DISSIPATION EFFECT ON A FREE CONVECTION FLOW PAST A POROUS VERTICAL PLATE

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Nomenclatures

N	Viscous dissipation parameter	T	Temperature of the fluid in the boundary layer
C_{fx}	Local skin friction coefficient	q_w	Heat flux at the surface
C_p	Specific heat at constant pressure	q_c	Conduction heat flux.
F	Dimensionless stream function	Pr	Prandtl number
u	Dimensionless velocity component along x-axis	x	Axis in the direction along the surface
V	Dimensionless velocity component along y-axis	y	Axis in the direction normal to the surface
G	Acceleration due to gravity	T_∞	Temperature of the ambient fluid
K	Thermal conductivity	T_w	Temperature at the surface
Nu_x	Local Nusselt number	V	Wall suction velocity

Greek symbols

θ_w	Surface temperature	ν	Kinematic viscosity
β	Coefficient of thermal	ρ	Density of the fluid
θ	Dimensionless temperature	μ	Viscosity of the fluid
ΔT	Equal to $T - T$	τ	Coefficient of skin friction
ξ	Similarity variable	τ_w	Shearing stress
η	Similarity variable	ψ	Non-dimensional stream function

Subscripts:

w wall conditions

∞ ambient temperature

Abstract:. *A Numerical study on the effect of dissipation on a steady free convection flow through a porous vertical plate is made. The relevant non-leaner boundary equations are made dimensionless using specific non-dimensional variables. The corresponding non-similar partial differential equations are solved using implicit finite difference method with Keller-Box scheme. The results are then presented graphically and discussed thereafter.*

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1. Introduction

The effects of viscous dissipation plays an important role in natural convection in various devices which are subject to large deceleration or which operate high rotative speeds . It is important in strong gravitational field processes in fluids internal to various bodies. . Free convection in presence of viscous dissipation has been drawn forth not only for its fundamental aspects but also for its significance in the contexts of space technology and processes involving high temperature. In the presence of viscous dissipation natural convection boundary layer flow from a porous vertical plate of a steady two dimensional viscous incompressible fluid has been investigated. In this analysis consideration had been given to grey gases. Over the work it is assumed that the surface temperature of the porous vertical plate T_w, is constant, where$T_w > T_\infty$. Here T_∞ is the ambient temperature of the fluid, T is the temperature of the fluid in the boundary layer, g is the acceleration due to gravity, the fluid is assumed to be a grey emitting and absorbing, but non scattering medium. In the present work variations in fluid properties are limited only to those density variations which affect the buoyancy terms.

Merkin [1] studied free convection with blowing and suction. Lin and Yu [2] studied free convection on a horizontal plate with blowing and suction. Hossain et al [3] studied the effect of radiation on free convection flow with variable viscosity from a porous vertical plate. Gebhart [4] concluded on effects of viscous dissipation in natural convection, Gebhart and Mollendorf [5] described viscous dissipation in external natural convection flows, Chowdhury and Islam [6] studied MHD free convection flow of visco- elastic fluid past an infinite porous plate, Hossain [7] explained viscous and Joule heating effects on MHD-free convection flow with variable plate temperature. Alam et al.[8] studied viscous dissipation effects on MHD natural convection flow over a sphere in the presence of heat generation. Hossain et al. [9] studied the effect of radiation on free convection flow from a porous vertical plate. They [9] analyzed a full numerical solution and found, an increase in Radiation parameter R_d causes to thin the boundary layer and an increase in surface temperature parameter causes to thicken the boundary layer. The presence of suction ensures that its ultimate fate if vertically increased is a layer of constant thickness. Vajravelu and Hadjinicolaou [10] studied the heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation.

Takhar and Soundalgekar [11] studied dissipation effects on MHD free convectionflow past a semi infinite vertical plate, Makinde and Ogulu [12] studied the effect of thermal radiation on the heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field. Alam et al.[13] viscous dissipation effects on MHD natural convection flow along a sphere. None of the aforementioned studies, considered the viscous dissipation effects on laminar boundary layer flow of the fluids along porous plate.

The present study deals with natural convection flow from a porous vertical plate in presence of viscous dissipation. The results will be obtained for different values of relevant physical parameters and will be shown in graphs as well as in tables.

The governing partial differential equations are reduced to locally non-similar partial differential forms by adopting some appropriate transformations. The transformed boundary layer equations are solved numerically using implicit finite difference scheme together with the Keller box technique [14]. Here, we have focused our attention on the evolution of the surface shear stress in terms of local skin friction and the rate of heat transfer in terms of local Nusselt number, velocity profiles as well as temperature profiles for selected values of parameters consisting of viscous dissipation parameter N, Prandtl number Pr. In order to check the accuracy of our numerical results the present results are compared with [9].

2. Formulation of the problem

We have investigated natural convection flow from a porous plate in presence of viscous dissipation. The fluid is assumed to be a grey, emitting and absorbing. Over the work it is assumed that the surface temperature of the porous vertical plate, T_w, is constant, where $T_w > T$. The physical configuration considered is as shown in Fig.1:

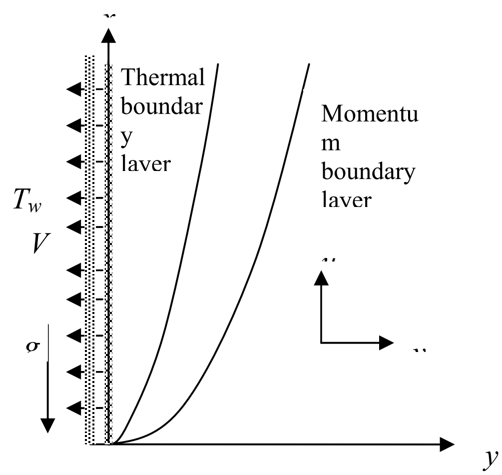


Fig. 1: Coordinate system and the physical model

The conservation equations for the flow characterized with steady, laminar and two dimensional boundary layers, under the usual Boussinesq approximation, the continuity, momentum and energy equations can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=\mu\frac{\partial^2 u}{\partial x^2}+\rho g\beta(T-T_\infty)$$

$$(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y})=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2.$$

The boundary conditions are

$$x=0, y>0, u=0, T=T_\infty.$$
$$y=0, x>0, u=0, v=-V,\ T=T_w$$
$$y\to\infty, x>0, u=0, T=T_\infty$$

where ρ is the density, k is the thermal conductivity, β is the coefficient of thermal expansion, ν is the reference kinematic viscosity $\nu=\mu/\rho$, μ is the viscosity of the fluid, C_p is the specific heat due to constant pressure. In order to reduce the complexity of the problem we introduce the following non-dimensional variables:

$$\eta=\frac{Vy}{\nu\xi}, \xi=V\left\{\frac{4x}{\nu^2 g\beta\Delta T}\right\}^{\frac{1}{4}},$$
$$\psi=V^{-3}\nu^2 g\beta\Delta T\xi^3\left\{f+\frac{\xi}{4}\right\}$$
$$\theta=\frac{T-T_\infty}{T_w-T_\infty},\ \theta_w=\frac{T_w}{T_\infty},$$

where, θ is the non-dimensional temperature function, θ_w is the surface temperature parameter.

Substituting (5), (6) into Equations (1), (2) and (3) leads to the following non-dimensional equations

$$f'''+\theta-2f'^2+3ff''+\xi f''=\xi\left(f'\frac{\partial f'}{\partial\xi}f''\frac{\partial f'}{\partial\xi}\right) \quad (7)$$

$$\frac{1}{pr}\frac{\partial}{\partial\eta}\left[\frac{\partial\theta}{\partial\eta}\right]+N\xi^4\left(f''\right)+3f\theta'+\xi\theta'=\xi\left(f'\frac{\partial\theta}{\partial\xi}\frac{\partial f}{\partial\xi}\theta'\right);\ (8)$$

where $Pr=\nu C_p/k$ is the Prandtl number and $N=\nu^2 g\beta\Delta T/\rho C_p V^4$ is the viscous dissipation parameter.

The boundary conditions (4) become

$$\begin{aligned}&f=0, f'=0,\ \theta=1\ \text{at}\ \eta=0\\&f'=0,\ \theta=0\ \text{as}\ \eta\to\infty\end{aligned}. \quad (9)$$

The solution of equations (7), (8) enable us to calculate the nondimensional velocity components $\bar{u}, \bar{v}$ from the following expressions

$$\bar{u}=\frac{\nu^2}{Vg\beta(T_w-T_\infty)}u=\xi^2 f'(\xi,\eta)$$
$$\bar{v}=\frac{v}{V}=\xi^{-1}(3f+\xi-\eta f'+\xi\frac{\partial f}{\partial\xi}) \quad (4)$$

In practical applications, the physical quantities of principle interest are the shearing stress τ_w and the rate of heat transfer in terms of the skin-friction coefficients C_{fx} and Nusselt number Nu_x respectively, which can be written as

$$Nu_x=\frac{\nu}{V\Delta T}(q_c)_{\eta=0},\ C_f=\frac{V}{g\beta\Delta T}(\tau)_{\eta=0}$$

$$\text{where}\ \tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{\eta=0}\ \text{and}\ q_c=-k\left(\frac{\partial T}{\partial y}\right)_{\eta=0} \quad (12)$$

q_c is the conduction heat flux. (5)

Using the Equations (6) and the boundary condition (9) into (11) and (12), we get (6)

$$C_{fx}=\xi f''(x,0)$$
$$Nu_x=-\xi^{-1}\theta'(x,0).$$

The values of the velocity and temperature distribution are calculated respectively from the following relations:

$$\bar{u}=\xi^2 f'(\xi,\eta),\qquad \theta=\theta(x,y).\quad (14)$$

3. Method of Solution

Solutions of the local non similar partial differential equation (7) to (8) subjected to the boundary condition (9) are obtained by using implicite finite difference method with Keller-Box Scheme, which has been described in details by Cebeci [15].

The solution methodology of equations (7) and (8) with the boundary condition given in eqn. (9) for the entire ξ values based on Keller – box scheme is proposed here . The scheme specifically incorporated a nodal distribution favoring the vicinity of the plate, enabling accuracy to be maintained in this region of steep gradient. In detail equations (7) and (8) are solved as a set of five simultaneous equations.

$$f''' + 3ff'' - 2(f')^2 + \theta - \xi f'' = \xi\left(f'\frac{\partial f'}{\partial \xi} - \frac{\partial f}{\partial \xi} f''\right) \quad (15)$$

and

$$\frac{1}{\Pr}\frac{\partial}{\partial \eta}\left[\frac{\partial \theta}{\partial \eta}\right] + N(f'')^2 \xi^4 + 3f\theta + \xi\theta = \xi\left(f'\frac{\partial \theta}{\partial \xi} - \frac{\partial f}{\partial \xi}\theta\right). \quad (16)$$

To apply the aforementioned method, we first convert Equations (15)-(16) into the following system of first order equations with dependent variables $u(\xi,\eta)$, $v(\xi,\eta)$, $p(\xi,\eta)$ and $g(\xi,\eta)$ as

$f' = u$, $u' = v$, $g = \theta$, and $\theta' = p$

$$v' + p_1 fv - p_2 u^2 + g - \xi v = \xi\left(u\frac{\partial u}{\partial \xi} - \frac{\partial f}{\partial \xi} v\right) \quad (18)$$

$$\frac{1}{\Pr}\frac{\partial}{\partial \eta}[p] + p_4 v^2 \xi^4 + \xi p + p_1 fp = \xi\left(u\frac{\partial g}{\partial \xi} - p\frac{\partial f}{\partial \xi}\right) \quad (19)$$

where

$p_1 = 3,\ p_2 = 2,\ p_4 = N.$ (20)

The corresponding boundary conditions are

$$f(\xi,0) = 0, u(\xi,0) = 0 \text{ and } g(\xi,0) = 0$$
$$u(\xi,\infty) = 0, g(\xi,\infty) = 0 \quad (21)$$

We now consider the net rectangle on the (ξ,η) plane and denote the net point by

$$\eta_0 = 0, \quad \eta_j = \eta_{j-1} + h_j, \quad j = 1,2,\ldots J$$
$$\xi^0 = 0, \quad \xi^n = \xi^{n-1} + k_n, \quad n = 1,2,\ldots N$$

We approximate the quantities (*f, u, v, p*) at the points (ξ^n, η_j) of the net by $(f_j^n, u_j^n, v_j^n, p_j^n)$ which we call net functions as.

$$\eta_{j-1/2} = \tfrac{1}{2}(\eta_j - \eta_{j-1})$$

$$\xi^{n-1/2} = \tfrac{1}{2}(\xi^n + \xi^{n-1})$$

$$g_j^{n-1/2} = \tfrac{1}{2}(g_j^n + g_j^{n-1}) \quad (22)$$

$$g_{j-1/2}^n = \tfrac{1}{2}(g_j^n + g_{j-1}^n).$$

Now we write the difference equations that are to approximate Equations (17) - (19) by considering one mesh rectangle for the mid point $(\xi^n, n_{j-1/2})$ to obtain

$$\frac{f_j^n - f_{j-1}^n}{h_j} = u_{j-1/2}^n \quad (23)$$

$$\frac{u_j^n - u_{j-1}^n}{h_j} = v_{j-1/2}^n \quad (24)$$

$$\frac{g_j^n - g_{j-1}^n}{h_j} = p_{j-1/2}^n \quad (25)$$

Similarly Equations (18) – (19) are approximated by centering about the midpoint $(\xi^{n-1/2}, n_{j-1/2})$. Centering the Equations (17) (22) about the point $(\xi^{n-1/2}, n)$ without specifying η to obtain the algebraic equations. The difference approximation to Equations (18)-(19) becomes

$$h_j^{-1}(v_j^n - v_{j-1}^n) + \{(p_1)_{j-1/2}^n + \alpha_n\}(fv)_{j-1/2}^n - \{(p_2)_{j-1/2}^n + \alpha_n\}(u^2)_{j-1/2}^n + g_{j-1/2}^n$$
$$-(\xi v)_{j-1/2}^n + \alpha_n\{f_{j-1/2}^n v_{j-1/2}^{n-1} - v_{j-1/2}^n f_{j-1/2}^{n-1}\} = R_{j-1/2}^{n-1}$$

where

$$L_{j-\frac{1}{2}}^{n-1} = (p_1)_{j-\frac{1}{2}}^{n-1}(fv)_{j-\frac{1}{2}}^{n-1} - (p_2)_{j-\frac{1}{2}}^{n-1}(u^2)_{j-\frac{1}{2}}^{n-1} + g_{j-\frac{1}{2}}^{n-1}$$
$$-(\xi p)_{j-\frac{1}{2}}^{n-1} + h_j^{-1}(v_j^{n-1} - v_{j-1}^{n-1})$$

And

$$R_{j-\frac{1}{2}}^{n-1} = -L_{j-\frac{1}{2}}^{n-1} + \alpha_n\left\{-(u^2)_{j-\frac{1}{2}}^{n-1} + (fv)_{j-\frac{1}{2}}^{n-1}\right\}$$

The corresponding boundary conditions (21) become

$$f_0^n = 0, \quad u_0^n = 0, \quad g_0^n = 1$$
$$u_J^n = 0, \quad g_J^n = 0$$

which just express the requirement for the boundary conditions to remain during the iteration process. Now we will convert the momentum and energy equations into system of linear equations and together with the boundary conditions can be written in matrix or vector form, where the coefficient matrix has a block tri-diagonal structure. The whole procedure, namely reduction to first order followed by central difference approximations, Newton's quasi-linearization method and the block Thomas algorithm, is well known as the Keller-box method.

$$\frac{1}{\Pr}[h_j^{-1}(p_j^n - p_{j-1}^n) + \xi_{j-\frac{1}{2}}^n p_{j-\frac{1}{2}}^n + (p_4\xi^4)_{j-\frac{1}{2}}^n \left(v^2\right)_{j-\frac{1}{2}}^n + (p_1)_{j-\frac{1}{2}}^n (f\,p)_{j-\frac{1}{2}}^n = -M_{j-\frac{1}{2}}^{n-1} + \alpha_n\,[-(ug)_{j-\frac{1}{2}}^{n-1}$$
$$+(f\,p)_{j-\frac{1}{2}}^{n-1}] + \alpha_n\,[(ug)_{j-\frac{1}{2}}^n - (f\,p)_{j-\frac{1}{2}}^n - u_{j-\frac{1}{2}}^n g_{j-\frac{1}{2}}^{n-1} + u_{j-\frac{1}{2}}^{n-1} g_{j-\frac{1}{2}}^n\} + p_{j-\frac{1}{2}}^n f_{j-\frac{1}{2}}^{n-1} - p_{j-\frac{1}{2}}^{n-1} f_{j-\frac{1}{2}}^n]$$
$$\frac{1}{\Pr}[h_j^{-1}(p_j^n - p_{j-1}^n) + \xi_{j-\frac{1}{2}}^n p_{j-\frac{1}{2}}^n + (p_4\xi^4)_{j-\frac{1}{2}}^n \left(v^2\right)_{j-\frac{1}{2}}^n + \{(p_1)_{j-\frac{1}{2}}^n + \alpha_n\}(f\,p)_{j-\frac{1}{2}}^n$$
$$-\alpha_n\,[\{(ug)_{j-\frac{1}{2}}^n - (ug)_{j-\frac{1}{2}}^{n-1} - u_{j-\frac{1}{2}}^n g_{j-\frac{1}{2}}^{n-1} + u_{j-\frac{1}{2}}^{n-1} g_{j-\frac{1}{2}}^n\} + p_{j-\frac{1}{2}}^n f_{j-\frac{1}{2}}^{n-1} - p_{j-\frac{1}{2}}^{n-1} f_{j-\frac{1}{2}}^n\}] = T_{j-\frac{1}{2}}^{n-1}$$

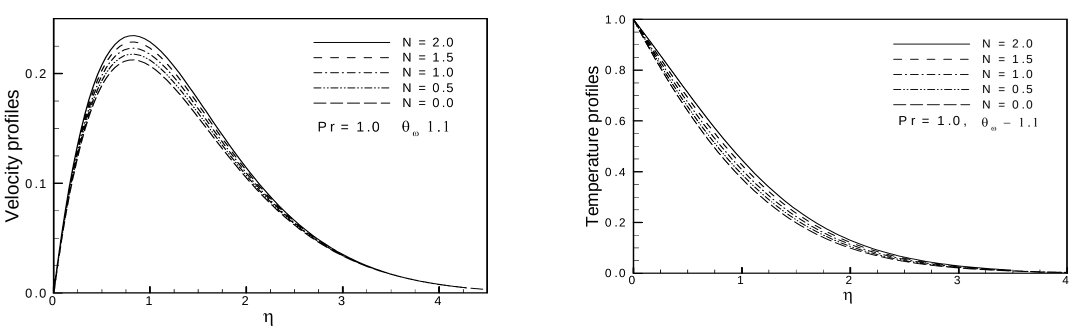
where

$$M_{j-\frac{1}{2}}^{n-1} = \frac{1}{\Pr}[h_j^{-1}(p_j^{n-1} - p_{j-1}^{n-1}) - [\xi_{j-\frac{1}{2}}^{n-1} p_{j-\frac{1}{2}}^{n-1} + (p_4\xi^4)_{j-\frac{1}{2}}^{n-1} \left(v^2\right)_{j-\frac{1}{2}}^{n-1} + (p_1)_{j-\frac{1}{2}}^{n-1} (f\,p)_{j-\frac{1}{2}}^{n-1}]$$
$$T_{j-\frac{1}{2}}^{n-1} = -M_{j-\frac{1}{2}}^{n-1} + \alpha_n [(f\,p)_{j-\frac{1}{2}}^{n-1} - (u\,g)_{j-\frac{1}{2}}^{n-1}]$$

4. Results and Discussion

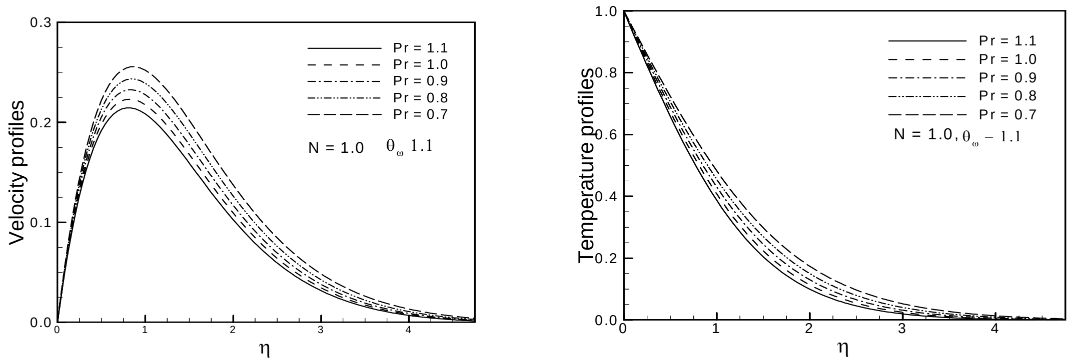
In this exertion natural convection flow on a porous vertical plate in presence of viscous dissipation is investigated. Numerical values of local rate of heat transfer are calculated in terms of Nusselt number Nu_x for the surface of the porous vertical plate from lower stagnation point to upper stagnation point, for different values of the aforementioned parameters and these are shown in tabular form in Table:1 and Table:2 and graphically in Fiqure 2-5. The effect for different values of viscous dissipation parameter N on local skin friction coefficient C_{fx} and the local Nusselt number Nu_x, as well as velocity and temperature profiles are displayed in Fig.2 and 5.The aim of these figures are to display how the profiles vary in ξ, the selected streetwise co-ordinate.

Figures 2(a)-2(b) display results for the velocity and temperature profiles, for different values of viscous dissipation parameter N with Prandtl number Pr = 1.0 surface temperature parameter θ_w = 1.1. It has been seen from Figures 2(a) and 2(b) that as the viscous dissipation parameter N increases, the velocity and the temperature profiles increase. The changes of velocity profiles in the η direction reveals the typical velocity profile for natural convection boundary layer flow, i.e., the velocity is zero at the boundary wall then the velocity increases to the peak value as η increases and finally the velocity approaches to zero (the asymptotic value). The maximum values of velocity are recorded to be 0.21244, 0.21770, 0.22315, 0.22879 at and 0.23464 at η = 0.83530 for N = 0.0, 0.5, 1.0, 1.5 and 2.0 respectively, the maximum values of velocity are recorded to be 0.23464. Here, it is observed that at η = 0.83530, the velocity increases by 21.46% as the viscous dissipation parameter N changes from 0.0 to 2.0. The changes of temperature profiles in the η direction also shows the typical temperature profile for natural convection boundary layer flow that is the value of temperature profile is 1.0 (one) at the boundary wall then the temperature profile decreases gradually along η direction.



(a) (b)

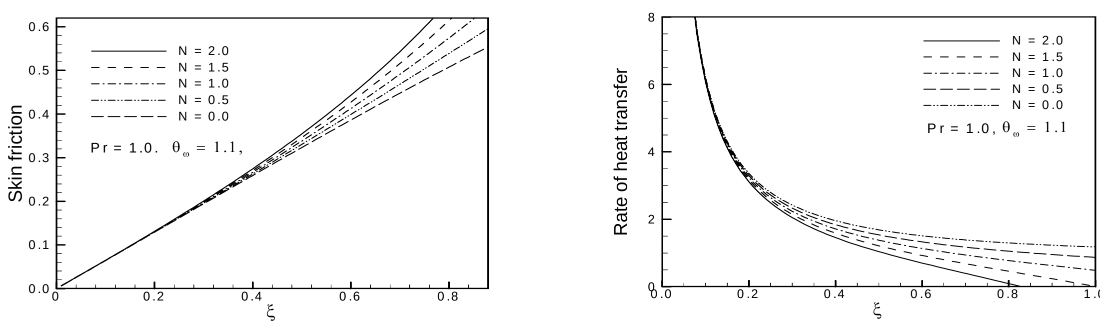
Fig. 2: (a) Velocity and (b) temperature profiles for different values of viscous dissipation parameter N with others fixed parameters.



(a) (b)

Fig. 3: (a) Velocity and (b) temperature profiles for different values of prandtl number *Pr* with others fixed parameters.

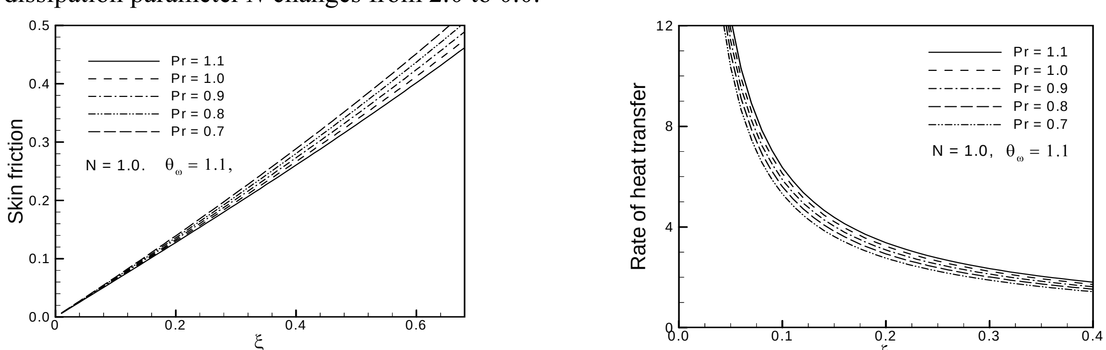
In Figures 3(a)-3(b), it is shown that when the Prandtl number *Pr* increases with $\theta_w = 1.1$ and $N = 1.0$, both the velocity and temperature profiles decrease.



(a) (b)

Fig. 4: (a) Skin friction and (b) rate of heat transfer for different values of viscous dissipation parameter *N* with others fixed parameters.

Fig. 6(a)-6(b) show that skin friction coefficient C_{fx} increases and heat transfer coefficient Nu_x decrease respectively for increasing values of viscous dissipation parameter *N* in case of Prandtl number $Pr = 1.0$ and surface temperature parameter $\theta_w = 1.1$. The values of skin friction coefficient C_{fx} and Nusselt number Nu_x are recorded to be 0.65876, 0.61401, 0.57413, 0.53873, 0.50750 and 0.09120, 0.45806, 0.77642, 1.05429, 1.29838 for N = 2.0, 1.5, 1.0, 0.5 and 0.0 respectively which occur at the same point $\xi = 0.8$. Here, it is observed that at $\xi = 0.8$, the skin friction increases by 29.80% and Nusselt number *Nu* decreases by 92.97% as the viscous dissipation parameter *N* changes from 2.0 to 0.0.



(a) (b)

Fig. 5: (a) Skin friction and (b) rate of heat transfer for different values of Prandtl number parameter *Pr* with others fixed parameters.

The variation of the local skin friction coefficient C_{fx} and local rate of heat transfer Nu_x for different values of Prandtl number Pr for $\theta_w = 1.1$ and $N = 1.0$ are shown in Figures 5(a)-5(b). We can observe from these figures that as the Prandtl number Pr increases, the skin friction coefficient decreases and rate of heat transfer increase.

Numerical results of skin friction and rate of heat transfer are calculated from equation (13) for the surface of the porous plate from lower stagnation point to upper stagnation point at $\xi = 0.01$ to $\xi = 0.71$. Numerical values of C_{fx} and Nu_x are depicted in Table .1.

Table 1: Skin friction coefficient and rate of heat transfer against ξ for different values of viscous dissipation parameter N with other controlling parameters $Pr = 1.0$, $\theta_w = 1.1$

ξ	N=0.0		N=0.5		N= 1.5		N=2.0	
	C_{fx}	Nu_x	C_{fx}	Nu_x	C_{fx}	Nu_x	C_{fx}	Nu_x
0.01	0.00642	57.686145	0.00642	57.6800615	0.00642	57.667895	0.00642	57.6618
0.11	0.07099	.68304	0.07107	.64827	0.07123	.57850	0.07130	05.5434
0.21	0.13593	3.22613	0.13646	3.16224	0.13753	3.03288	0.13808	9
0.31	0.20093	2.36292	0.20263	2.26958	0.20613	2.07782	0.20796	2.96739
0.41	0.26562	1.92712	0.26960	1.80397	0.27793	1.54562	0.28230	1.97953
0.51	0.32963	1.66724	0.33738	1.51383	0.35403	1.18303	0.36300	
0.61	0.39256	1.49672	0.40600	1.31256	0.43588	0.90142	0.45245	1.41005
0.71	0.45402	1.37777	0.47549	1.16209	0.52531	0.66010	0.55389	1.00441
								0.67106
								0.36565

Here in the above table the values of skin friction coefficient C_{fx} and Nusselt number Nu_x are recorded to be 0.45402, 0.47549, 0.52531, 0.55389 and 1.37777, 1.16209, 0.66010, 0.36565 for N = 0.0, 0.5, 0.5 and 2.0 respectively which occur at the same point $\xi = 0.71$. Here, it is observed that at $\xi = 0.71$, the skin friction increases by 21.99% and Nusselt number Nu decreases by 73.46% as the viscous dissipation parameter N changes from 0.0 to 2.0.

5. Comparison of the Results

In order to verify the accuracy of the present work, the values of Nusselt number and skin friction for $N = 0$, $R_d = 0.05$.$Pr = 1.0$ and various surface temperatures θ_w=1.1, θ_w=2.5 at different positions of ξ are compared with Hossain et al. [9] as presented in Table 2. The results are found to be in excellent agreement.

Table 2: Comparison of the present result with Hossain et al

ξ	θw= 1.1				θw = 2.5			
	Hossain		Present		Hossain		Present	
	C_{fx}	Nu_x	C_{fx}	Nu_x	C_{fx}	Nu_x	C_{fx}	Nu_x
0.1	0.0655	6.4627	0.06535	6.48306	0.0709	8.0844	0.07078	8.10360
0.2	0.1316	3.4928	0.13138	3.50282	0.1433	4.2858	0.14313	4.29682
0.4	0.2647	2.0229	0.26408	2.03018	0.2917	2.4003	0.29120	2.40669
0.6	0.3963	1.5439	0.39519	1.55522	0.4423	1.7863	0.44145	1.78912
0.8	0.5235	1.3247	0.52166	1.32959	0.5922	1.4860	0.59080	1.48991
1.0	0.6429	1.1995	0.64024	1.20347	0.7379	1.1098	0.73590	1.31822
1.5	0.8874	1.0574	0.88192	1.06109	1.0613	1.1098	1.05693	1.11262

6. Conclusion

Natural convection flow on a porous vertical plate in presence of viscous dissipation has been investigated for different values of relevant physical parameters including Prandtl number *Pr,* and viscous dissipation parameter *N*.

- Significant effects of viscous dissipation parameter N on velocity and temperature profiles as well as on skin friction and the rate of heat transfer have been found in this investigation but the effect of viscous dissipation parameter N on rate of heat transfer is more significant. An increase in the values of viscous dissipation parameter N leads to increase both the velocity and the temperature profiles, the local skin friction coefficient C_{fx} increases at different position of η and the local rate of heat transfer Nu_x decreases at different position of ξ.
- The increase in Prandtl number Pr leads to decrease in all the velocity profile, the temperature profile, the local skin friction coefficient C_{fx} but the local rate of heat transfer Nu_x increase.

References

[1] J. H. Merkin, Free convection with blowing and suction, International journal of heat and mass transfer, 15, pp. 989-999, 1972.

[2] H. T. Lin, W. S. Yu, Free convection on horizontal plate with blowing and suction. Transactions on ASME journal of Heat Transfer 110, 793-796, 1988.

[3] M. Anwar Hossain, Khalil Khanafer, Kambiz Khafai. The effect of radiation on free convection flow with variable viscosity from a porous vertical plate, International Journal of Thermal science, 40, pp. 115-124, 1999.

[4] B. Gebhart, Effects of viscous dissipation in natural convection, *J. Fluid Mech.*, 14(2), pp. 225–232, 1962.

[5] B. Gebhart, J. Mollendorf, Viscous dissipation in external natural convection flows, *J. Fluid Mech.*, 38(1), pp. 97–107, 1969.

[6] M.K. Chowdhury, M.N. Islam, MHD free convection flow of visco- elastic fluid past an infinite porous plate, *Heat and Mass Transfer*, 36(5), pp. 439–447, 2000.

[7] M.A. Hossain, Viscous and Joule heating effects on MHD-free convection flow with variable plate temperature, *Int. J. Heat Mass Transfer*, 35(12), pp. 3485-3487, 1992.

[8] Md.M. Alam, M.A. Alim, Md.M.K. ChowdhuryViscous Dissipation Effects on MHD Natural Convection Flow over a Sphere in the Presence of Heat Generation Nonlinear Analysis: Modelling and Control, 2007, Vol. 12, No. 4, 447–459..

[9] M. A. Hossain, M. A. Alim, D. A. S. Rees, The effect of radiation on free convection flow from a porous vertical plate, International Journal of Heat and Mass Transfer, 42, pp. 81-91, 1999.

[10] K. Vejravelu, A. Hadjinicolaou, Heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation. Int. comm. Heat Transfer, 20, 417-430, 1993.

[11] H. S. Takhar, V. M. Soundalgekar, Dissipation effects on MHD free convectionflow past a semi infinite vertical plate, Applide Scientific research 36, pp 163-171, 1980.

[12] O. D. Makinde, A. Ogulu: The effect of thermal radiation on the heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field. Chemical Engineering Communications, Vol. 195, No. 12, 1575 -1584, 2008.

[13] Md.M. Alam, M.A. Alim, Md.M.K. Chowdhury, Viscous dissipation effects on MHD natural convection flow along a sphere, Journal of Mechanical Engineering , V. 36, pp. 44-48, 2006.

[14] H.B. Keller, Numerical methods in boundary layer theory, Annual Review of Fluid Mechanics, 10, pp. 417-433, 1978.

[15] T. Cebeci, P. Bradshaw, Physical and Computational Aspects of Convective Heat Transfer, Springer, New York, 1984.