A FAST ADAPTIVE ECHO CANCELLATION TECHNIQUE WITH A LEAKY PROPORTIONATE NLMS ALGORITHM

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Abstract: *A modified version of proportionate normalized least mean square (PNLMS) algorithm that achieves faster convergence in time domain with a marginal increment in implementation complexity than the existing one is proposed. The proposed algorithm, leaky PNLMS (LPNLMS), mainly differs from PNLMS by virtue of a leaky factor which is dependent on the algorithm step size parameter and speeds up the convergence behaviour. We apply the proposed algorithm in case of an adaptive echo canceller. The performance of the proposed algorithm is examined in the said application with respect to the mean square error (MSE) and bit error rate (BER) curves.*

Keywords: *Bit error rate, Fast convergence, Leaky factor, Mean square error, Proportionate NLMS algorithm.*

1. INTRODUCTION

In general, adaptive signal processing algorithms are self adjusting in nature. They adjust themselves according to a reference signal, which may be internally generated from the received signal or is sent as a training sequence by the transmitter itself. Such algorithms find their use in wide variety of applications in time varying systems [1] starting from medical instrumentation to channel equalization. The essential difference between the various applications of the adaptive filtering arise the manner in which the desired response is extracted. In this context, four basic classes of adaptive filtering applications can be formulated. The functions of these four classes are classified as identification, inverse modelling, prediction and interference cancelling [2].

The adaptive signal processing uses certain time varying algorithms to achieve the necessary requirements in real time. The simplest algorithm used, from structural view point, is least mean square (LMS) algorithm [3] which minimizes the mean square error (MSE) between desired response and the input signal by using a simple updating model. However, the LMS algorithm has a drawback of not considering the input signal variations in the tap weight correction factor leading towards the problem of gradient noise amplification. .An improved version of LMS, called normalized LMS (NLMS) algorithm [4] , alleviates this problem by taking into consideration the input signal variations. The comparison of performance characteristics with respect to convergence and computational complexity between these two algorithms i s given in[5]. Although NLMS is preferred over LMS for its convergence rate, it still cannot performance adequately when large memory applicants like echo cancelling comes into consideration.. To achieve faster convergence for such cases, a new algorithm called proportionate NLMS(PNLMS) [6] is proposed recently which is also based on MSE criterion. However, the novelty of PNLMS algorithm [7] lies in the fact that an adaptive individual learning rate is assigned to each tap weight of the filter according to some criterion ,there by achieving faster convergence Though there is a marginal increase in computational complexity, the tap weights are fast converging when compared to conventional algorithms like NLMS. In this paper, we propose a faster variant of PNLMS algorithm by introducing a leaky factor into it. The leaky factor, In this paper, we propose a faster variant of PNLMS algorithm by introducing a leaky factor into it. The leaky factor, which is dependent on the step size parameter for its convergence range, speeds up the overall convergence behaviour of the new algorithm. We find the range of this leaky factor experimentally and apply the proposed algorithm in a generic framework of adaptive echo cancelation. The performance of the proposed algorithm is examined in the said application with respect to the mean square error (MSE) and bit error rate (BER) curves. This new algorithm can be called leaky PNLMS algorithm.

This paper is organized as follows. In Section 2 we deal with fundamentals of PNLMS algorithm.Section3 introduces the proposed leaky PNLMS algorithm along with its corresponding computational counting with respect to PNLMS. Experimental results are provided in Section 4. This paper is concluded by summarizing the present work in Section 5.

2. FUNDAMENTALS OF PNLMS ALGORITHM

The proportionate normalized least mean squares (PNLMS) algorithm [6] is a new scheme that exploits the sparseness of impulse responses to achieve significantly faster adaptation than the conventional LMS and NLMS algorithms. Estimation quality is not sacrificed in attaining this faster convergence as well as there is only a modest increase in computational complexity.

The PNLMS algorithm differs from the NLMS algorithm fundamentally in that the available adaptation energy is distributed unevenly over N taps. In other words, the novelty of PNLMS algorithm lies in the fact that an adaptive individual learning rate is assigned to each tap weight of the filter according to some criterion, thereby attaining faster convergence [7]. In the contexts of PNLMS,various notations used in the sequel are as follows.Tap-input vector: **u(n)**= $\left[u_1(n), u_2(n), \ldots\ldots, u_k(n), \ldots\ldots, u_n(n)\right]^T$,tap weight vector: **W**(n)= $\left[w_1(n), w_2(n), \ldots\ldots, w_k(n), \ldots\ldots, w_n(n)\right]^T$, estimation error: e(n)=d(n)-y(n) where y(n) and d(n) are output response and desired responseof the filter respectively, μ : the constant learning rate scalar with convergence range

$$0 < \mu < \frac{2}{trR} \tag{1}$$

Input correlation matrix:R=E[u(n)u^T(n)], time varying positive definite learning rate matrix: $\Gamma(n)$, and, $N \times N$ Identity matrix: **I**.

The two principal operations, filtering and weight updating, respectively, in PNLMS algorithm are given as

$$y(n) = u^T(n)w(n) \text{ and} \tag{2}$$

$$w(n) = w(n-1) + \mu\Gamma(n)u(n)e(n) \tag{3}$$

where

$$\Gamma(n) = \frac{G(n)}{uT(n)G(n)u(n) + \alpha}. \tag{4}$$

In (4), α is small positive scalar and

$$\mathbf{G}(n) = \begin{pmatrix} \frac{g_1(n)}{\bar{g}(n)} & 0 & .. & 0 \\ 0 & \frac{g_2(n)}{\bar{g}(n)} & .. & 0 \\ .. & .. & .. & .. \\ 0 & 0 & .. & \frac{g_N(n)}{\bar{g}(n)} \end{pmatrix}$$

In order to have an idea about how each $g_k(n), k = 1,2,3,\ldots\ldots, N$,is computed,let us define two new two parameters $l_k.and.l_k'$ as

$$l_k = \max(\{|w_1(n)|........|w_N(n)|\} \tag{6}$$

and $l_k' = \max\{\delta, l_k\}$. (7)

with the help of(7),each $g_k(n)$ is defined as

$$g_k(n) = \max\{\rho l_k', |\omega_k(n-1)\}|$$

with $\bar{g}(n)$ being the mean of all $g_k(n)$ at time n i, e....

$$\bar{g}(n) = \frac{1}{N}\sum_{n=1}^{N} g_k(n).$$

Note that in (7) and (8), for calculating g_k (n), ρ and δ play the role of regularization where both are a small positive quantity and δ prevents division by zero at initialization and ρ prevents the stalling in the case where all coefficients are zero [9].

It should also be noted that when α is zero and G(n) = I, the above algorithm reduces to NLMS algorithm.

The normalized term $\frac{g_k(n)}{\bar{g}(n)}$ mainly distributes different amount of energy to different coefficients being estimated and removes the sensitivity of maladjustment noise to the exact shape of G(n).

3. THE PROPOSED LEAKY PNLMS ALGORITHM

In the weight update equation of PNLMS algorithm (3), we introduce a leaky factor (γ), which is dependent on μ, to improve the convergence characteristics of the algorithm. The resulting weight up date equation of the new algorithm, may be called leaky PNLMS (LPNLMS), is:

$$w(n) = (1 - \mu\gamma)w(n-1) + \mu\Gamma(n)u(n)e(n).$$

TABLE 1 COMPUTATIONAL COMPLEXITIES OF NLMS, FLMS AND LFLM S ALGORITHMS

Algorithms	Computational Complexity		Storage Requirement
	Multiplication	Addition	
NLMS	3N+1	3N	N
FLMS	$9\log_2 N+6$	4N	N
Leaky FLMS	$10\log_2 N+8$	5N	N

All the other equations and notations remain same for the proposed algorithm as is given for PNLMS. By introducing γ , a marginal increment occurs both in computational complexity as well as in mean square error. However, the overall performance is enhanced to a significant extent with a faster convergence rate. It has been found experimentally that the convergence range of the proposed leaky factor is

$$0 \le \gamma \le \frac{2}{\mu}. \qquad (11)$$

Beyond which, the proposed algorithm starts diverging. As the value of γ increases, the error rate increases as well as convergence rate. We have to fix for a certain error rate and accordingly γ is adjusted to obtain faster convergence. A judicial adjustment for both γ and μ is thus required for a specified error rate.

A. A comment on computational complexities

The computational complexities of the algorithms mentioned above namely NLMS, PNLMS and LPNLMS algorithms are provided in Table 1 (where N is the length of the filter). For example, if we take PNLMS algorithm, there will be 4N additions, 5N + 1 multiplications and the remaining are for divisions and sorting. Equation (2) takes N additions to complete the matrix multiplication and (4) takes 3N additions to complete matrix multiplication in denominator. Equation (3) takes another N addition for completing the remaining multiplication. Multiplications can be explained in the same way. Before entering the filter the signal is stored in N element block for further usage in the processing. This block serves as the storage block for passing the signal. This explains the storage requirement. We enlist all these in Table 1. Note that, for all the algorithms given in the table, the storage requirement is same.

B. A few remarks about ρ and δ

To develop the intuition for PNLMS algorithm assumes for a moment ρ is zero, in which case, the value of δ is immaterial. With ρ equal to zero the gain distributors g_o (n) are proportional to the currenttap-weightvector sample $|w_k (n-1)|$. The PNLMS algorithm update equation differs from NLMS algorithm update equation only g(n) . The average of these terms is necessarily one. Tap weights that are being equalized as far from zero get significantly more update energy than those currently being equalized close to zero. If one were to truly parameterize the PNLMS algorithm with ρ equal to zero, then if a weight $w_k (n-1)$ ever happened at some time k equal to zero, it would stuck at zero forever after. The variable ρ is associated with a band aid introduced to circumvent this problem. The first argument to the maximum function clamps the gain distributors g_k (n) associated with small tap-weight vector samples to a fraction ρ of the gain distributor associated with largest tap-weight vector sample. The parameter δ controls a final band aid to avoid problems when all coefficients are zero as it occurs after every reset. For our case, we take $\delta = \rho = 0.01$ which is sufficient for the purpose.

4. RESULTS AND DISCUSSIONS

The MSE plot of PNLMS and LPNLMS algorithms are given here in Fig. 1. The algorithm is testedon5000 test data, averaged on 100 independent trials. The various values of variables used here areμ=0.1,α=0.01,$\rho = \delta = 0.01$. The channel impulse response used here is h(n) = [0.7 , 0.6, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.2]. For the use of this algorithm in echo cancellation the duration of the echo is taken as 0.04ms. The value of γ determines the fastness of the algorithm with which it converges. Here γ value is taken as 0.5.

We can observe that the convergence rate is significantly increased in the case of LPNLMS algorithm in comparison with PNLMS algorithm. We can also see that the mean square error is increased slightly which is the cost paid for the sake of faster convergence.

The BER curves of PNLMS and LPNLMS algorithms are given in Fig. 2. The BER is calculated for a test data of 5000 samples, averaged on 100 independent trials for a SNR range of 0 to 20 dB. They both are closely matched most of the time. So the BER is slightly increased in the case of LPNLMS algorithm.

The convergence characteristics of w_1 for PNLMS and LPNLMS algorithm are shown in Fig. 3. In the steady state, $w_1 = 0.7$. It is clearly seen in the figure that Leaky PNLMS algorithm achieves faster convergence in comparison with PNLMS algorithm.

Next, in order to investigate the relationship of MSE and BER with respect to γ , we have taken several values of γ within its convergence range. Fig. 4 shows the MSE plots for LPNLMS algorithm for different such γ values. From the figure, it can be commented that faster convergence is achieved by using higher values of γ.

In Fig. 5, we show BER plot for LPNLMS algorithm for different γ values. Thus, from the figure, we can say that the error value increases for various values of γ . As γ increases the error values is also increased. Hence the selection of γ is done judiciously such that error and fastness are both considered.

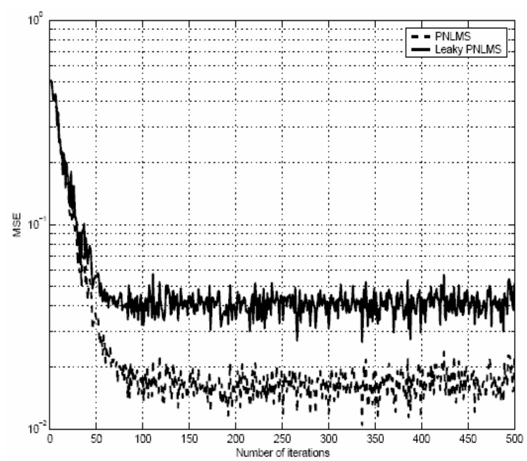

Fig. 1 MSE plots for PNLMS and Leaky PNLMS algorithms with $\mu = 0.1$ and $\gamma = 0.5$

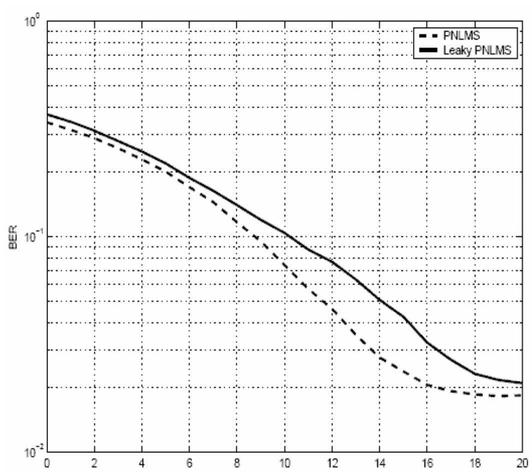

Fig. 2 BER curves for PNLMS and Leaky PNLMS algorithms with $\mu = 0.1$ and $\gamma = 0.5$

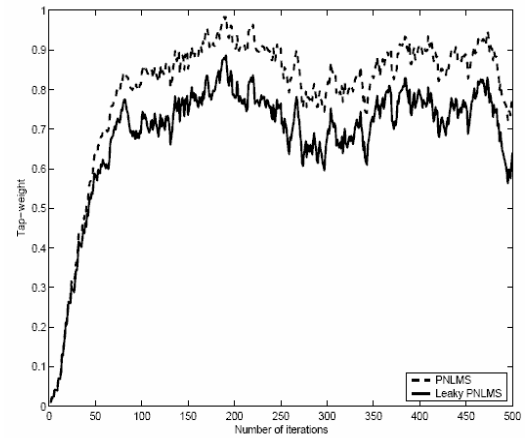

Fig. 3 Convergence characteristics of w_1 for PNLMS and Leaky PNLMS algorithms

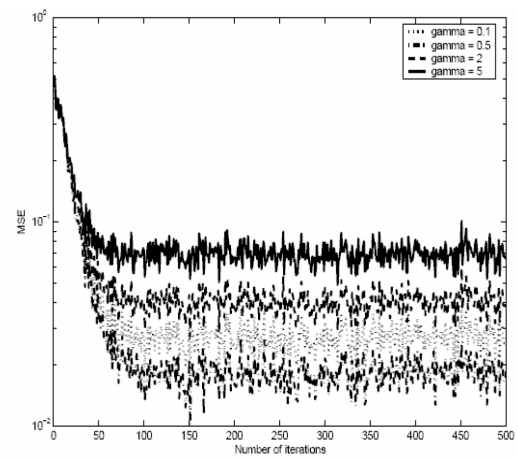

Fig. 4 MSE plots of Leaky PNLMS algorithms for different γ values

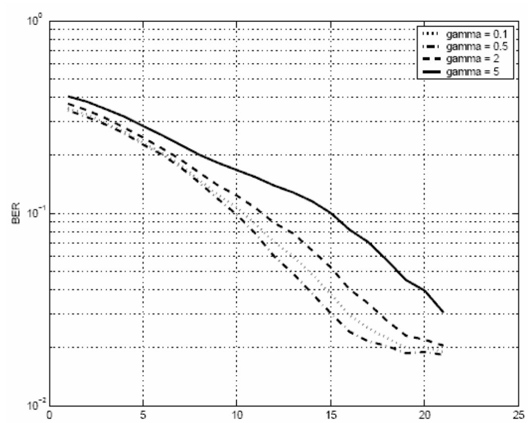

Fig. 5 BER curves for Leaky PNLMS algorithms for different γ values

5. CONCLUSION

In this paper, we have proposed a faster variant of PNLMS algorithm by introducing a step size parameter dependent leaky factor into it. The leaky factor speeds up the overall convergence behaviour of the new algorithm. We found the range of this leaky factor experimentally and apply the proposed algorithm in a generic framework of adaptive echo cancellation to evaluate the performance characteristics of the proposed LPNLMS algorithm. It has been observed that the proposed algorithm has faster convergence with respect to PNLMS algorithm(Fig.1)but the error rate and computational complexity increase slightly (Fig. 2) because the leaky f actor forces the filter taps to keep learning with a fractional energy in order to prevent the noise from building up. From Fig.3 we can see that LPNLMS has faster convergence characteristics when compared with PNLMS algorithm. The fastness with which LPNLMS algorithm converges for different values of γ is given in Fig. 4. In the same way, the increase in error value with different values of γ is given in Fig. 5. Although the algorithm has been applied in a general framework of echo cancellation, this can be used efficiently for other time domain applications a s well like channel equalization, system identification etc.

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