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ON NYORRAUF TRANSFORMATION ALGORITHM TO FLIGHT CREW SCHEDULING PROBLEMS

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Abstract: In this work, we applied NyorRauf Transformation Algorithm to a flight crew scheduling problem to assess its impact on crew scheduling models. The result yielded a solution that is implementable by airlines. It was concluded that, generally speaking, the transformation algorithm is efficient in the Karmarkar Interior Point scheme for solving Linear Programming Problems.

Keywords: Flight Crew, Flight Crew Scheduling Problem, NyorRauf Transformation Algorithm, Karmarkar Interior Point scheme

1. Introduction

According to Balaji and Ellis [1], the airline industry is characterized by some of the largest scheduling problems of any industry. The problem of crew scheduling involves the optimal allocation of crews to flights. Balaji and Ellis [1] argued that, over the last two decades the magnitude and complexity of crew scheduling problems have grown enormously and airlines are depending more and more on automated mathematical procedures as a practical necessity. Michael [4] also reiterated that, One major problem for airlines is the scheduling of their flight crews.

The airline industry is severely unionized and there are stringent limitations on how to use a crew. For example, there are rules on how many hours a crew must be in the air in a day; and there are restrictions on the number of hours a crew can be away from their home base before they must stopover in a hotel. But crew Overheads are the second largest operating expense an airline has (after gasoline). Therefore, there is an opening to work with a hard problem influenced by enormous potential cost savings (Michael [4]).

According to Karla and Manfred [3], the air scheduling problem is one that has been studied almost continually for the past 40 years. It is obvious that, the problem is much more important today since costs for flying personnel of organizations or companies or major government parastatals have so much grown and are second largest cost (next to fuel) of the total operating costs for airlines. As a result of this, even small percentage savings amount to substantial amounts.

NyorRauf Transformation Algorithm is a transformation procedure that converts an (LP) problem from standard maximization form to a Karmarkar form. Karmarkar assumes that the LP is given as:

\[ \text{Minimize} \quad Z = CX \]
\[ \text{Subject to} \quad AX = 0 \]

All the constraints are homogeneous equations except for the constraint

\[ 1X = \sum_{j=1}^{n} x_j = 1, \]

which defines an n-dimensional simplex. The validity of Karmarkar algorithm rests on satisfying two conditions:

1. \( x = \left( \frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n} \right) \) satisfies \( AX = 0 \)
2. \( \min z = 0 \) \hspace{1cm} (Taha [6]).
1.1 NyorRauf Transformation Procedure.

Given the original LP problem:
Maximize $Z = CX$
Subject to $AX \leq b$
$X \geq 0$

NyorRauf has the following steps for the transformation:

**Step 1.** Convert the constraints inequalities of the original LP problem into equations by augmenting slack or surplus variables appropriately as:
\[
\sum_{j=1}^{n+1} X_j \leq U_i; \quad i = 1, 2, \ldots, m
\]
where $n$ is the number of $X$ variables and $m$ is the number of constraints in the original LP.

**Step 2.** Define the augmented equations in step 1 leaving out the coefficient of $X$ but considering all the $X_j$ as:
\[
\sum_{j=1}^{n+1} X_j \leq U_i; \quad i = 1, 2, \ldots, m
\]
where $m$ is the number of constraint equations.

$U$ is the value that is sufficiently large so as not to eliminate any feasible point from the solution space.

**Step 3.** For each standardized constraint $i$, obtain
\[
X_j = L.U.I. \left[ \frac{b_i}{a_j} \right]
\]
if fractional part, by setting other $n$ variables equal to zero.

Where L.U.I. is Least Upper Integers.

**Step 4.** Obtain $U_i = \sum_{j=1}^{n+1} X_j$

**Step 5.** Obtain $U_{val}$ from
\[
U_{val} = G.L.I. \left( \frac{\sum_{i=1}^{m} U_i}{m} \right)
\]
where G.L.I. is Greatest Lower Integers.

**Step 6.** Thus, step 2 becomes
\[
\sum_{j=1}^{n+1} X_j \leq \frac{U_{val}}{U_{val}}
\]

**Step 7.** Augment again the defined constraint in step 6 to have
\[
\sum_{j=1}^{n+1} X_j = \frac{U_{val}}{U_{val}}
\]

**Step 8** Homogenize the Right Hand Side (RHS) of each augmented constraints in step 1 by
\[
\left( \sum_{j=1}^{n+2} a_j X_j \right) U_{val} = b \left( \sum_{j=1}^{n+2} X_j \right)
\]
where $a$ is the coefficient of $X$ in the original LP problem and $b$ is the RHS constraint of the original LP. Thus,
\[
\left( \sum_{j=1}^{n+2} a_j X_j \right) U_{val} - b \left( \sum_{j=1}^{n+2} X_j \right) = 0
\]

**Step 9.** Ensure that the sum of the coefficient of the LHS equals zero by adding artificial variables where necessary.

**Step 10.** Penalize the artificial variables introduced in step 7 in the objective function.

**Step 11.** Define new variables for the objective function as:
\[
Y_i = \frac{X_i}{U_{val}}
\]

**Step 12.** Substitute the new variables as defined in step 11 into the constraints to maintain consistency; hence the transformed karmarkar algorithm (Omolehin et al., [5]).

2. Application

The aim of this paper is to apply NyorRauf Transformation Algorithm in order to assess its impact on flight crew scheduling models. The flight scheduling problem under consideration in section 3 is in a standard LP form. It was a case study of IRS [2] airline, which was formulated first as an integer program then its LP dual was standardized to this state.

2.1 Flight Crew Scheduling Mathematics

We have $n$ flights and assign $m$ crews. One
possibility is to define decision variables
\( y_{ij} \), \( 1 \leq i \leq m, 1 \leq j \leq m \); Where
\[
y_{ij} = \begin{cases} 
1 & \text{flight } j \text{ has a crew } i \\
0 & \text{otherwise}
\end{cases}
\]

To cover flight \( j \), we introduce a constraint of the form:
\[
\sum_{i=1}^{n} y_{ij} \geq 1
\]
for each flight \( j \). A crew pairing problem can be visualized as:

**Given:**

i. A set of scheduled flight;
ii. Safety and working rules;
iii. Minimum or maximum credited hours per crew base.

**Find** least-cost feasible crew pairings

**Subject to**

i. Each flight is covered by an active crew
ii. The maximum or minimum credited hours per crew base is represented (Tran, [7]).

3. The Flight Crew Scheduling Problem

The Flight Crew Scheduling problem given below is in its standard form which NyorRauf can be applied. Slack variables \( x_{12} \sim x_{67} \) are added to the left hand side of the inequalities to obtain the equality constraints.

**Maximize**

\[
Z = x_{1} + x_{7} + x_{8} + x_{17} + x_{14} = 245
\]

**Subject to:**

\[
x_{1} + x_{15} + x_{32} = 128 \quad (3.1)
\]

\[
x_{1} + x_{7} + x_{16} + x_{33} = 181 \quad (3.2)
\]
\[ x_8 + x_{12} + x_{14} + x_{25} + x_{63} = 326 \]  \hspace{1cm} (3.32)  
\[ x_9 + x_{14} + x_{25} + x_{64} = 301 \]  \hspace{1cm} (3.33)  
\[ x_{14} + x_{28} + x_{65} = 292 \]  \hspace{1cm} (3.34)  
\[ x_4 + x_{15} + x_{21} + x_{23} + x_{26} + x_{66} = 280 \]  \hspace{1cm} (3.35)  
\[ x_{13} + x_{29} + x_{67} = 154 \]  \hspace{1cm} (3.36)  
\[ x_j \geq 0; (j = 1 \ldots 31) \]  \hspace{1cm} (3.37)  

4. NyorRauf Transformation of the LP

Using the standardized problem above, we begin the transformation into NyorRauf form as:

Define
\[ x_1 + x_2 + x_3 + \ldots + x_{67} \leq U \]  
From equation (3.1),
\[ x_1 + x_{15} + x_{32} = 128 \]  
If \( x_1 = x_{15} = 0 \), then \( x_{32} = 128 \).
If \( x_1 = x_{32} = 0 \), then \( x_{15} = 128 \).
If \( x_1 = x_{15} = 0 \), then \( x_{32} = 128 \).
Since there is no fractional part,
\[ \sum_{i=1}^{36} U_i \]  
will not be applied here.
\( U_1 = 128 + 128 + 128 = 384 \).
\( U_2 = 128 \times 3 = 384 \).

Since there is no fractional part,
From equation (3.2),
\[ x_1 + x_2 + x_3 + x_4 \leq 181 \]  
If \( x_1 = x_2 = x_3 = 0 \), then \( x_4 = 181 \).
If \( x_1 = x_2 = x_4 = 0 \), then \( x_3 = 181 \).
If \( x_1 = x_3 = x_4 = 0 \), then \( x_2 = 181 \).
Again, since there is no fractional part,
\[ \sum_{i=1}^{36} U_i \]  
will not be applied here.
\( U_2 = 181 \times 4 = 724 \).

Similarly, solving equations (3.3)–(3.36) as above, the following \( U_j \) are obtained:
\[ U_3 = 1225 \]  
\[ U_4 = 1680 \]  
\[ U_5 = 2688 \]  
\[ U_6 = 3115 \]  
\[ U_7 = 584 \]  
\[ U_8 = 840 \]  
\[ U_9 = 1225 \]  
\[ U_{10} = 2094 \]  
\[ U_{11} = 2870 \]  
\[ U_{12} = 504 \]  
\[ U_{13} = 812 \]  
\[ U_{14} = 1535 \]  
\[ U_{15} = 2208 \]  
\[ U_{16} = 246 \]  
\[ U_{17} = 744 \]  
\[ U_{18} = 1235 \]  
\[ U_{19} = 408 \]  
\[ U_{20} = 788 \]  
\[ U_{21} = 396 \]  
\[ U_{22} = 1080 \]  
\[ U_{23} = 1205 \]  
\[ U_{24} = 840 \]  
\[ U_{25} = 788 \]  
\[ U_{26} = 378 \]  
\[ U_{27} = 651 \]  
\[ U_{28} = 996 \]  
\[ U_{29} = 744 \]  
\[ U_{30} = 1124 \]  
\[ U_{31} = 744 \]  
\[ U_{32} = 1304 \]  
\[ U_{33} = 1204 \]  
\[ U_{34} = 876 \]  
\[ U_{35} = 1680 \]  
\[ U_{36} = 462 \]  

But \( U_{val} = \text{G.L.I.} \left( \frac{\sum_{i=1}^{36} U_i}{m} \right) = \text{G.L.I.} \left( \frac{40382}{36} \right) = 1121 \)

Equation (3.38) becomes,
\[ x_1 + x_2 + x_3 + \ldots + x_{67} \leq 1121 \]  \hspace{1cm} (3.39)

Augmenting again,
\[ x_1 + x_2 + x_3 + \ldots + x_{67} + x_{68} = 1121 \]  \hspace{1cm} (3.40)

By multiplying the right hand side of equations (3.1) to (3.36) respectively by \( x_1 + x_2 + x_3 + \ldots + x_{67} + x_{68} \), we obtain the constraints of NyorRauf form as follows:

From equation (3.1)
\[ x_1 + x_{15} + x_{32} = 128 \]  
\[ \cdots \]  
\[ \frac{x_1 + x_2 + x_3 + \ldots + x_{67} + x_{68}}{1121} \]  
\[ = 128x_1 + 128x_2 \]  \hspace{1cm} But
\[ 1121x_1 + 1121x_{15} + 1121x_{32} + 128x_3 + \cdots + 128x_{67} + 128x_{68} + 993x_{15} - 128x_{16} - 993x_{31} - \cdots - 128x_{68} + 993x_{32} - 128x_{33} - 128x_{34} = 0 \]
Thus
\[ -128 x_{10} - 128 x_{11} + 993 x_{12} - 128 x_{13} - - 128 x_{14} + 5341 x_{15} = 0 \]

From equation (3.2)
\[
\begin{aligned}
    x_t + x_s + x_{10} + x_{11} = 181 \left( \frac{x_8 + x_9 + \ldots + x_{21} + x_8}{1121} \right) = 181 x_t \\
    1121 x_t + 1121 x_s + 1121 x_{10} + 1121 x_{11} = 181 x_s + 181 x_{10} + 181 x_{11} + 940 x_{12} \\
    940 x_t - 181 x_s - - 181 x_{10} - 181 x_{11} + 940 x_{12} - - 181 x_{13} = 0 \\
    \text{But} \\
    940 - 905 + 940 - 1448 + 940 - 2896 + 940 - 6335 = -7824 \\
    \text{Thus} \\
    940 x_t - 181 x_s - - 181 x_{10} + 940 x_{11} - 181 x_{12} - - 181 x_{13} - - 181 x_{14} = 0 \\
    \text{Similarly, solving equations (3.3)-(3.6) as above, equations (3.43)-(3.76) are obtained respectively as follows:} \\
    876 x_t - 245 x_s - - 245 x_{10} + 876 x_{11} - 245 x_{12} - - 245 x_{13} - - 245 x_{14} + 876 x_{15} - - 245 x_{16} + 876 x_{17} - 245 x_{18} - - 245 x_{19} + 876 x_{20} - - 245 x_{21} + 876 x_{22} - 245 x_{23} - - 245 x_{24} + 876 x_{25} - - 245 x_{26} + 11055 x_{27} = 0 \\
    841 x_t - 280 x_s - - 280 x_{10} + 841 x_{11} + 841 x_{12} - - 280 x_{13} - - 280 x_{14} + 841 x_{15} - - 280 x_{16} + 841 x_{17} - 280 x_{18} - - 280 x_{19} + 841 x_{20} - - 280 x_{21} + 12311 x_{22} = 0 \\
    737 x_t - 384 x_s - - 384 x_{10} + 737 x_{11} + 737 x_{12} - - 384 x_{13} - - 384 x_{14} + 737 x_{15} - - 384 x_{16} + 737 x_{17} - 384 x_{18} - - 384 x_{19} + 737 x_{20} - - 384 x_{21} + 18265 x_{22} = 0 \\
    676 x_t - 445 x_s - - 445 x_{10} + 676 x_{11} - 445 x_{12} + 676 x_{13} + 676 x_{14} - - 445 x_{15} + 676 x_{16} + 445 x_{17} - - 445 x_{18} + 676 x_{19} - - 445 x_{20} + 676 x_{21} - - 445 x_{22} + 22412 x_{23} = 0 \\
    -146 x_t + 975 x_s - - 146 x_{10} - - 146 x_{11} + 975 x_{12} - - 146 x_{13} - - 146 x_{14} + 975 x_{15} - - 146 x_{16} - - 146 x_{17} + 975 x_{18} - - 146 x_{19} + 975 x_{20} - - 146 x_{21} + 975 x_{22} - - 146 x_{23} + 16x_{24} - 975 x_{25} - - 146 x_{26} - - 146 x_{27} + 975 x_{28} - - 146 x_{29} - - 146 x_{30} + 16x_{31} - - 146 x_{32} + 975 x_{33} - - 146 x_{34} - - 146 x_{35} + 5444 x_{36} = 0 \\
    -210 x_t + 911 x_s - - 210 x_{10} - - 210 x_{11} + 911 x_{12} + 210 x_{13} + 210 x_{14} + 911 x_{15} - - 210 x_{16} + 911 x_{17} - - 210 x_{18} + 976 x_{19} = 0 \\
    -245 x_t + 876 x_s - - 245 x_{10} - - 245 x_{11} + 876 x_{12} - - 245 x_{13} - - 245 x_{14} + 876 x_{15} - - 245 x_{16} + 876 x_{17} - - 245 x_{18} + 876 x_{19} - - 245 x_{20} + 876 x_{21} - - 245 x_{22} + 11055 x_{23} = 0 \\
    -349 x_t + 772 x_s - - 349 x_{10} - - 349 x_{11} - - 349 x_{12} + 772 x_{13} + 349 x_{14} - - 349 x_{15} - - 349 x_{16} - - 349 x_{17} + 772 x_{18} - - 349 x_{19} - - 349 x_{20} + 349 x_{21} - - 349 x_{22} + 772 x_{23} = 0 \\
    + 772 x_{24} - - 349 x_{25} - - 349 x_{26} - - 349 x_{27} + 17006 x_{28} = 0 \\
    (3.42) \\
    -410 x_t + 711 x_s - - 410 x_{10} - - 410 x_{11} + 711 x_{12} - - 410 x_{13} + 711 x_{14} - - 410 x_{15} - - 410 x_{16} + 711 x_{17} + 711 x_{18} + 410 x_{19} - - 410 x_{20} + 410 x_{21} - - 410 x_{22} + 410 x_{23} - - 410 x_{24} + 20033 x_{25} = 0 \\
    -168 x_t + 168 x_s + 935 x_{10} - - 168 x_{11} + 935 x_{12} - - 168 x_{13} + 935 x_{14} - - 168 x_{15} + 935 x_{16} - - 168 x_{17} - - 168 x_{18} + 935 x_{19} + 935 x_{20} - - 168 x_{21} + 168 x_{22} + 168 x_{23} - - 168 x_{24} + 168 x_{25} + 168 x_{26} + 8064 x_{27} = 0 \\
    -203 x_t - - 203 x_s + 918 x_{10} - - 203 x_{11} - - 203 x_{12} + 918 x_{13} - - 203 x_{14} - - 203 x_{15} + 918 x_{16} + 203 x_{17} + 918 x_{18} - - 203 x_{19} + 203 x_{20} - - 203 x_{21} + 203 x_{22} - - 203 x_{23} + 203 x_{24} - - 203 x_{25} + 203 x_{26} + 93208 x_{27} = 0 \\
    -307 x_t - - 307 x_s + 814 x_{10} - - 307 x_{11} - - 307 x_{12} + 814 x_{13} - - 307 x_{14} + 307 x_{15} - - 307 x_{16} + 814 x_{17} + 307 x_{18} - - 307 x_{19} + 307 x_{20} - - 307 x_{21} + 307 x_{22} - - 307 x_{23} + 307 x_{24} + 307 x_{25} + 307 x_{26} + 15271 x_{27} = 0 \
\]
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\[-368x_1 - 368x_2 + 753x_3 - 368x_4 - \cdots - 349x_{10} + 753x_{10} + 753x_{11} - 368x_{12} + 753x_{13} + 753x_{14} - 368x_{15} - 753x_{16} + 368x_{17} - 753x_{18} + 368x_{19} - 753x_{20} - 368x_{21}\]

\[-368x_{22} + 753x_{23} - 368x_{24} - \cdots - 349x_{30} \]

\[-368x_{68} + 18298x_{69} = 0 - 82x_1 - 82x_3 - 82x_4 + 1039x_5 - 82x_6 - \cdots - 82x_{17} + 1039x_{18} - 82x_{19} - \cdots - 82x_{26} + 1039x_{27} - 82x_{28} + \cdots - 82x_{68} + 2213x_{69} = 0 - 186x_1 - 186x_3 - 186x_4 + 935x_5 - 186x_6 = 0 - 186x_{12} + 935x_{13} - 186x_{14} - \cdots - 186x_{19} + 935x_{20} - 186x_{21} - \cdots - 186x_{47} + 935x_{48} - 186x_{49} - \cdots - 186x_{68} + 8164x_{69} = 0 - 247x_1 - 247x_3 - 247x_4 + 874x_5 - 347x_6 \]

\[-247x_{12} + 874x_{13} + 874x_{14} - 247x_{15} - \cdots - 247x_{23} + 874x_{24} - \cdots - 247x_{48} + 874x_{49} - 247x_{50} - \cdots - 247x_{69} + 11438x_{70} = 0 - 136x_1 - 136x_3 + 985x_4 - 136x_5 - 136x_6 + 985x_7 - 136x_8 - \cdots - 136x_{47} + 985x_{48} - 136x_{49} - \cdots - 136x_{68} + 985x_{69} - 136x_{70} = 0 - 197x_1 - \cdots - 197x_4 + 924x_5 - 197x_6 - \cdots - 197x_{13} + 924x_{14} - 197x_{15} - \cdots - 197x_{19} + 924x_{20} - 197x_{21} - \cdots - 197x_{38} + 924x_{39} - 197x_{40} - \cdots - 197x_{56} + 995x_{57} - 126x_{58} - \cdots - 126x_{66} + 5205x_{67} = 0 - 217x_1 - \cdots - 217x_{13} + 905x_{14} - 217x_{15} - \cdots - 217x_{29} + 905x_{30} - 217x_{31} - \cdots - 217x_{57} + 905x_{58} - 217x_{59} - \cdots - 217x_{68} + 11393x_{69} = 0 - 249x_1 - \cdots - 249x_{13} + 872x_{14} - 249x_{15} - \cdots - 249x_{19} + 872x_{20} - 249x_{21} \]

\[-249x_{23} + 872x_{24} - \cdots - 249x_{58} + 872x_{59} - 249x_{60} - \cdots - 249x_{68} + 12448x_{69} = 0 - 186x_1 - \cdots - 186x_{13} + 935x_{14} - 186x_{15} - \cdots - 186x_{29} + 935x_{30} - 186x_{31} - \cdots - 186x_{59} + 935x_{60} - 186x_{61} - \cdots - 186x_{68} + 8164x_{69} = 0 \]
\[-281x_1 - \cdots - 281x_9 + 840x_{10} - 281x_{11} \]
\[-281x_{22} + 840x_{23} - 281x_{24} \]
\[-281x_{36} + 840x_{27} - 281x_{28} - \cdots - 281x_{60} + 840x_{61} \]
\[-281x_{62} - \cdots - 281x_{68} + 14624x_{98} = 0 \]
\[-186x_1 - \cdots - 186x_9 + 935x_{10} - 186x_{11} \]
\[-186x_{12} + 935x_{13} - 186x_{14} \]
\[-\cdots - 186x_{16} + 935x_{19} - 186x_{20} - \cdots - 186x_{61} + 935x_{62} \]
\[-186x_{63} - \cdots - 186x_{68} + 8164x_{99} = 0 \]
\[-326x_1 - \cdots - 326x_9 + 795x_{10} - 326x_{11} \]
\[-326x_{12} + 795x_{13} - 326x_{14} \]
\[-\cdots - 326x_{24} + 795x_{25} - 326x_{26} - \cdots - 326x_{62} + 795x_{63} - 326x_{64} - \cdots - 326x_{98} + 16563x_{100} = 0 \]
\[-301x_1 - \cdots - 301x_9 + 820x_{10} - 301x_{11} \]
\[-\cdots - 301x_{12} + 820x_{13} - 301x_{14} - \cdots - 301x_{24} + 820x_{25} - 301x_{26} - \cdots - 301x_{63} + 820x_{64} - 301x_{65} - \cdots - 281x_{68} - 15984x_{101} = 0 \]
\[-292x_1 - \cdots - 292x_9 + 829x_{10} - 292x_{11} \]
\[-\cdots - 292x_{27} + 829x_{28} - 292x_{29} - \cdots - 292x_{64} + 829x_{65} - 292x_{66} - \cdots - 292x_{68} + 16493x_{102} = 0 \]
\[-280x_1 - \cdots - 280x_9 + 841x_{10} - 280x_{11} \]
\[-\cdots - 280x_{12} + 841x_{13} - 280x_{14} - \cdots - 280x_{20} + 841x_{21} - 280x_{22} + 841x_{23} - 280x_{24} - 280x_{25} + 841x_{26} - 280x_{27} - \cdots - 280x_{55} + 841x_{56} - 280x_{57} - 280x_{58} + 12314x_{103} = 0 \]
\[-154x_1 - \cdots - 154x_{12} + 967x_{13} \]
\[-154x_{14} - \cdots - 154x_{26} + 967x_{29} \]
\[-154x_{30} - \cdots - 154x_{66} + 967x_{67} - 154x_{68} \]
\[+ 7109x_{104} = 0 \]

To transform the objective function, we have:
\[y_i = \frac{x_i}{U_{val}} \]

But $U_{val}$ is 1121. Thus,
\[y_i = \frac{x_i}{1121} \quad ; \quad x_i = 1121y_i \]

Hence, the original objective function becomes
\[1121y_1 + 1121y_2 + \cdots + 1121y_{31} \quad (3.77) \]

Equations (3.41) to (3.76) are the transformed constraints. Penalizing the artificial variables $y_{69}$ to $y_{104}$ in the transformed constraints using the big-M method gives us the transformed problem as:

**minimize**

\[z = 1121y_1 + 1121y_2 + 1121y_3 + \cdots + 1121y_{31} + 23000y_{69} + 23000y_{70} + \cdots + 23000y_{104} \]

**subject to**

\[993y_1 - 128y_2 - \cdots - 128y_{14} + 993y_{15} \]
\[-128y_{16} - \cdots - 128y_{31} + 993y_{32} - 128y_{33} - \cdots - 128y_{68} + 5341y_{69} = 0 \]
\[940y_1 - 181y_2 - \cdots - 181y_{16} + 940y_{17} - 181y_{18} - \cdots - 181y_{32} + 940y_{33} - 181y_{34} - \cdots - 181y_{68} + 7824y_{70} = 0 \]
\[876y_1 - 245y_2 - \cdots - 245y_{6} + 876y_{7} + \]
\[876y_{7} - 245y_{9} - \cdots - 245y_{16} + 876y_{17} - 245y_{18} - \cdots - 245y_{33} + 876y_{34} - 245y_{35} - \cdots - 245y_{68} + 11055y_{71} = 0 \]
\[841y_1 - 280y_2 - \cdots - 280y_{6} + 841y_{7} + \]
\[841y_{7} - 280y_{9} - \cdots - 280y_{10} + 841y_{11} - 280y_{12} - \cdots - 280y_{16} + 841y_{17} - 280y_{18} - \cdots - 280y_{34} + \]
\[841y_{35} - 280y_{36} - \cdots - 280y_{68} + 12025y_{72} = 0 \]
\[737y_1 - 384y_2 - \cdots - 384y_{6} + 737y_{7} + 737y_{8} - 384y_{9} - \cdots - 384y_{10} + 737y_{11} - 384y_{12} + 737y_{13} - 384y_{14} - 384y_{16} + 737y_{19} - 384y_{20} - \cdots - 384y_{35} + 737y_{36} - 384y_{37} - \cdots - 384y_{68} + 18265y_{73} = 0 \]
$676y_1 - 445y_2 - \cdots - 445y_6 + 676y_7 + 676y_8$

$- 445y_9 - 445y_{10} + 676y_{11} - 445y_{12} + 676y_{13} + 676y_{14} - 445y_{15} - \cdots - 445y_{36} + 676y_{37}$

$- 445y_{38} - \cdots - 445y_{68} + 22412y_{74} = 0$

$- 146y_1 + 975y_2 - 146y_3 - \cdots - 146y_{15} + 975y_{16} - 146y_{17} - \cdots - 146y_{19} + 975y_{20} - 146y_{21}$

$\cdots - 146y_{37} + 975y_{38} - 146y_{39} - \cdots - 146y_{68} + 5444y_{75} = 0$

$- 210y_1 + 911y_2 - 210y_3 - \cdots - 210y_7 + 911y_8$

$- 210y_9 - \cdots - 210y_{16} + 911y_{17} - 210y_{18} - \cdots - 210y_{38} + 911y_{39} - 210y_{40} - \cdots - 210y_{68} + 976y_{76} = 0$

$- 245y_1 + 876y_2 - 245y_3 - \cdots - 245y_7 + 876y_8$

$- 245y_9 - 245y_{10} + 876y_{11} - 245y_{12} - \cdots - 245y_{17} + 876y_{18} - 245y_{19} - \cdots - 245y_{39}$

$+ 876y_{40} - 245y_{41} - \cdots - 245y_{68} + 11055y_{77}$

$= 0$

$- 349y_1 + 772y_2 - 349y_3 - \cdots - 349y_7$

$+ 772y_8 - 349y_9 - 349y_{10} + 772y_{11}$

$- 349y_{12} + 772y_{13} - 349y_{14} - \cdots - 349y_{18} + 772y_{19} - 349y_{20}$

$- \cdots - 349y_{40} + 772y_{41} - 349y_{42} - \cdots - 349y_{68} + 17006y_{78} = 0$

$- 410y_1 + 711y_2 - 410y_3 - \cdots - 410y_7$

$+ 711y_8 - 410y_9 - 410y_{10} + 711y_{11} - 410y_{12} + 711y_{13} + 711y_{14} - 410y_{15} - \cdots - 410y_{19} + 711y_{20} - 410y_{21} - \cdots - 410y_{41} + 711y_{42} - 410y_{43} - \cdots - 410y_{68} + 20033y_{79} = 0$

$- 168y_1 - 168y_2 + 953y_3 - 168y_4 - \cdots - 168y_{16} + 953y_{17} - 168y_{18} - \cdots - 168y_{22} + 953y_{23}$

$+ 168y_{24} - \cdots - 168y_{68} + 8161y_{80} = 0$

$- 203y_1 - 203y_2 + 918y_3 - 203y_4 - \cdots - 203y_{10} + 918y_{11} - 203y_{12} - 203y_{13} + 918y_{14} - 203y_{15}$

$\cdots - 203y_{43} + 918y_{44} - 203y_{45} - \cdots - 203y_{68} + 9320y_{81} = 0$

$- 307y_1 - 307y_2 + 814y_3 - 307y_4 - \cdots - 307y_{10} + 814y_{11} - 307y_{12} + 814y_{13} - +307y_{14} - \cdots - 307y_{18} + 814y_{19} - 307y_{20} - \cdots - 307y_{44}$

$+ 814y_{45} - 307y_{46} - \cdots - 307y_{68} + 15271y_{82} = 0$

$- 368y_1 - 368y_2 + 753y_3 - 368y_4 - \cdots - 349y_{10} + 753y_{11} - 368y_{12} + 753y_{13} + 753y_{14} - 368y_{15} - \cdots - 368y_{45} + 753y_{46} - 368y_{47} - \cdots - 368y_{68} + 18298y_{83} = 0$

$- 82y_1 - 82y_2 - 82y_3 + 1039y_4 - 82y_5 - \cdots - 82y_{17} + 1039y_{18} - 82y_{19} - \cdots - 82y_{46} + 1039y_{47}$

$- 82y_{48} - \cdots - 82y_{68} + 2213y_{84} = 0$

$- 186y_1 - 186y_2 - 186y_3 + 935y_4 - 186y_5 - \cdots - 186y_{12} + 935y_{13} - 186y_{14} - \cdots - 186y_{19} + 935y_{20} - 186y_{21} - \cdots - 186y_{40} + 935y_{41} - \cdots - 186y_{68} + 7974y_{85} = 0$

$- 247y_1 - 247y_2 - 247y_3 + 874y_4 - 347y_5 - \cdots - 247y_{12} + 874y_{13} + 874y_{14} - 247y_{15} - \cdots - 247y_{19} + 874y_{20} - 247y_{21} - \cdots - 247y_{38} + 874y_{39} - 247y_{40} - \cdots - 247y_{68} + 11438y_{86} = 0$

$- 136y_1 - \cdots - 136y_{4} + 985y_{5} - 136y_{6} - \cdots - 136y_{18} + 985y_{19} - 136y_{20} - \cdots - 136y_{49} + 985y_{50} - 136y_{51} - \cdots - 136y_{68} + 5885y_{87} = 0$

$- 197y_1 - \cdots - 197y_{4} + 924y_{5} - 197y_{6} - \cdots - 197y_{13} + 924y_{14} - 197y_{15} - \cdots - 197y_{19} + 924y_{20} - 197y_{21} - \cdots - 197y_{50} + 924y_{51} - 197y_{52} - \cdots - 197y_{68} + 8912y_{88} = 0$
Using TORA (an operations Research Software),
the following result was obtained:

\[
\text{Objective value} = 2296.72
\]

\[
y_{\text{consider}} = \begin{cases} 
1 & \text{for } y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}, y_{11}, y_{12}, y_{13}, y_{14}, y_{15}, y_{16}, y_{17}, y_{18}, y_{19}, y_{20}, y_{21}, y_{22}, y_{23}, y_{24}, y_{25}, y_{26}, y_{27}, y_{28}, y_{29}, y_{30}, y_{31}, y_{32}, y_{33}, y_{34}, y_{35}, y_{36}, y_{37}, y_{38}, y_{39}, y_{40}, y_{41}, y_{42}, y_{43}, y_{44}, y_{45}, y_{46}, y_{47}, y_{48}, y_{49}, y_{50}, y_{51}, y_{52}, y_{53}, y_{54}, y_{55}, y_{56}, \vspace{1mm} \\
0 & \text{otherwise} 
\end{cases}
\]

6. Conclusion

The application of NyorRauf Transformation Algorithm to flight crew scheduling as demonstrated in this work has performed well and yielded a solution that is implementable by airlines. Generally speaking, this transformation algorithm is efficient in the Karmarkar Interior Point scheme for solving LPs.

References