NATURAL CONVECTION IN SQUARE ENCLOSURE WITH ADIABATIC CYLINDER AT CENTER AND DISCRETE BOTTOM HEATING

Sumon Saha[1], Goutam Saha[2] and Md. Quamrul Islam[1]
[1]Department of Mechanical Engineering
Bangladesh University of Engineering and Technology (BUET), Dhaka-1000, Bangladesh.
[2]Department of Arts and Sciences
Ahsanullah University of Science and Technology (AUST), Dhaka-1215, Bangladesh.
Email: sumonsaha@me.buet.ac.bd, ranamath06@gmail.com, quamrul@me.buet.ac.bd

Abstract: *Natural convection in two-dimensional square enclosure containing adiabatic cylinder at the center has been studied numerically using finite element method. In the present study, top wall is considered adiabatic, two vertical walls are maintained at constant cold temperature, a constant heat flux is embedded at the bottom wall and the non-heated parts of the bottom wall are considered adiabatic. The aim of this work is to demonstrate the capabilities of this numerical methodology for handling such problems. The finite element formulations of the dimensionless governing equations with the associated boundary conditions are solved by a nonlinear coupled solution algorithm using three-nodded triangular element discretization scheme for all the field variables. The Grashof number is varied from* 10^3 *to* 10^6 *and Prandtl number is taken as 0.71. The effect of heat source length on the fluid flow and heat transfer process in the enclosure are analyzed. Results are presented in the form of streamline and isotherm plots.*

Keywords: *Natural convection, square enclosure, heat flux, finite element method, adiabatic cylinder.*

1. Introduction

Natural convection is observed as a result of the motion of the fluid due to density changes arising from heating process. The movement of the fluid in free convection results from the buoyancy forces imposed on the fluid when its density in the proximity of the heat transfer surface is described as a result of thermal expansion of the fluid in a non-uniform temperature distribution. Convection heat transfer is dependent on the movement of the fluid and the development of the flow of the fluid is also influenced by the shape of the heat transfer surfaces. Both numerical and experimental methods have been used to obtain the solution of heat transfer and fluid flow problems. Although experimental methods are more realistic, they are costly and time consuming due to fabrication of prototypes and necessary instrumentation. On the other hand, numerical methods can offer considerable savings in design time and costs. Effective cooling of electronic components has become increasingly important as power dissipation and component density continue to increase substantially with the fast growth of electronic technology. It is very important that such cooling systems are designed in the most efficient way and the power requirement for the cooling is minimized. The electronic components are treated as heat sources embedded on flat surface [1, 2]. In many applications natural convection is the only feasible mode of cooling of the heat source. Besides cooling of the electronic components, there are numerous other practical applications of natural convective cooling in rectangular enclosures with various combinations of the temperature gradients, cavity aspect ratios, placement of the heat source and cold surfaces etc.

Following the pioneering numerical work of Chu *et al.* [3] on two-dimensional, laminar natural convection cooling of a single, isothermal flush-mounted heater on a vertical wall inside an air filled rectangular enclosure; the heat transfer problem of natural convection in a discretely heated enclosure is of great research interest as indicated by the considerable research activities on this subject. A natural convection heat transfer experiment in a tall vertical rectangular enclosure (aspect ratio 16.5) with an array of eleven discrete flush-heaters was performed by Keyhani *et al.* [4]. It was found that the discrete heating in the enclosure resulted in a significantly augmented local heat transfer rate over that for an enclosure with the uniformly heated vertical wall. A follow-up study [5] for a vertical enclosure aspect ratio 4.5 with three flush heaters further revealed that the temperature of the heaters was strongly affected by the stratification of fluid inside the enclosure. Moreover, the effects of enclosure width and Prandtl number on natural convection liquid cooling of discrete flush heaters in a tall enclosure cooled from the top were investigated experimentally and numerically [6, 7]. Ahmed and Yovanovich [8] performed a numerical study to examine the influence of discrete heat source location on natural convection heat transfer in a vertical square enclosure.

The problem of convective heat transfer in an enclosure has been studied extensively because of the wide application of such process. Ostrach [9] provided a comprehensive review article and extensive bibliography on natural convection in cavities. Anderson and Lauriat [10] studied the natural convection in a vertical square cavity heated from bottom and cooled from one side. Convection in a similar configuration where the bottom wall of the rectangular cavity was partially heated with cooling from one side was studied by November and Nansteel [11]. It was reported that the heated fluid layer near the bottom wall remained attached up to the turning corner. Ganzarolli and Milanez [12] performed numerical study of steady natural convection in rectangular enclosures heated from below and symmetrically cooled from the sides. The size of the cavity was varied from square to shallow where the cavity width was varied from 1 to 10 times of the height. The heat source, which spanned the entire bottom wall was either isothermal or at constant heat flux condition. Aydin and Yang [13] and Sharif and Mohammad [14] numerically investigated the natural convection of air in a vertical square cavity with localized isothermal and isoflux heating from below and symmetrical cooling from sidewalls. The top wall and the non-heated parts of the bottom wall were considered adiabatic. The length of the symmetrically placed isothermal heat source at the bottom was varied. Two counter rotating vortices were formed in the flow domain due to natural convection. The average Nusselt number at the heated part of the bottom wall was increased with increasing Rayleigh number as well as with the increase of the length of the heat source.

Buoyancy driven flow and heat transfer between a cylinder and its surrounding medium has been a problem of considerable importance. This problem has a wide range of applications. Energy storage devices, crop dryers, crude oil storage tanks, heat exchangers and spent fuel storage of nuclear power plants are a few to mention. Larson *et al.* [15] carried out experimental study of temperature field around a heated horizontal cylindrical body in an isothermal rectangular enclosure. Roychowdhury *et al.* [16] analyzed the natural convective flow and heat transfer features for a heated cylinder kept in a square enclosure with different thermal boundary conditions. Elepano and Oosthuizen [17] carried out numerical study of natural convective flow in an enclosure containing a heated cylinder and a cooled upper surface. A majority of the available studies [18, 19, 20] deal with natural convective flow and heat transfer around a circular cylinder kept inside an enclosure. However, almost no study has been found on the natural convection process with adiabatic cylinder inside a discretely heated enclosure having cold side walls.

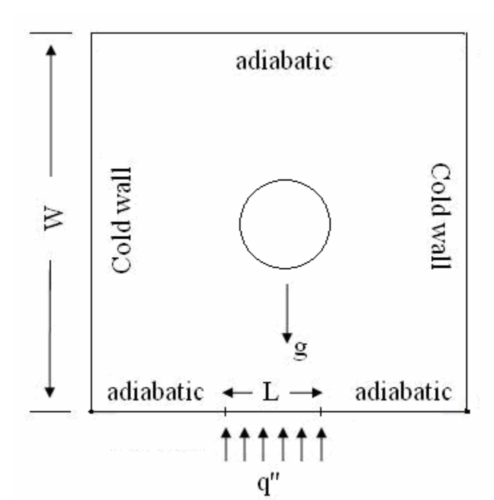


Fig. 1 Schematic diagram of the physical system

The geometry of the problem under consideration is illustrated in Fig. 1. It consists of an insulated cylinder at the center of the square enclosure (W × W), whose vertical sidewalls are kept at a constant cold temperature T_c, the bottom wall has an embedded heat source with constant heat flux q'' and length L and the remaining parts of the bottom wall and the entire upper wall are kept adiabatic. The Grashof number is varied from 10^3 to 10^6 and the ratio of the heating element to the cavity width, L/W is varied from 0.2 to 0.8.

2. Mathematical Model

Natural convection is governed by the differential equations expressing conservation of mass, momentum and energy. The present flow is considered steady, laminar, incompressible and two-dimensional. The viscous dissipation term in the energy equation is hereby neglected. The Boussinesq approximation is invoked for the fluid properties to relate density changes to temperature changes, and to couple in this way the temperature field to the flow field. Then the governing equations for steady natural convection can be expressed in the dimensionless form as,

$$U\frac{\partial U}{\partial X}+V\frac{\partial V}{\partial Y}=0 \quad (1)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \quad (2)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Gr\,\theta \quad (3)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{Pr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \quad (4)$$

where, X and Y are the coordinates varying along horizontal and vertical directions respectively, U and V are the velocity components in the X and Y directions respectively, θ is the temperature and P is the pressure.

The non-dimensional Grashof and Prandtl numbers are indicated by Gr and Pr respectively and they are defined as

$$Gr=\frac{g\beta\Delta t\,W^3}{\upsilon^2} \text{ and } Pr=\frac{\upsilon}{\alpha} \quad (5)$$

The dimensionless parameters in the above equations are defined as follows:

$$X=\frac{x}{W},Y=\frac{y}{W},U=\frac{uW}{\upsilon},V=\frac{vW}{\upsilon},$$
$$P=\frac{pW^2}{\rho\upsilon^2},\theta=\frac{T-T_c}{\Delta t},\ \Delta t=\frac{q''W}{k}. \quad (6)$$

where, ρ, β, υ, α and g are the fluid density, coefficient of volumetric expansion, kinematic viscosity, thermal diffusivity and gravitational acceleration respectively.

The boundary conditions for the present problem are mentioned below.

For top and bottom walls:

$U = V = 0$

$$\frac{\partial\theta}{\partial Y}=\begin{cases}-1, & 0.5-\frac{\varepsilon}{2}\le X\le 0.5+\frac{\varepsilon}{2}, Y=0\\ 0, & \text{otherwise}\end{cases}$$

For right and left vertical walls:

$U = V = 0,\ \theta = 0$

For cylinder surface:

$$U=V=0,\ \frac{\partial\theta}{\partial S}=0$$

Also the dimensionless heat flux at the bottom wall is 1/Pr. The average Nusselt number [14] can be obtained respectively as

$$Nu=-\frac{1}{\varepsilon}\int_0^{\varepsilon}\frac{1}{\theta_s(X)}dX \quad (7)$$

where, $\theta_s(X)$ is the local dimensionless temperature of the heat source.

3. Numerical Procedure

Numerical solutions for the governing equations with the associated boundary conditions are obtained using finite element method. The three node triangular element is used in this paper for the development of the finite element equations. A mixed finite element model is implemented, with two types of triangular Lagrange elements i.e. first one is with an element of linear velocity and pressure interpolations for continuity and momentum equations and the second one is with an element of a quadratic basis velocity and temperature interpolations for energy equation. The relative tolerance for the error criteria is 10^{-6}. As the dependent variables vary greatly in magnitude, manual scaling of the dependent variables is used to improve numerical convergence. The nonlinear equations are solved iteratively using Broyden's method with an LU-solution for a nearby Grashof number. The numerical simulations are performed varying the number of elements of the grid in order to increase the accuracy and efficiency for the solutions. Non-uniform grids of triangular element are employed in the analysis with denser grids clustering in regions near the heat sources, the cylinder and the enclosed walls.

Table 1 Comparison of the results for various grid dimensions (Gr = 10^6 and ε = 0.2)

Elements	Nu
1970	16.229
2902	16.854
3540	16.379
4608	16.487
4828	16.534
6394	16.581
12606	16.581

Table 2 Comparison of the average Nusselt number of the square enclosure for ε = 0.2

Gr	Average Nusselt Number, Nu		
	Sharif and Mohammad [14]	Present Work	Error (%)
10^3	5.927	5.939	0.21
10^4	5.946	5.954	0.13
10^5	7.124	7.117	0.10
10^6	11.342	11.226	1.01

4. Results and Discussion

The working fluid is chosen as air with Prandtl number, Pr = 0.71. The normalized length of the constant flux heat source at the bottom wall, ε is varied from 0.2 to 0.8. For each value of ε, the Grashof number, Gr is varied from 10^3 to 10^6. To test and assess grid independence of the present solution scheme, many numerical runs are performed for higher Grashof number as shown in Table 1. These experiments reveal that a non-uniform spaced grid of 6394 elements for the solution domain is adequate to describe correctly the flow and heat transfer processes inside the enclosure. In order to validate the numerical model, the results are compared with those reported by Sharif and Mohammad [14] for square enclosure with Gr = 10^3 to 10^6 and ε = 0.2. In Table 2, a comparison of the average Nusselt number of the square enclosure is presented. The agreement is found to be excellent with a maximum discrepancy of about 1.01%, which validates the present computations indirectly.

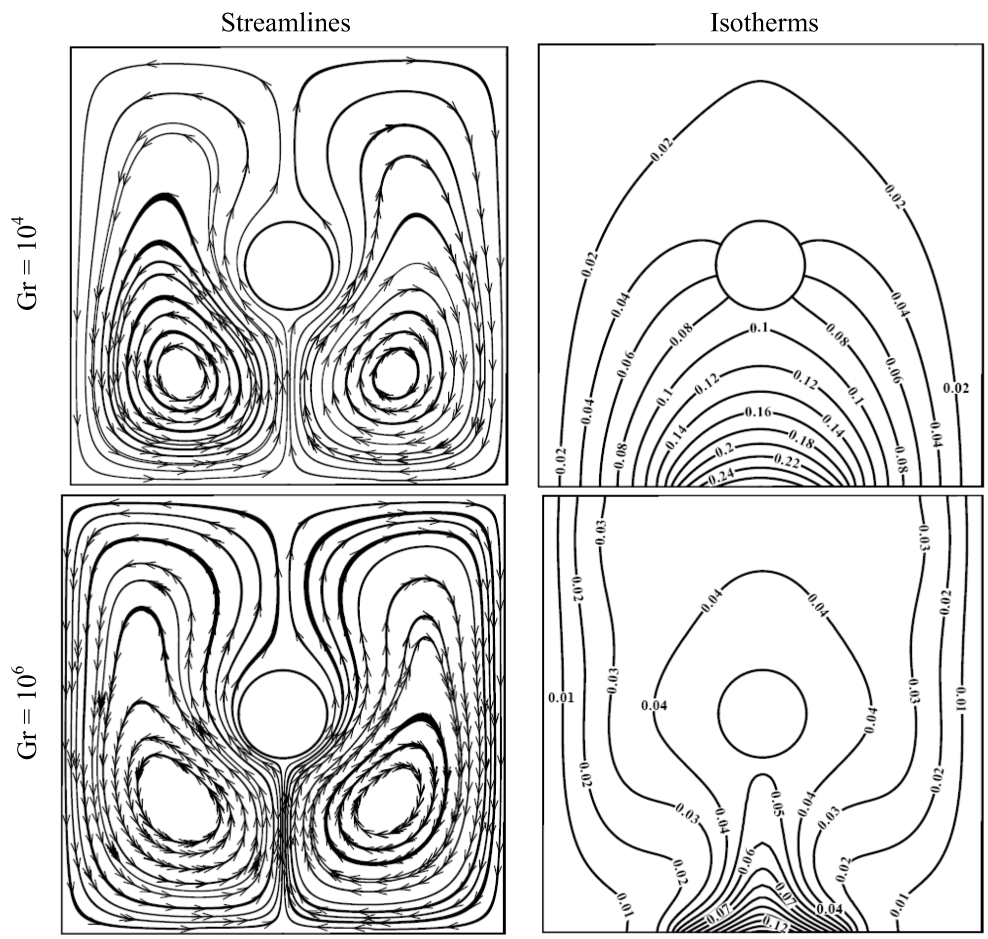


Fig. 2 Variation of streamlines and isotherms of the enclosure for ε = 0.

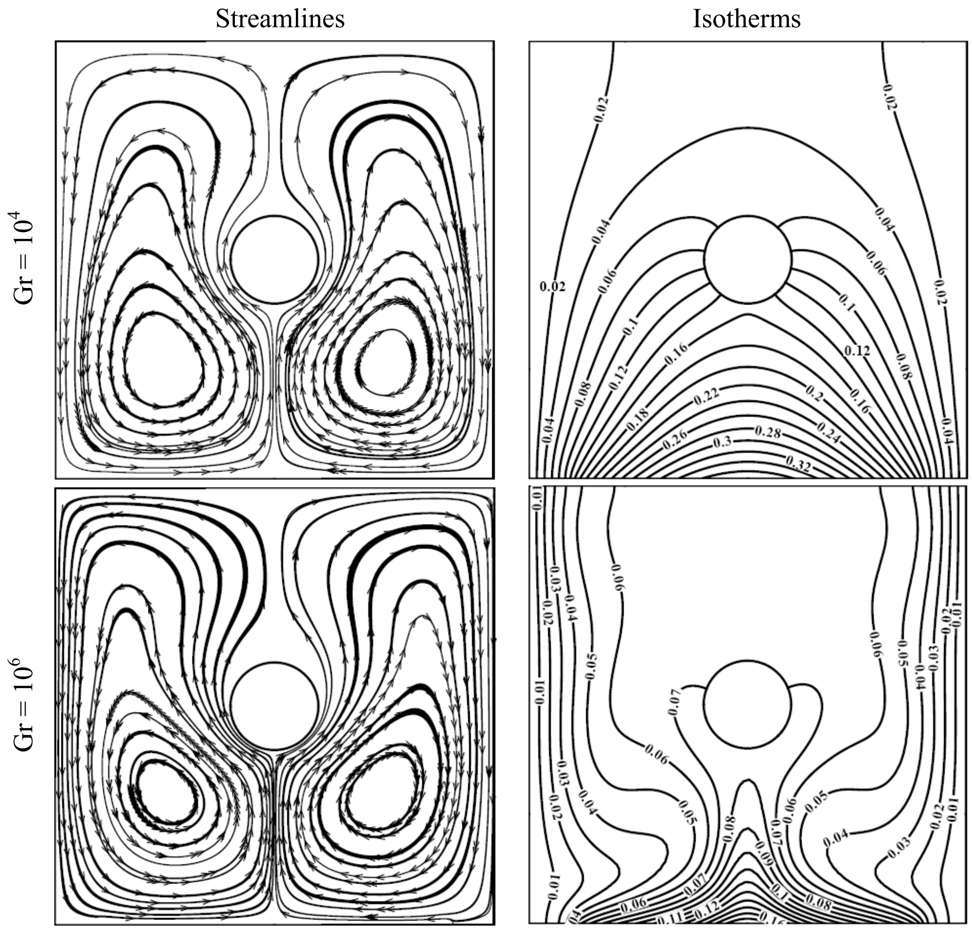


Fig. 3 Variation of streamlines and isotherms of the enclosure for ε = 0.8

The flow and temperature fields in terms of computed streamlines and isotherms for two representative values of the dimensionless source length, ε = 0.4 and 0.8 are presented in Figs. 2 and 3. Plots are shown only for two different values of the Grashof number, Gr = 10^4 and 10^6 for each case. For Gr = 10^4, two large rotating cells are observed. The solution is symmetric about the vertical midline due to the symmetry of the problem geometry and boundary conditions. In each case, the flow descends downwards along the cold isothermal vertical sidewalls and turns horizontally to the central region after hitting the bottom wall. The flow then rises along the vertical symmetry axis and gets blocked at the centrally positioned adiabatic cylinders, which deflects the flow away from the vertical center line towards the cold sidewalls. Thus a pair of counter rotating rolls is formed in the flow domain. Visual examination of the streamlines does not reveal any significant difference among the different cases. However, noticeable difference is observed in the isotherms plots. The convection region adjacent to the heat source becomes thinner and denser producing higher temperature gradients with increasing Gr. Similar behavior is also observed for cases with other values of ε and Gr.

The natural convection becomes more predominant with the increase of ε because the energy transport increases due to the increased area of the heated part. Increasing the value of ε makes the range of natural convection regime larger. Since the isotherm plots change when Gr changes, it is the parameter of focus in the analysis for all cases. At low values of Gr ≤ 10^4, diffusion among the fluid particle plays the main role in the heat transport. For Gr > 10^4, the buoyancy begins to influence the heat transport and isotherms with high values tend to concentrate near the heat source surface. This trend keeps on growing as Gr increases. The temperature profiles along the vertical middle section of the enclosure for a range of Gr indicate that the temperature decreases from the bottom to top more abruptly due to the presence of the adiabatic cylinder at the center. For any value of ε, the temperature contours changes from an orderly oriented pattern along the adiabatic cylinder to a concentrated layer around the heat source, as the Grashof number increases.

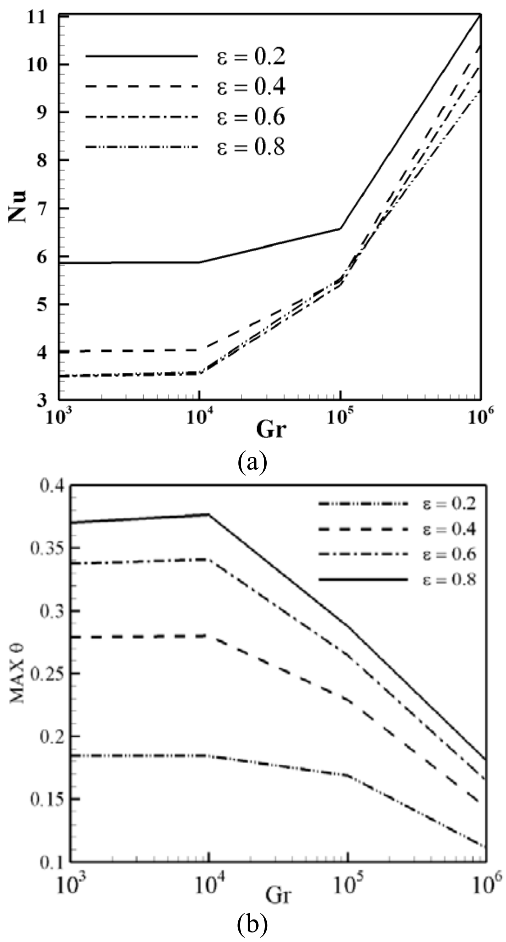


Fig. 4 Variation of the average Nusselt number and the maximum temperature at the heated surface with Grashof number

Figure 4(a) shows the variation of the average Nusselt number with Gr for different ε. It is shown that the average Nusselt number increases with Gr, while keeping the other parameters fixed. It may be concluded that both Gr and ε enhance the heat transfer rate separately. At lower value of Gr, i.e. less heat flux from the heat source, Nu appears to increase slightly with the increase of Gr. This phenomenon indicates the fact that buoyancy effect does not dominate the heat transport mechanism for small value of Gr. In Fig. 4(b), the maximum dimensionless surface temperature θ_{max} on heat source is presented. An increase in parameter Gr results in a decrease in the dimensionless temperature θ_{max}. Similarly, decreasing the value of ε also gives rise to decrease θ_{max} at the heat source surface. When comparing with the plot in Fig. 4(a), it is found that an increase in Nusselt number, Nu i.e. an increase in heat transfer rate from the source, generally leads to a decrease in the dimensionless surface temperature θ_{max}.

5. Conclusion

In the present study, the thermal and hydrodynamic interactions inside a discretely heated square enclosure having centrally placed insulated cylinder have been rigorously investigated. Attention is focused particularly on the effect of discrete heat source size on the flow structure and heat transfer characteristics for the system at various Gr. The following conclusions can be drawn from the present research work.

(i) The fluid that is heated next to the hot bottom surface rises and replaces the cooled fluid next to the cold side walls that is falling, thus giving rise to clockwise and counter-clockwise rotating vortices.

(ii) Comparing the results of the cases with lower Grashof number, the maximum Nusselt number is obtained for ε = 0.2 which is significantly differed from other ε ≥ 0.4.

(iii) For Gr ≤ 10^4, there is a marginal change in Nusselt number for all discrete heat source sizes.

(iv) Although the average Nusselt number increases with the increase of Gr over 10^4 for all ε values, there is a noticeable difference exists at a particular value of ε = 0.2.

(v) For all values of ε except ε = 0.2, the Nusselt number variation does not differ much within the range of $5\times10^4 \leq Gr \leq 5\times10^5$.

The installation or placement of adiabatic cylinder into the system may cause the main flow to separate at the central region of the enclosure, which would substantially reduce the effectiveness of heat transfer enhancement compare to the enclosure without cylinder [14]. This is due to the fact that the cylinder blocks the movement of the fluid and weakens the primary vortices.

References

[1] F. P. Incropera, "Convection heat transfer in electronic equipment cooling", *ASME J. Heat Transfer*, Vol. 110, 1988, pp. 1097-1111.

[2] Y. Jaluria, "Natural convection cooling of electronic equipment in natural convection: fundamental and applications", 1985, pp. 961-986.

[3] H. H. S. Chu, S. W. Churchill, and C. V. S. Patterson, "The effects of heaters size, location, aspect ratio, and boundary condition on two-dimensional, laminar natural convection in rectangular enclosure", *ASME J. Heat Transfer*, Vol. 98, 1976, pp. 194-201.

[4] M. Keyhani, V. Prasad, R. Shen, and T. T. Wong, "Free Convection Heat Transfer from Discrete Heat Sources in a Vertical Cavity". *In Natural and Mixed Convection in Electronic Equipment Cooling, ASME HTD*, Vol. 100, 1988, pp. 13-24.

[5] K. Keyhani, V. Prasad, and R. Cox, "An experimental study of natural convection in a vertical cavity with discrete heat sources", *ASME J. Heat Transfer*, Vol. 110, 1988, pp. 616-624.

[6] R. Carmona, and M. Keyhani, "The cavity width effect on immersion cooling of discrete flush-heaters on one vertical wall of an enclosure cooled from the top", *ASME J. Electronic Packaging*, Vol. 111, 1989, pp. 268-276.

[7] V. Prasad, M. Keyhani, and R. Shen, "Free convection in a discretely heated vertical enclosure: effects of Prandtl number and cavity size", *ASME J. Electronic Packaging*, Vol. 112, 1990, pp. 63-74.

[8] G. Refai Ahmed, and M. M. Yovanovich, "Influence of discrete heat source location on natural convection heat transfer in a vertical square enclosure", *ASME J. Electronic Packaging*, Vol. 113, 1991, pp. 268-274.

[9] S. Ostrach, "Natural convection in enclosures", *Int. J. Heat Transfer*, Vol. 110, 1988, pp. 1175-1190.

[10] R. Anderson, and G. Lauriat, "The horizontal natural convection boundary layer regime in a closed cavity", *in: Proceedings of the Eighth International Heat Transfer Conference*, Vol. 4, 1986, pp. 1453-1458.

[11] M. November and M. W. Nansteel, "Natural convection in rectangular enclosures heated from below and cooled along one side", *Int. J. Heat Mass Transfer*, Vol. 30, 1986, pp. 2433-2440.

[12] M. M. Ganzarolli, and L. F. Milanez, "Natural convection in rectangular enclosures heated from below and symmetrically cooled from the sides", *Int. J. Heat Mass transfer*, Vol. 38, 1995, pp. 1063-1073.

[13] O. Aydin, and J. Yang, "Natural convection in enclosures with localized heating from below and symmetrically cooling from sides", *Int. J. Numer. Methods Heat Fluid Flow*, Vol. 10, 2000, pp. 518-529.

[14] M. A. R. Sharif, and T. R. Mohammad, "Natural convection in cavities with constant flux heating at the bottom wall and isothermal cooling from the sidewalls", *Int. J. of Thermal Sciences*, Vol. 44, 2005, pp. 865-878.

[15] D. W. Larson, D. K. Gartling, W. P. Schimmel, "Natural convection studies in nuclear spent-fuel shipping casks: computation and experiment", *J. Energy*, Vol. 2, No. 3, 1993.

[16] D. G. Roychowdhury, S. K. Das, T. S. Sundararajan, "Numerical simulation of natural convective heat transfer and fluid flow around a heated cylinder inside an enclosure", *Heat Mass Transfer*, Vol. 38, 2002, pp. 565–576.

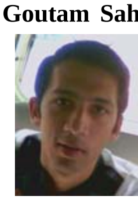
[17] A. R. Elepano, P. H. Oosthuizen, "Free convective heat transfer from a heated cylinder in an enclosure with a cooled upper surface", in: L. C. Wrobel, C. A. Brabbia, A. J. Nowak (Eds.), *Advanced Computational Methods in Heat Transfer 2, Computational Mechanics Publication*, Southampton, Boston, 1990.

[18] T. H. Kuehn, R. J. Goldstein, "An experimental study and theoretical study of natural convection in the annulus between horizontal concentric cylinders", *J. Fluid Mech.*, Vol. 74, 1976, pp. 695–719.

[19] U. Projahn, H. Rieger, H. Beer, "Numerical analysis of laminar natural convection between concentric and eccentric cylinders", *Numer. Heat Transfer*, Vol. 4, 1981, pp. 131–146.

[20] K. Sasaguchi, K. Kuwabara, K. Kusano, H. Kitagawa, "Transient cooling of water around a cylinder in a rectangular cavity – a numerical analysis of the effect of the position of the cylinder", *Int. J. Heat Mass Transfer*, Vol. 41, 1998, pp. 3149–3156.

Sumon Saha received his B.Sc. and M.Sc. in Mechanical Engineering from Bangladesh University of Engineering and Technology, Dhaka, Bangladesh in November 2004 and 2007 respectively. He is now in the department of Mechanical Engineering, BUET, Bangladesh as a lecturer. He has participated in several National and International conferences for presenting his research papers and already published 9 papers in international journals. His research interests include CFD, convection mechanism, mass and energy transfer, numerical analysis, digital measurement system, thermal buckling etc. He is a member of the Institution of Engineers, Bangladesh (IEB) and also a member of Bangladesh Society of Mechanical Engineers (BSME).



Goutam Saha received his B.Sc. (Hons) degree (1st Class 2nd position) and M.S. (Thesis) (1st Class 1st position) in Mathematics from University of Dhaka. Now he is working as a Lecturer in the Department of Arts and Sciences of Ahsanullah University of Science and Technology (AUST), Dhaka, Bangladesh. His field of interest is real function theory, complex analysis, differential equations, finite element analysis, computational fluid dynamics and heat transfer.

Md. Quamrul Islam is currently working as a

Professor in the department of Mechanical Engineering at Bangladesh University of Engineering and Technology (BUET), Dhaka. He worked as the Dean of the faculty of Mechanical Engineering and Head of the department Mechanical Engineering of BUET. Mr. Islam obtained B.Sc. Engg., M.Sc. Engg., and Ph.D. in Mechanical Engineering. He published 122 research papers in International/National journals and proceedings. He is also the author/co-author of nine text books in Mechanical Engineering. He received certificate of Merit from the Institution of Engineers, India in 2001. He received **Prime Minister's Gold medal** in 2005 and also received best paper award from the Institution of Engineers, Bangladesh in 2006. He was awarded the honorary fellowship of Bangla Academy in 2007. His areas of interest include Fluid Mechanics, Heat Transfer and Renewable Energy. He is fellow/member of 10 professional societies.