \cdots - 292x_{13} + 829x_{14} - 292x_{15} - \cdots - 292x_{27} + 829x_{28} - 292x_{29} - \cdots - 292x_{64} + 829x_{65} - 292x_{66} - \cdots - 292x_{68} + 16493x_{102} = 0$$

$$-280x_1 - \cdots - 280x_3 + 841x_4 - 280x_5 - \cdots - 280x_{14} + 841x_{15} - 280x_{16} - \cdots - 280x_{20} + 841x_{21} - 280x_{22} + 841x_{23} - 280x_{24} - 280x_{25} + 841x_{26} - 280x_{27} - \cdots - 280x_{65} + 841x_{66} - 280x_{67} - 280x_{68} + 12314x_{103} = 0$$

$$-154x_1 - \cdots - 154x_{12} + 967x_{13} - 154x_{14} - \cdots - 154x_{28} + 967x_{29} - 154x_{30} - \cdots - 154x_{66} + 967x_{67} - 154x_{68} + 7109x_{104} = 0$$

Transforming the Objective Function,

$$y_i = \frac{x_i}{U_{val}}$$

But U_{val} is 1121. Thus,

$$y_i = \frac{x_i}{1121} \quad ; \quad x_i = 1121y_i$$

Hence, the original objective function becomes

$$1121\,y_1 + 1121\,y_2 + 1121\,y_3 + \cdots 1121\,y_{31} \quad (3.77)$$

Equations (3.41) to (3.76) are the transformed constraints. Penalizing the artificial variables y_{69} to y_{104} in the transformed constraints using the big-M method gives us the transformed problem as:

minimize z

$$= 1121y_1 + 1121y_2 + 1121y_3 + \cdots 1121y_{31} + 23000y_{69} + 23000y_{70} + \cdots + 23000y_{104}$$

subject to

$$993y_1 - 128y_2 - \cdots - 128y_{14} + 993y_{15} - 128y_{16} - \cdots - 128y_{31} + 993y_{32} - 128y_{33} - \cdots - 128\,y_{68} + 5341\,y_{69} = 0$$

$$940y_1 - 181y_2 - \cdots - 181y_6 + 940y_7 - 181y_8 - \cdots - 181y_{15} + 940y_{16} - 181y_{17} - \cdots - 181y_{32} + 940y_{33} - 181y_{34} - \cdots - 181y_{68} + 7824y_{70} = 0$$

$$876y_1 - 245y_2 - \cdots - 245y_6 + 876y_7 + 876y_8 - 245y_9 - \cdots - 245y_{16} + 876y_{17} - 245y_{18} - \cdots - 245y_{33} + 876y_{34} - 245y_{35} - \cdots - 245y_{68} + 11055y_{71} = 0$$

$$841y_1 - 280y_2 - \cdots - 280y_6 + 841y_7 + 841y_8 - 280y_9 - \cdots - 280y_{10} + 841y_{11} - 280y_{12} - \cdots - 280y_{16} + 841y_{17} - 280y_{18} - \cdots - 280y_{34} + 841y_{35} - 280y_{36} - \cdots - 280y_{68} + 12025y_{72} = 0$$

$$737y_1 - 384y_2 - \cdots - 384y_6 + 737y_7 + 737y_8 - 384y_9 - \cdots - 384y_{10} + 737y_{11} - 384y_{12} + 737y_{13} - 384y_{14} - \cdots - 384y_{18} + 737y_{19} - 384y_{20} - \cdots - 384y_{35} + 737y_{36} - 384y_{37} - \cdots - 384y_{68} + 18265\,y_{73} = 0$$

$$\begin{gathered}676y_1-445y_2-\cdots-445y_6+676y_7+676y_8\\ -445y_9-445y_{10}+676y_{11}-445y_{12}+676y_{13}+\\ 676y_{14}-445y_{15}-\cdots-445y_{36}+676y_{37}\\ -445y_{38}-\cdots-445y_{68}+22412y_{74}=0\end{gathered}$$

$$\begin{gathered}-146y_1+975y_2-146y_3-\cdots-146y_{15}\\ +975y_{16}-146y_{17}-\cdots-146y_{19}+975y_{20}\\ -146y_{21}\\ -\cdots-146y_{37}+975y_{38}-146y_{39}-\cdots-\\ 146y_{68}+5444y_{75}=0\end{gathered}$$

$$\begin{gathered}-210y_1+911y_2-210y_3-\cdots-210y_7+911y_8\\ -210y_9-\cdots-210y_{16}+911y_{17}-210y_{18}-\cdots\\ -210y_{38}+911y_{39}-210y_{40}-\cdots-210y_{68}+9796y_{76}=0\end{gathered}$$

$$\begin{gathered}-245y_1+876y_2-245y_3-\cdots-245y_7+876y_8\\ -245y_9-245y_{10}+876y_{11}-245y_{12}-\cdots\\ -245y_{17}+876y_{18}-245y_{19}-\cdots-245y_{39}\\ +876y_{40}-245y_{41}-\cdots-245y_{68}+11055y_{77}\\ =0\end{gathered}$$

$$\begin{gathered}-349y_1+772y_2-349y_3-\cdots-349y_7\\ +772y_8-349y_9-349y_{10}+772y_{11}\\ -349y_{12}+772y_{13}-\\ 349y_{14}-\cdots-349y_{18}+772y_{19}-349y_{20}\\ -\cdots-349y_{40}+772y_{41}-349y_{42}-\cdots\\ -349y_{68}+\\ 17006\,y_{78}=0\end{gathered}$$

$$\begin{gathered}-410y_1+711y_2-410y_3-\cdots-410y_7\\ +711y_8-410y_9-410y_{10}+711y_{11}-\\ 410y_{12}+711y_{13}+\\ 711y_{14}-410y_{15}-\cdots-410y_{19}+711y_{20}-\\ 410y_{21}-\cdots-410y_{41}+711y_{42}-410y_{43}-\cdots-\\ 410\,y_{68}+20033\,y_{79}=0\end{gathered}$$

$$\begin{gathered}-168y_1-168y_2+953y_3-168y_4-\cdots\\ -168y_{16}+953y_{17}-168y_{18}-\cdots-168y_{42}\\ +953y_{43}-\\ 168\,y_{44}-\cdots-168\,y_{68}+8161\,y_{80}=0\end{gathered}$$

$$\begin{gathered}-203y_1-203y_2+918y_3-203y_4-\cdots\\ -203y_{10}+918y_{11}-203y_{12}-203y_{17}\\ +918y_{18}-203y_{19}\\ -\cdots-203y_{43}+918y_{44}-203y_{45}-\cdots\\ -203y_{68}+9320y_{81}=0\end{gathered}$$

$$\begin{gathered}-307y_1-307y_2+814y_3-307y_4-\cdots-307y_{10}\\ +814y_{11}-307y_{12}+814y_{13}-+307y_{14}-\cdots\\ -307y_{18}+814y_{19}-307y_{20}-\cdots-307y_{44}\\ +814y_{45}-307y_{46}-\cdots-307y_{68}+15271y_{82}=0\end{gathered}$$

$$\begin{gathered}-368y_1-368y_2+753y_3-368y_4-\cdots\\ -349y_{10}+753y_{10}+753y_{11}-368y_{12}\\ +753y_{13}+\\ 753y_{14}-368y_{15}-\cdots-368y_{19}+753y_{20}\\ -368y_{21}-\cdots-368y_{45}+753y_{46}-368y_{47}-\cdots\\ -368\,y_{68}+18298\,y_{83}=0\end{gathered}$$

$$\begin{gathered}-82y_1-82y_2-82y_3+1039y_4-82y_5\\ -\cdots-82y_{17}+1039y_{18}-82y_{19}-\cdots-82y_{46}\\ +1039y_{47}\\ -82\,y_{48}-\cdots-82\,y_{68}+2213\,y_{84}=0\end{gathered}$$

$$\begin{gathered}-186y_1-186y_2-186y_3+935y_4-186y_5\\ -\cdots-186y_{12}+935y_{13}-186y_{14}-\cdots-186y_{19}+\\ 935y_{20}-186y_{21}-\cdots-186y_{47}+935y_{48}\\ -186y_{49}-\cdots-186y_{68}+7974y_{85}=0\end{gathered}$$

$$\begin{gathered}-247y_1-247y_2-247y_3+874y_4-347y_5\\ -\cdots-247y_{12}+874y_{13}+874y_{14}-247y_{15}-\cdots\\ -247y_{19}+874y_{20}-247y_{21}-\cdots-247y_{48}\\ +874y_{49}-247y_{50}-\cdots-247y_{68}+11438y_{86}=0\end{gathered}$$

$$\begin{gathered}-136y_1-\cdots-136y_4+985y_5-136y_6\\ -\cdots-136y_{18}+985y_{19}-136y_{20}-\cdots\\ -136y_{49}+\\ 985\,y_{50}-136\,y_{51}-\cdots-136\,y_{68}+5885\,y_{87}=0\end{gathered}$$

$$\begin{gathered}-197y_1-\cdots-197y_4+924y_5-197y_6-\cdots\\ -197y_{13}+924y_{14}-197y_{15}-\cdots-197y_{19}\\ +924y_{20}-197y_{21}-\cdots-197y_{50}+924y_{51}\\ -197y_{52}-\cdots-197y_{68}+8912y_{88}=0\end{gathered}$$

$$-132y_1 - \cdots -132y_5 + 989y_6 - 132y_7 -$$
$$\cdots -132y_{19} + 989y_{20} - 132y_{21} - \cdots -132y_{51}$$
$$+989\,y_{52} - 132\,y_{53} - \cdots - 132\,y_{68} + 5613\,y_{89} = 0$$

$$905y_1 - 216y_2 - \cdots - 216y_6 + 905y_7$$
$$-216y_8 - \cdots - 216y_{14} + 905y_{15} - 216y_{15}$$
$$-216y_{16} - \cdots$$
$$-216y_{20} + 905y_{21} - 216y_{22} - \cdots - 2167y_{52}$$
$$+905y_{53} - 216y_{54} - \cdots - 216y_{68} + 9021y_{90} = 0$$

$$-241y_1 + 880y_2 - 241y_3 - \cdots - 241y_7$$
$$+880y_8 - 241y_9 - \cdots - 241y_{14} + 880y_{15}$$
$$-241y_{16} - \cdots$$
$$-241y_{21} + 880y_{22} - 241y_{23} - \cdots - 2417y_{53}$$
$$+880y_{54} - 241y_{55} - \cdots - 241y_{68} + 10783y_{91} = 0$$

$$-210y_1 - 210y_2 + 911y_3 - 210y_4 - \cdots$$
$$-210y_{15} + 911y_{16} - 210y_{17} - \cdots - 210y_{22}$$
$$+911y_{23} - 210y_{24} - \cdots - 210y_{54} + 911y_{55}$$
$$-210y_{56} - \cdots - 210y_{68} + 9796y_{92} = 0$$

$$-197y_1 - \cdots -197y_4 + 924y_5 - 197y_6 - \cdots$$
$$-197y_{13} + 924y_{14} - 197y_{15} - \cdots -197y_{19}$$
$$+924y_{20} - 197y_{21} - \cdots -197y_{55} + 924y_{56}$$
$$-197y_{57} - \cdots -197y_{68} + 8912y_{93} = 0$$

$$-126y_1 - \cdots -126y_{13} + 995y_{14} - 126y_{15}$$
$$-\cdots -126y_{30} + 995y_{31} - 126y_{32} - \cdots$$
$$-126y_{56} + 995y_{57} - 126y_{58} - \cdots -126y_{68}$$
$$+5205y_{94} = 0$$

$$-217y_1 - \cdots - 217y_{13} + 905y_{14} - 217y_{15}$$
$$-\cdots - 217y_{29} + 905y_{30} - 217y_{31} - \cdots$$
$$-217y_{57} + 905y_{58} - 217y_{59} - \cdots - 217y_{68}$$
$$+11390y_{95} = 0$$

$$-249y_1 - \cdots - 249y_4 + 872y_5 - 249y_6$$
$$-\cdots - 249y_{15} + 872y_{16} - 249y_{17} - \cdots - 249y_{23}$$
$$+872y_{24} - 249y_{25} - \cdots - 249y_{58} + 872y_{59}$$
$$-249y_{60} - \cdots - 249y_{68} + 12448y_{96} = 0$$

$$-186y_1 - \cdots -186y_4 + 935y_5 - 186y_6 - \cdots$$
$$-186y_{17} + 935y_{18} - 186y_{19} - \cdots -186y_{28}$$
$$+935y_{29} - 186y_{30} - \cdots -186y_{59} + 935y_{60}$$
$$-186y_{61} - \cdots -186y_{68} + 8164y_{97} = 0$$

$$-281y_1 - \cdots - 281y_9 + 840y_{10} - 281y_{11}$$
$$-\cdots - 281y_{22} + 840y_{23} - 281y_{24}$$
$$-\cdots - 281y_{26}$$
$$+840y_{27} - 281y_{28} - \cdots - 281y_{60} + 840y_{61}$$
$$-281y_{62} - \cdots - 281y_{68} + 14624y_{98} = 0$$

$$-186y_1 - \cdots -186y_3 + 935y_4 - 186y_5 - \cdots$$
$$-186y_{12} + 935y_{13} - 186y_{14} - \cdots -186y_{18}$$
$$+935y_{19} - 186y_{20} - \cdots -186y_{61} + 935y_{62}$$
$$-186y_{63} - \cdots -186y_{68} + 8164y_{99} = 0$$

$$-326y_1 - \cdots - 326y_7 + 795y_8 - 326y_9$$
$$-\cdots - 326y_{11} + 795y_{12} - 326y_{13} + 795y_{14}$$
$$-326y_{15} - \cdots$$
$$-326y_{24} + 795y_{25} - 326y_{26} - \cdots$$
$$-326y_{62} + 795y_{63} - 326y_{64} - \cdots - 326y_{68}$$
$$+16563y_{100} = 0$$

$$-301y_1 - \cdots - 301y_8 + 820y_9 - 301y_{10} - \cdots$$
$$-301y_{13} + 820y_{14} - 301y_{15} - \cdots - 301y_{24}$$
$$+820y_{25} - 301y_{26} - \cdots - 301y_{63} + 820y_{64}$$
$$-301y_{65} - \cdots - 281y_{68} + 15984y_{101} = 0$$

$$-292y_1 - \cdots - 292y_{13} + 829y_{14} - 292y_{15}$$
$$-\cdots - 292y_{27} + 829y_{28} - 292y_{29} - \cdots$$
$$-292y_{64} + 829y_{65} - 292y_{66} - \cdots - 292y_{68}$$
$$+16493y_{102} = 0$$

$$-280y_1 - \cdots - 280y_3 + 841y_4 - 280y_5 -$$
$$\cdots - 280y_{14} + 841y_{15} - 280y_{16} - \cdots - 280y_{20}$$
$$+841y_{21} - 280y_{22} + 841y_{23} - 280y_{24} - 280y_{25}$$
$$+841y_{26} - 280y_{27} - \cdots - 280y_{65} + 841y_{66}$$
$$-280\,y_{67} - 280\,y_{68} + 12314\,y_{103} = 0$$

$$-154y_1 - \cdots -154y_{12} + 967y_{13} - 154y_{14}$$
$$-\cdots -154y_{28} + 967y_{29} - 154y_{30} - \cdots$$
$$-154\,y_{66} + 967\,y_{67} - 154\,y_{68} + 7109\,y_{104} = 0$$

5. Tora Result

Using TORA (an operations Research Software), the following result was obtained:

Objective value = **2296.72**

$$y_{variables} = \begin{cases} 1 \ for \ y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, \\ y_{10}, y_{11}, y_{12}, y_{13}, y_{14}, y_{15}, y_{16}, \\ \quad y_{17}, y_{18}, y_{19}, y_{20}, y_{21}, y_{22}, y_{23}, y_{24}, y_{25}, y_{26}, y_{27}, y_{28}, y_{29}, y_{30}, \\ \quad y_{31}, y_{32}, y_{33}, y_{34}, y_{35}, y_{36}, \\ \\ 0 \ otherwise \end{cases}$$

6. Conclusion

The application of NyorRauf Transformation Algorithm to flight crew scheduling as demonstrated in this work has performed well and yielded a solution that is implementable by airlines. Generally speaking, this transformation algorithm is efficient in the Karmarkar Interior Point scheme for solving LPs.

References

[1] G. Balaji and L. J. Ellis, *Airline crew scheduling: state-of-the-art*. Annals of Operations Research, 140 (1) (2005), 305-337.

[2] http://www.nigeria.to/airlines/IRS/IRS.php5.

[3] L. H. Karla and P. Manfred, *Solving airline crew scheduling problems by branch-and-cut*, Management Science 39 (6) (1999), 657-682.

[4] A. T. Michael, *Airline Crew Scheduling*, (1996). Retrieved from http://mat.gsia.cmu.edu/ classes /mstc/ decomp/ node5.html on May, 2015.

[5] J. O. Omolehin, K. Rauf, N. Nyor and A. A. Owolabi, *Optimal Characterization of the Transformation procedure of Karmarkar Algorithm*, Global Journal of Mathematics, 2 (2) (2015), 128-137. www.gpcpublishing.com.

[6] H. A. Taha(2010), *Operations research - An Introduction*. Prentice-hall, Inc. London.

[7] V. H. Tran (2013). *Airline Crew Scheduling Problem*. http://www.cse.hcmut.edu.vn/hoai