

Techniques of Integration: An Alternative Tool for Solving Problems of Financial Transactions Concerning Simple Interest

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***Abstract:** Money has time value. The value of Tk today is more worthy than the value of Tk 1 tomorrow as today's money can be invested to earn positive returns in future. This economic principle recognizes that the value of money is affected with the passage of time. Problems of financial transactions concerning simple interest are usually solved by algebraic formula. This paper attempts to solve such problems using techniques of integration and makes a comparison with the results obtained by the classical algebraic method and finds no error. The proposed techniques could be implemented in the practical field as it provides error-free result.*

***Key Words:** Integration, financial transactions, simple interest, time value of money.*

1. Introduction

The value of money changes over time due to the current preference of money over future. Several macro-economic variables expedite like inflation, interest rates, exchange rates etc. are the contributing factors for this current preference. Interest rate accommodates most of the macro economic anomalies to define the way of how money is losing its value over the period. Thus, the idea of simple interest is an important concept in finance literature for the people who deal with money. The same amount of money today is more valuable than in the future as the value of money is always decreasing. So, the timing of cash outflows and inflows has important economic consequences in any type of financial decision making process. It may be unanimously said that the financial decision making is based on the core concept of interest with different dimensions. Say, for example, a current cash flow may be required to be converted into the value such cash flow may generate with a future reference of time (future value) and vice versa. In a very simple scenario, this is done by considering simple interest though now a days a good number of financial institutions have started more frequent compounding to attract more investors. Problems concerning the calculation of simple interest and future value are often solved by algebraic formulae. This paper attempts to solve such problems using integration techniques. From this point of view the study has somewhat got diagnostic format and tries to establish a formula related to integration techniques regarding the solutions of the problems of calculating financial transactions concerning simple interest.

2. Objectives

In line with the basic focus of the paper, it seeks to compute the future value (at $t = n$) of a certain sum of money under integration technique. It also shows the computation of present value (at $t=0$) of a future sum of money. And finally, the paper will also be

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helpful to know the rate (simple interest) for which your investment is growing. By using the integration techniques the above mentioned problems are being solved here that is perceived to be the addition of this paper and the motivation point for study. So the prime objective of this study is to develop and propose a methodology that involves alternative formula to solve problems of financial transactions concerning simple interest. Compound interest is not considered here for the sake of simplicity of presentation and to avoid some complexity and confusion that may arise in applying integration to compound interest situation. The result of the integration confirms the accuracy which is shown again re-computing the same thing under traditional algebraic equations.

3. Literature Review

Interest is the price paid for the use of borrowed money or money earned by depositing funds (Arthur & Sheffrin, 2003). Everyone in business becomes involved in transactions where interest rates affect the amount to be paid or received. The value of the money is changed considering that interest rate together with time; that is why it is called Time Value of Money (Gitman, 2004). The calculation has been simplified by the time through different formulations in mathematics and economics. The formulas, for example, PV, FV, RATE, NPER, PMT, are programmed into most financial calculators and several spreadsheet functions (Hovey, 2005). In these formulations, interest rate is used to mean discounting which refers to the common issue that the present value a future sum will be less than its explicit value. Numerous experimental studies have lent credence to the hyperbolic discounting model, which posits that individuals are impatient about immediate or near-term consumption decisions, but are relatively more patient over future consumption (Klemick & Yesuf, 2008). Individual rates of time preference have important policy implications in developing countries, from savings to investment to conservation decisions.

Pender (1996) conducted an experiment resulting that discount rates are significantly higher than prevailing interest rates, consistent with the hypothesis of constrained credit access among the study population. Discount rates increased with the time horizon (up to one year), indicating hyperbolic preferences. Time discounting experiments in the Bolivian rainforest used candy as the form of payment, classifying participants as impatient if they preferred one candy early in the interview instead of multiple candies at the end of the interview (Godoy et al., 1998). Thus the common understanding is that the present is more preferred and all future cash flows are discounted at a certain rate to take decision with economic justification. Modern business relies hundred percent on the time value of money and no one can move a step neglecting the changing rate. Next section presents the application of algebraic formula and integration using simple interest rate.

3.1 Simple Interest and the Future Value

Interest rates are generally quoted in percentage form and, for use in calculations, must be converted to the equivalent decimal value by dividing the percentage by 100. For example, $I = 8.5\% = 0.085$

Unless otherwise stated, a quoted rate is a rate per year. Thus Tk 1 at 8 percent means that interest of Tk 0.08 will be earned in a year, and Tk 100 at this rate provides Tk 8

(100×0.08) of interest in one year. Interest on Tk 100 at 8 percent for 9 month is interest for 9/12 years. That is, the amount of interest will be Tk 6 as computed below:

Interest	=	Principal	×	Rate	×	Time (in years)
I	=	P	×	i	×	n
Tk 6	=	100	×	0.08	×	9/12

Thus, the algebraic formula for simple interest is: $I = Pin$ (Prichett & Saber, 2004)

Now the interest is added back to the principal and the sum is called the future value (F). Thus, the future value under algebraic formulation of future value is given as,

$$F = P + I$$

$$= P + Pin = P(1 + in)$$

The algebraic formula for calculating future value assuming simple interest is $F = P(1 + in)$ (Prichett & Saber, 2004)

‘A dollar in hand today is worth more than a dollar to be received in the future because, if you had it now, you could invest it, earn interest, and end up with more than one dollar in the future. The process of going from today’s values, or present values (PVs), to future values (FVs) is called compounding’ (Brigham et al., 2001) that is simply opposite to discounting as said earlier. To illustrate, let us assume that we have deposited Tk 100 in a bank that pays 5 percent interest per year. How much we have at the end of one year may be computed by using the algebraic equation for calculating future value as presented below:

$$F = P(1 + in) = Tk 100(1 + 0.05) = Tk 105$$

3.2 Differential Equations and Techniques of Integration

The methodology of differential equations which are going to introduce is widely applied in solving rate problems. A differential equation is one that contains a differential or a derivative.

If $y = f(x)$ is a function, where for any value of x , we have a unique value of y , then

$\frac{dy}{dx} = f'(x)$, and $\frac{dy}{dx}$ is not a fraction with dy as numerator and dx as denominator, but a

single symbol meaning the same as $f'(x)$. However, when we write $\int f'(x) dx$, dx appears alone. To achieve consistency, we define dy to be $f'(x) dx$. That is, if

$\frac{dy}{dx} = f'(x)$, then $dy = f'(x) dx$.

Thus for example, if $\frac{dy}{dx} = 2x$, then $dy = 2x dx$ is a differential equation that is solved by integrating the left with respect to y and the right with respect to x . That is,

$$\int dy = \int 2x dx$$

$$\Rightarrow y + c_1 = x^2 + c_2 \quad \Rightarrow y = x^2 + c_2 - c_1$$

in as much as c_2 and c_1 are arbitrary constants, $c_2 - c_1$ is another arbitrary constant, which we may call C. We shall follow the practice of writing only one constant for one integration step, so the solution is $y = x^2 + C$.

A solution such as this, which contains the constant of integration, is called the general solution of the differential equation, and each value for C yields a particular solution of the differential equation. Thus, $y = x^2 + 10$ is one particular solution, and there are infinitely many particular solutions (Prichett & Saber, 2004).

Differential equation in explicit form:

$$\frac{dy}{dx} = \frac{y}{x}; x \neq 0$$

$$\Rightarrow dy = \frac{y}{x} dx \quad \Rightarrow \frac{dy}{y} = \frac{dx}{x}$$

Integrating the equation, we have -

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\Rightarrow \ln y = \ln x + C.$$

This solves the equation in terms of $\ln y$, but not explicitly in terms of y . Further, we write,

$$\ln y - \ln x = C$$

$$\Rightarrow \ln y/x = C.$$

We next apply the definition of the natural logarithm and write $y/x = e^C$ in as much as e is a constant and C is a constant, e^C is also a constant, which may be denoted by K . Hence we have $y/x = K$ or $y = Kx$ as the explicit general solution. This result is a straight line of slope K that passes through the origin, and a line through the origin is the only function whose slope can be found by dividing the y -coordinate of any one of its points by the x -coordinate; that is, slope is equal to $\frac{dy}{dx} = \frac{y}{x}$, which is the initial differential equation.

Observe, however, that x cannot be equal to zero, so the differential equation, and therefore the general solution is not defined for $x = 0$ (Prichett & Saber, 2004).

4. Establish the Formula

Step 1: Determine the slope of the formula

Let the present value = P, Interest rate = i , Time = t years and Future value = F

To compute the future value (F), we have gone for a differential equation. To this end, the rate of increase (i) has Taka/year as its unit of measurement. If this rate continued for dt years, where dt therefore has years as its unit of measurement, then, i(Taka/year).dt years = i dt Taka. So i dt Taka represents the change in the amount in time dt years. Calling this change in amount in to the account is dy, and we write the differential equation as $\frac{dy}{dt}=i$ (Constant), which is the slope of a straight line. The equation finally implies that **dy = i dt**.

Step 2: General solution of the differential equation

Now to derive the general solution of the differential equation as found in step 1, we have to follow the following steps:

$$dy = i dt$$

$$dy = (\text{Interest rate}). (\text{Present value}). dt$$

$\Rightarrow dy = 0.0i(P).dt$, where dy denotes change of amount after t years, P represents present value and i refers to interest rate which is 0.0i.

$$\Rightarrow \int dy = \int 0.0i(P).dt$$

$$\Rightarrow y = 0.0i(P)t + C \dots\dots\dots(1)$$

As (Interest rate).(Present Value) is a constant term, it will not effect the integration. The mode of thinking is just illustrated precisely only when the rate function is linear. This is the general solution.

Step 3: Particular solution of the differential equation

To derive the particular solution, the following steps may be illustrative:

Let the present value of the amount be y = P at t = 0 years.

$$\therefore P = 0.0i(P).0 + C$$

$$\Rightarrow P = C$$

Hence the particular solution is;

$$\therefore F = y(t) = 0.0i(P).t + P$$

$$\Rightarrow F = 0.0i(P).t + P \dots\dots\dots(2)$$

So the steps that are required to calculate the future value are:

Identify the slope that is the interest rate

Set dy = (Interest rate) (Present value) dt

By integrating both sides we have -

$$y = (\text{Interest rate}) (\text{Present value}) t + C$$

Compute the value of C when t = 0

Determine the formula,

$$Y = (\text{Interest rate}) (\text{Present value}) t + P$$

(Source: Shil, N.C, Parvez, M. 2010. "Life Cycle Costing: Techniques and Applications, Choosing the Most Economic Project" VDM Verlag Dr. Muller, Germany, p.16-17)

Illustration 4.1: Find the future value if Tk 10,000 is invested at 10 percent for 3 months.

Given: Present Amount (P) = Tk 10, 000

Interest rate (I) = 10 percent

Time (t) = 3 months ($\frac{1}{4}$ years)

According to our rule we get, slope = interest rate = 10% = 0.10/ year

Hence, $dy = (\text{Interest rate}) (\text{Present value}) dt$ implies $dy = (0.10) (10,000) dt$

$$\int dy = \int 0.10(10000).dt \Rightarrow y = 1000t + C$$

By applying the initial condition, we have $y = 10000$, when $t = 0$

implies $C = 10000$

Hence, we have $F = y (t) = Tk 1000 t + \10000

When $t = \frac{1}{4}$ years, $F = Tk 1000 (\frac{1}{4}) + Tk 10000 = Tk 10250$ (F = Future Amount)

Using algebraic formula we get,

$$\text{Future Amount} = P (1+in) = Tk 10000(1 + 0.10/4) = Tk 10250$$

Here error in amount is 0.

Illustration 4.2: Find the present value of Tk 2000 at 9 percent due 6 months from now.

Given: Future Amount (F) = Tk 2000

Interest rate (i) = 9%

Time (t) = 6 months = $\frac{1}{2}$ years

Present Amount (P) = ?

According to our rule we get, slope = interest rate = 9% = 0.09/ year

We have

$$dy = (\text{Interest rate}) (\text{Present value}) dt$$

$$dy = (0.09) (P) dt$$

$$\Rightarrow \int dy = \int (0.09) (P) .dt$$

$$\Rightarrow y = 0.09Pt + C$$

By applying the initial condition, we have $y = P$, when $t = 0$

that implies $C = P$

Hence, we have $F = y(t) = .09Pt + P$

$$\Rightarrow \text{Tk } 2000 = (0.09)1/2 P + P$$

$$\Rightarrow P = \text{Tk } 1913.87$$

Using algebraic formula we get,

$$\text{Future Amount} = P(1 + in)$$

$$\Rightarrow \text{Tk } 2000 = P(1 + 0.09 \times 1/2)$$

$$\Rightarrow P = \text{Tk } 1913.87$$

Here error in amount is 0.

Illustration 4.3: At what rate of interest will an investment of Tk 1500 for 5 years grow to the amount of Tk 3000?

Given: Future Amount (F) = Tk 3000

Present Amount (P) = Tk 1500

Time (t) = 5 years

Interest rate (i) = ?

We have,

$$dy = (\text{Interest rate})(\text{Present value}) dt$$

$$dy = i(P)dt \quad \Rightarrow \int dy = \int i(P).dt$$

$$\Rightarrow y = iPt + C \quad \Rightarrow 3000 = i(1500)t + C$$

$$\Rightarrow 3000 = i(1500)t + 1500 \quad (\text{when } t = 0, y = p = 1500)$$

$$\Rightarrow 1500 = i(1500)(5)$$

$$\Rightarrow i = 1/5 = 0.20$$

$$\Rightarrow i = 20\%$$

Using algebraic formula we get

$$\text{Future Amount} = P(1 + in)$$

$$\Rightarrow 1 + 5i = 3000/1500$$

$$\Rightarrow 5i = 1$$

$$\Rightarrow i = 20\%$$

Here error in interest rate is 0.

Illustration 4.4: Someone placed Tk 100 in an employees' savings account that pays 8 percent simple interest. How long will it be, until the investment amounts to Tk 150?

Given: Future Amount (F) = Tk 150

Present Amount (P) = Tk 100

Interest rate (i) = 8%

Time (t) = ?

According to our rule we get, slope = interest rate = 8% = 0.08/ year

$$dy = (0.08)(100) dt$$

$$\Rightarrow \int dy = \int (0.08)(100).dt \Rightarrow y = 8t + C$$

By applying the initial condition, we have $y = \text{Tk } 100$, when $t = 0$ that implies $C = \text{Tk } 100$

Hence, we have $F = y(t) = 8t + 100$

$$\Rightarrow 150 = 8t + 100$$

$$\Rightarrow 8t = 50$$

$$\Rightarrow t = 6.25 \text{ years}$$

Using algebraic formula we get,

$$\text{Future Amount} = P(1 + in)$$

$$\Rightarrow 150 = 100(1 + 0.08n)$$

$$\Rightarrow n = 6.25 \text{ years}$$

Here error in time is 0.

5. Results and Discussions

The above formula used to calculate the future value of simple interest is newly derived. The formula is a new one and established by using basic techniques of Integration. This paper tries to solve the problems relating to financial transactions concerning simple interest with more advanced mathematical formula instead of classical algebraic formula and provides exact result. As the error is zero in every case, these equations may be used for doing the same financial functions. However, the derived formula is only used to calculate the future value, present value, interest rate and periods considering only the simple interest. Compound interest is not taken into consideration. This may be considered as a limitation of the study and left for the further researchers who like to extend their researches in this area.

7. Conclusion

The paper is methodology based and the author has tried to establish a new methodology regarding solving problems of future value formula for simple interest. As the formula provides exact result, the formula could be implemented in practical problems.

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