

A study of Performance Analysis of Image Compression Methods and Denoising Natural Image

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A Thesis Report Submitted to the Department of Electronics and Telecommunication Engineering in Partial Fulfillment of the Degree of Bachelor of Science in Electronics and Telecommunication Engineering.

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APPROVAL

This thesis report “**A study of Performance Analysis of Image Compression Methods and Denoising Natural Image**” is submitted by Md. Ashikur Rahman Sarkar, Md. Mahmudul Hasan And Md. Abdul Khalek to the Department of Electronics and Telecommunication Engineering, in Daffodil International University, has been accepted as satisfactory for the partial fulfillment of the requirements for the degree of Bachelor of Science in Electronics and Telecommunication Engineering. The presentation was held on 27-29 August 2012.

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DECLARATION

It's hereby declared that except for the contents where specific references have been made to the work of others, the studies contained in this thesis are the result of investigation carried out by the authors under the supervision of **Assistant Professor Ms. Shahina Haque, Department of Electronics and Telecommunication Engineering**, Daffodil International University, Dhaka. No part of this thesis has been submitted to any other university or other educational establishments for a degree, diploma or other qualifications.

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ABSTRACT

Image compression is in high demand as it reduces the computational time and consequently the cost in image storage and transmission. The basis for image compression is to remove redundant and unimportant data while to keep the compressed image quality in an acceptable range. Image compression is an application of data compression on digital images which is in highly demand as it reduces the computational time and consequently the cost in image storage and transmission. Wavelet Transform (WT) is used for compressing a natural image, De noise. Several wavelets (Haar, Daubechies, Coiflet, Symmlet and Dmey) are used for compressing and Denoising image. As a measure of performance Root Mean Square Error (RMSE), Peak Signal to Noise Ratio (PSNR), Compression Ratio (CR) are calculated between the original and compressed images. It is observed from our study that Haar performs better (lowest RMSE, highest PSNR and CR) than any other tested wavelets for natural Image. JPEG compression method performs 48.85% better than Haar wavelet.

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Chapter 1

Introduction

1.1 Image

An image is essentially a 2-D signal processed by the human visual system. The signals representing images are usually in analog form. However, for image processing, storage and transmission, they are converted from analog to digital form. A digital image is basically a 2-D array of pixels. Images are formed of the significant part of data, particularly in remote sensing, biomedical and video conferencing applications. The use of and dependence on information and computers continue to grow, so does our need for efficient ways of storing and transmitting large amounts of data.

1. [djj.ee.ntu.edu.tw/NewImageCompression.docx]

1.2 Image Compression

Image Compression is different from data compression (binary data). When we apply techniques used for binary data compression to the images, the results are not optimal. In Lossless compression the data (binary data such as executables, documents etc) are compressed such that when decompressed, it give an exact replica of the original data. They need to be exactly reproduced when decompressed. On the other hand, images need not be reproduced 'exactly'. An approximation of the original image is enough for most purposes, as long as the error between the original and the compressed image is tolerable. Lossy compression techniques can be used in this area. This is because images have certain statistical properties, which can be exploited by encoders specifically designed for them. Also, some of the finer details in the image can be sacrificed for the sake of saving a little more bandwidth or storage space. In images the neighboring pixels are correlated and therefore contain redundant information. Before we compress an image, we first find out the pixels, which are correlated. The fundamental components of compression are redundancy and irrelevancy reduction. Redundancy means duplication and Irrelevancy means the parts of signal that will not be noticed by the signal receiver, which is the Human Visual System

(HVS). Image compression addresses the problem of reducing the amount of data required to represent a digital image. It is a process intended to yield a compact representation of an image, thereby reducing the image storage/transmission requirements. Compression is achieved by the removal of one or more of the three basic data redundancies.

1. Coding Redundancy
2. Interpixel Redundancy
3. Psychovisual Redundancy

Coding redundancy is present when less than optimal code words are used. Inter pixel redundancy results from correlations between the pixels of an image. Psychovisual redundancy is due to data that is ignored by the human visual system (i.e. visually non essential information). Image compression techniques reduce the number of bits required to represent an image by taking advantage of these redundancies. An inverse process called decompression (decoding) is applied to the compressed data to get the reconstructed image. The objective of compression is to reduce the number of bits as much as possible, while keeping the resolution and the visual quality of the reconstructed image as close to the original image as possible.

2./djj.ee.ntu.edu.tw/NewImageCompression.docx]

1.3 Principle Intensity level representation of Image

Images require much storage space, large transmission bandwidth and long transmission time. The only way currently to improve on these resource requirements is to compress images, such that they can be transmitted quicker and then decompressed by the receiver. In image processing there are 256 intensity levels (scales) of grey. 0 is black and 255 is white. Each level is represented by an 8-bit binary number so black is 00000000 and white is 11111111. An image can therefore be thought of as grid of pixels, where each pixel can be represented by the 8-bit binary value for grey-scale.



Figure 1-1 : Intensity level representation of image

The resolution of an image is the pixels per square inch. (So 256 dpi means that a pixel is 1/256th of an inch). To digit is a one-inch square image at 256 dpi requires $8 \times 256 \times 256 = 524288$ storage bits Using this representation it is clear that image data compression is a great advantage if many images are to be stored, transmitted or processed. According to "Image compression algorithms aim to remove redundancy in data in a way which makes image reconstruction possible." This basically means that image compression algorithms try to exploit redundancies in the data; they calculate which data needs to be kept in order to reconstruct the original image and therefore which data can be thrown away. By removing the redundant data, the image can be represented in a smaller number of bits, and hence can be compressed. But what is redundant information? Redundancy reduction is aimed at removing duplication in the image. According to there are two different types of redundancy relevant to images: (i) Spatial Redundancy correlation between neighboring pixels. (ii) Spectral Redundancy - correlation between different color planes and spectral bands. Where there is high correlation, there is also high redundancy, so it may not be necessary to record the data for every pixel.

3.[www.scribd.com/.../PRINCIPLES-OF-USING-TRANSFORM-AS-SO...]

1.4 Image compression techniques

The image compression techniques are broadly classified into two categories depending whether or not an exact replica of the original image could be reconstructed using the compressed image. These are:

1. Lossless technique
2. Lossy technique

1.4.1 Lossless compression technique

In lossless compression techniques, the original image can be perfectly recovered from the compressed (encoded) image. These are also called noiseless since they do not add noise to the signal (image). It is also known as entropy coding since it uses statistics/decomposition techniques to eliminate/minimize redundancy. Lossless compression is used only for a few applications with stringent requirements such as medical imaging.

Following techniques are included in lossless compression:

1. Run length encoding
 2. Huffman encoding
 3. LZW coding
 4. Area coding
4. [djj.ee.ntu.edu.tw/New**Image**Compression.docx]

1.4.2 Lossy compression technique

Lossy schemes provide much higher compression ratios than lossless schemes. Lossy schemes are widely used since the quality of the reconstructed images is adequate for most applications. By this scheme, the decompressed image is not identical to the original image, but reasonably close to it.

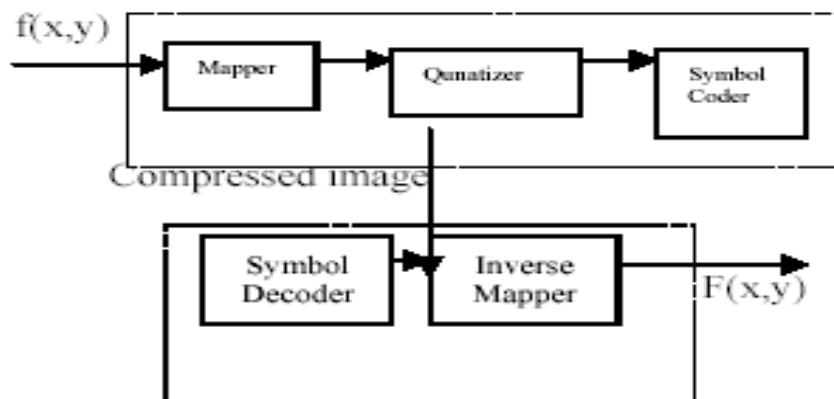


Figure 1-2: Outline of lossy compression techniques

As shown above the outline of lossy compression techniques. In this prediction – transformation – decomposition process is completely reversible. The quantization process results in loss of information. The entropy coding after the quantization step, however, is lossless. The decoding is a reverse process. Firstly, entropy decoding is applied to compressed data to get the quantized data. Secondly, dequantization is applied to it & finally the inverse transformation to get the reconstructed image.

Major performance considerations of a lossy compression scheme include:

1. Compression ratio
2. Signal - to - noise ratio
3. Speed of encoding & decoding.

Lossy compression techniques includes following schemes

1. Transformation coding
2. Vector quantization
3. Fractal coding
4. Block Truncation Coding
5. Subband coding

5.[www.rimtengg.com/coit2007/proceedings/pdfs/43.pdf]

1.5 Image Compression and Reconstruction

Three basic data redundancies can be categorized in the image compression standard.

1. Spatial redundancy due to the correlation between neighboring pixels.
2. Spectral redundancy due to correlation between the color components.
3. Psycho-visual redundancy due to properties of the human visual system.

The spatial and spectral redundancies are present because certain spatial and spectral patterns between the pixels and the color components are common to each other, whereas the psycho-visual redundancy originates from the fact that the human eye is insensitive to certain spatial frequencies. The principle of image compression algorithms are (i) reducing the redundancy in the image data and (or) (ii) producing a reconstructed image from the original image with the introduction of error that is insignificant to the intended applications.

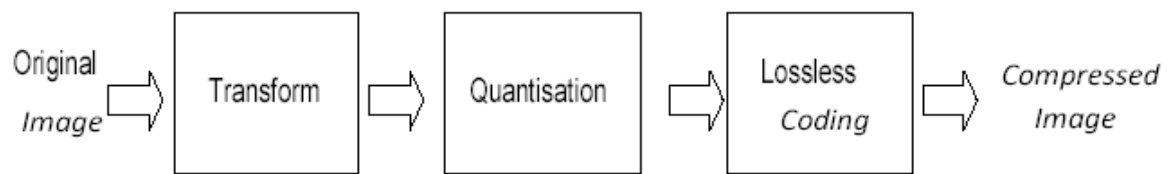


Figure 1-3: Image compression System

The problem faced by image compression is very easy to define, as demonstrated in figure

1-3. First the original digital image is usually transformed into another domain, where it is highly de-correlated by using some transform. This decorrelation concentrates the important image information into a more compact form. The compressor then removes the redundancy in the transformed image and stores it into a compressed file or data stream. In the second stage, the quantization block reduces the accuracy of the transformed output in accordance with some pre-established fidelity criterion. Also this stage reduces the psycho-visual redundancy of the input image. Quantization operation is a reversible process and thus may be omitted when there is a need of error free or lossless compression. In the final stage of the data compression model the symbol coder creates a fixed or variable-length code to represent the quantized output and maps the output in accordance with the code. Generally a variable-length code is used to represent the mapped and quantized data set. It assigns the shortest code words to the most frequently occurring output values and thus reduces coding redundancy. The operation in fact is a reversible one. The decompression reverses the compression process to produce the recovered image as shown in figure 1-4. The recovered image may have lost some information due to the compression, and may have an error or distortion compared to the original image.

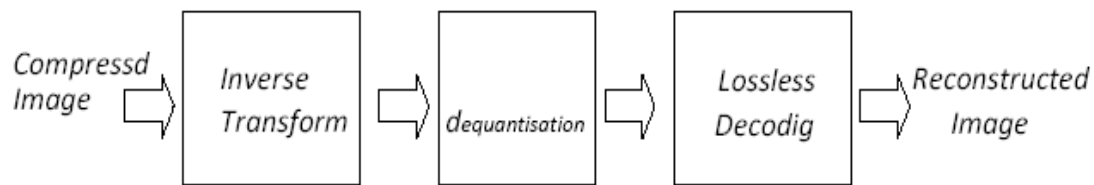


Figure 1-4 : Image decompression System

6.[ethesis.nitrkl.ac.in/.../Image_Compression_using_DCT_%26_DWT.pdf]

1.6 Why Need for Image Compression:

With the advance development in Internet and multimedia technologies, the amount of information that is handled by computers has grown exponentially over the past decades. This information requires large amount of storage space and transmission bandwidth that the current technology is unable to handle technically and economically. One of the possible solutions to this problem is to compress the information so that the storage space and transmission time can be reduced. A common Characteristic that can be found in most images is that they contain redundant information. This redundant information can be classified as :

- Spatial redundancy - Correlation between neighboring pixels values
- Spectral redundancy - Correlation between different spectral bands

1.7 Benefits of compression

- It provides a potential cost savings associated with sending less data over switched Telephone network where cost of call is really usually based upon its duration.
- It not only reduces storage requirements but also overall execution time.
- It also reduces the probability of transmission errors since fewer bits are transferred.
- It also provides a level of security against illicit monitoring.

The motivation for the compression of images is illustrated through the use of Figure 1-3. This figure shows the storage size, transmission bandwidth, and transmission time needed for various types of uncompressed images. It is clear from these values, that images require much storage space, large transmission bandwidths, and long transmission times. With the present state of technology, the only solution is to compress images before their storage and transmission. Then, at the receiver end, the

compressed images can be decompressed.

7.[djj.ee.ntu.edu.tw/NewImageCompression.docx]

Chapter 2

Theory of Wavelet Compression

2.1 Wavelet transform:

Wavelets are mathematical functions that cut up data into different frequency components, and then study each component with a resolution matched to its scale. They have advantages over traditional Fourier methods in analyzing physical situations where the signal contains discontinuities and sharp spikes. Wavelets were developed independently in the fields of mathematics, quantum physics, electrical engineering, and seismic geology. Interchanges between these fields during the last ten years have led to many new wavelet applications such as image compression, turbulence, human vision, radar, and earthquake prediction. We describe the history of wavelets beginning with Fourier, compare wavelet transforms with Fourier transforms, state properties and other special aspects of wavelets, and finish with some interesting applications such as image compression, musical tones, and de-noising noisy data. 8. [en.wikipedia.org/wiki/Wavelet]

2.2 History of wavelet transform

In the history of mathematics, wavelet analysis shows many different origins. Much of the work was performed in the 1930s, and, at the time, the separate efforts did not appear to be parts of a coherent theory.

Pre-1930

Before 1930, the main branch of mathematics leading to wavelets began with Joseph Fourier (1807) with his theories of frequency analysis, now often referred to as Fourier synthesis. He asserted that any 2π -periodic function $f(x)$ is the sum of its Fourier series.

$$\alpha_0 + \sum_{k=1}^{\infty} (\alpha_k \cos kx + b_k \sin kx) \quad (1)$$

The coefficients α_0 , α_k , and b_k are calculated by

$$\alpha_0 = \frac{1}{2\pi} \int_0^{2\pi} f(x) dx, \quad \alpha_k = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos(kx) dx, \quad b_k = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin(kx) dx$$

Fourier's assertion played an essential role in the evolution of the ideas mathematicians had about the functions. He opened up the door to a new functional universe.

After 1807, by exploring the meaning of functions, Fourier series convergence, and orthogonal systems, mathematicians gradually were led from their previous notion of frequency analysis to the notion of scale analysis. That is, analyzing $f(x)$ by creating mathematical structures that vary in scale. How? Construct a function, shift it by some amount, and change its scale. Apply that structure in approximating a signal. Now repeat the procedure. Take that basic structure, shift it, and scale it again. Apply it to the same signal to get a new approximation. And so on. It turns out that this sort of scale analysis is less sensitive to noise because it measures the average fluctuations of the signal at different scales.

Haar (1909). One property of the Haar wavelet is that it *has* compact support, which means that it vanishes outside of a finite interval. Unfortunately, Haar wavelets are not continuously differentiable which somewhat limits their applications.

The 1930s

In the 1930s, several groups working independently researched the representation of functions using scale-varying basis functions. Understanding the concepts of basis functions and scale-varying basis functions is key to understanding wavelets.

By using a scale-varying basis function called the Haar basis function (more on this

later) Paul Levy, a 1930s physicist, investigated Brownian motion, a type of random signal. He found the Haar basis function superior to the Fourier basis functions for studying small complicated details in the Brownian motion.

Another 1930s research effort by Little wood, Paley, and Stein involved computing the energy of a function $f(x)$:

$$\text{energy} = \frac{1}{2} \int_0^x |f(x)|^2 dx \quad (2)$$

The computation produced different results if the energy was concentrated around a few points or distributed over a larger interval. This result disturbed the scientists because it indicated that energy might not be conserved. The researchers discovered a function that can vary in scale *and* can conserve energy when computing the functional energy. Their work provided David Marr with an effective algorithm for numerical image processing using wavelets in the early 1980s.

1960-

1980

Between 1960 and 1980, the mathematicians Guido Weiss and Ronald R. Coifman studied the simplest elements of a function space, called *atoms*, with the goal of finding the atoms for a common function and finding the "assembly rules" that allow the reconstruction of all the elements of the function space using these atoms. In 1980, Grossman and Morlet, a physicist and an engineer, broadly defined wavelets in the context of quantum physics. These two researchers provided a way of thinking for wavelets based on physical intuition.

Post-

1980

In 1985, Stephane Mallat gave wavelets an additional jump-start through his work in digital signal processing. He discovered some relationships between quadrature mirror filters, pyramid algorithms, and orthonormal wavelet bases (more on these later). Inspired in part by these results, Y. Meyer constructed the first non-trivial

wavelets. Unlike the Haar wavelets, the Meyer wavelets are continuously differentiable; however they do not have compact support. A couple of years later, Ingrid Daubechies used Mallat's work to construct a set of wavelet orthonormal basis functions that are perhaps the most elegant, and have become the cornerstone of wavelet applications today.

9.[www.cis.udel.edu/~amer/CISC651/IEEEwavelet.pdf]

2.3 Three type of wavelet transform

There are three type of wavelet transform. Such as the continuous, semidiscrete and discrete time analysis. The distinction among the various types of WT depends on the way in which the scale and shift parameters are discretized. In this section we will look closer at the three type of these possibilities.

2.3.1 Continuous wavelet transform

For CWT the parameters vary in a continuous fashion. This representation offers the maximum freedom in the choice of the analysis wavelet. The only requirement is that the wavelet satisfies an admissibility condition, in particular it must have zero mean. The condition is also crucial to be CWT invertible on its range. The inverse transform

is given

byrelation:

$$f(x) = \frac{1}{K_\psi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} C(a,b) \psi(a,b) \frac{dad b}{a^2}$$

and ψ satisfies the admissibility condition:

$$K_\psi = \int_{-\infty}^{+\infty} \frac{|\hat{\psi}(w)|^2}{w} dw < \infty$$

where $\hat{\psi}$ is the FT of ψ

From an intuitive point of view, the CWT consists of calculating a -resemblance index between the signal and the wavelet (recall the definition of autocorrelation function.)

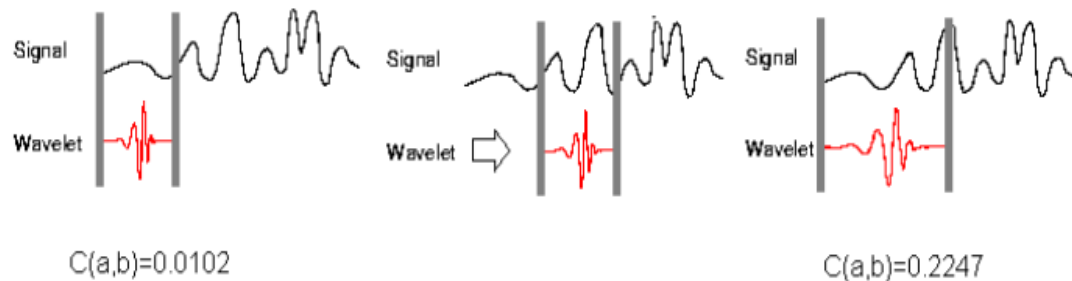


Figure 2-1 : The demonstration of CWT according to the equation

The algorithm of CWT can be described as following:

1. We take a wavelet and compare it to a section at the start of the original signal.
2. We calculate a coefficient $C(a,b)$, that represents how closely correlated the wavelet is with this section of the signal. The higher C is, the more the similarity and note that the results will depend on the shape of the wavelet we choose.
3. We shift the wavelet to the right and repeat steps 1 and 2 until we have covered the whole signal.
4. We scale the wavelet and repeat steps 1 through 3.

One example of CWT coefficients of Gold Standard signal is shown in figure 2-1.

2.3.2 Semi discrete wavelet transform:

In practice, it is often more convenient to consider WT for some discretized values a and b . For example the dyadic scales $a = 2^{-j}$ and integer shifts $b = 2^{-j}k$ with $k \in \mathbb{Z}$, let's call the scheme semi discrete wavelet transform (SWT). The transform will be reversible if the corresponding set of templates defines a wavelet frame. In other words, the wavelet must be designed such that:

$$A\|f\|^2 \leq \sum_{a,b} |\langle f, \psi(a,b) \rangle|^2 \leq B\|f\|^2$$

Where A and B are two positive constants called frame bounds. Notice, that we must still integrate to get wavelet coefficients, the $f(t)$ is still a continuous function.

2.3.3 Discrete wavelet transform

Here, we have discrete function $f(n)$ and the definition of discrete wavelet transform (DWT) is given by

$$C(a,b) = C(j,k) = \sum_{n \in \mathbb{Z}} f(n) \psi_{j,k}(n)$$

where $\psi_{j,k}$ is a discrete wavelet defined as:

$$\psi_{j,k}(n) = 2^{-j/2} \psi(2^{-j}n - k)$$

The parameters a, b are defined in such a way that $a = 2^{-j}$, $b = 2^{-j}k$. Sometimes the analysis is called dyadic as well. The inverse transform is defined in a similar way like If the frame bounds in are such that $A=B=1$, then the transformation is orthogonal.

$$f(n) = \sum_{j \in \mathbb{Z}} \sum_{k \in \mathbb{Z}} C(j,k) \psi_{j,k}(n)$$

2.4 Parameters and Equations

It is natural to raise the question of how much an image can be compressed and still preserve sufficient information for a given clinical application. This section discusses some parameters used to measure the trade-off between image quality and compression ratio. Compression ratio is defined as the nominal bit depth of the original image in bits per pixel (bpp) divided by the bpp necessary to store the compressed image. For each compressed and reconstructed image, an error image was calculated. From the error data, mean square error (MSE), root mean square error (RMSE), signal to noise ratio (SNR), and peak signal to noise ratio (PSNR) were calculated.

10.[cyber.felk.cvut.cz/gerstner/biolab/bio_web/.../WaveletTheory.pdf]

2.4.1 Mean square error

Mean Square Error measures the error with respect to the center of the image values, i.e. the mean of the pixel values of the image, and by averaging the sum of squares of the error between the two images.

$$MSE(u, v) = \frac{1}{MN} \sum_{m=1}^M \sum_{n=1}^N | u(m, n) - v(m, n) |^2$$

Where, $u(m, n)$ and $v(m, n)$ represent two images of size $m \times n$. In our case u is the original image and v is the reconstructed image. A lower value of MSE signifies lesser error in the reconstructed image.

2.4.2 Root mean square error:

The Root Mean Square Error (RMSE) is the square root of mean square error. It quantifies the average sum of distortion in each pixel of the reconstructed image.

$$RMSE = \sqrt{MSE}$$

The RMSE portrays the average change in a pixel caused by the image-processing algorithm.

2.4.3 Peak Signal to Noise Ratio:

Peak Signal to Noise Ratio (PSNR) measures the estimates of the quality of reconstructed image compared with an original image and is a standard way to measure image fidelity. Here ‘_signal’ is the original image and ‘_noise’ is the error in reconstructed image resulted due to compression and decompression. PSNR is a single number that reflects the quality of reconstructed image and is measured in decibels (db).

$$PSNR = 20 \log_{10} \left(\frac{S}{RMSE} \right)$$

Where S is the maximum pixel value and RMSE is the Root Mean Square Error of the image. The actual value of PSNR is not meaningful but the comparison between two values between different reconstructed images gives one measure of quality. As seen from inverse relation between MSE and PSNR, low value of MSE/RMSE translates to a higher value of PSNR, thereby signifying that a higher value of PSNR is better.

11.[icmsm2009.um.edu.my/filebank/published_article/1778/112.pdf]

2.5 EZW method

The EZW algorithm was one of the first algorithms to show the full power of wavelet-based image compression. It was introduced in the groundbreaking paper of Shapiro. We shall describe EZW in some detail because a solid understanding of it will make it much easier to comprehend the other algorithms we shall be discussing. These other algorithms build upon the fundamental concepts that were first introduced with EZW.

Our discussion of EZW will be focused on the fundamental ideas underlying it; we shall not use it to compress any images. That is because it has been superseded by a far superior algorithm, the SPIHT algorithm. Since SPIHT is just a highly refined version of EZW, it makes sense to first describe EZW. EZW stands for Embedded Zero tree Wavelet. We shall explain the terms Embedded, and Zero tree, and how they relate to Wavelet-based compression. An embedded coding is a process of encoding the transform magnitudes that allows for progressive transmission of the compressed image. Zero trees are a concept that allows for a concise encoding of the positions of significant values that result during the embedded coding process. We shall first discuss embedded coding, and then examine the notion of zero trees. The embedding process used by EZW is called bit-plane encoding.

12.[www.uwec.edu/walkerjs/media/imagecompchap.pdf]

2.6 Wavelets and compression

Wavelets are useful for compressing signals but they also have far more extensive uses. They can be used to process and improve signals, in fields such as medical imaging where image degradation is not tolerated they are of particular use. They can be used to remove noise in an image, for example if it is of very fine scales, wavelets can be used to cut out this fine scale, effectively removing the noise.

Digital image is represented as a two-dimensional array of coefficients, each coefficient representing the brightness level in that point. We can differentiate between coefficients as more important ones, and lesser important ones. Most natural images have smooth color variations, with the fine details being represented as sharp

edges in between the smooth variations. Technically, the smooth variations in color can be termed as low frequency variations and the sharp variations as high frequency variations. The low frequency components (smooth variations) constitute the base of an image, and the high frequency components (the edges which give the detail) add upon them to refine the image, thereby giving a detailed image. Hence, the smooth variations are demanding more importance than the details. Separating the smooth variations and details of the image can be done in many ways. One such way is the decomposition of the image using a Discrete Wavelet Transform (DWT). Digital image compression is based on the ideas of sub band decomposition or discrete wavelet transform (DWT). In fact, wavelets refer to a set of basis functions, which is defined recursively from a set of scaling coefficients and scaling functions. The DWT is defined using these scaling functions and can be used to analyze digital images with superior performance than classical short-time Fourier-based techniques, such as the DCT. The basic difference wavelet-based and Fourier-based techniques is that short- time Fourier-based techniques use a fixed analysis window, while wavelet-based techniques can be considered using a short window at high spatial frequency data and a long window at low spatial frequency data. This makes DWT more accurate in analyzing image signals at different spatial frequency, and thus can represent more precisely both smooth and dynamic regions in image. The compressor includes forward wavelet transform, Quantizer, and Lossless entropy encoder. The corresponding decompressed is formed by Lossless entropy decoder, de-quantizer and an inverse wavelet transform. Wavelet- based image compression has good compression results in both rate and distortion sense.

2.7 Wavelet Compression Techniques:

There are many different forms of data compression. This investigation will concentration transform coding and then more specifically on Wavelet Transforms. Image data can be represented by coefficients of discrete image transforms. Coefficients that make only small contributions to the information contents can be omitted. Usually the image is split into blocks of 8x8 or 16x16

pixels, then each block is transformed separately. However this does not take into account any correlation between blocks, and creates "blocking artifacts", which are not good if a smooth image is required. However wavelets transform is applied to entire images, rather than sub images, so it produces no blocking arte facts. This is a major advantage of wavelet compression over other transform compression methods.

2.8 Thresholding in Wavelet Compression

For some signals, many of the wavelet coefficients are close to or equal to zero. Thresholding can modify the coefficients to produce more zeros. In Hard thresholding any coefficient below a threshold is set to zero. This should then produce many consecutive zeros which can be stored in much less space, and transmitted more quickly by using entropy coding compression. An important point to note about Wavelet compression is explained by Aboufadel "The use of wavelets and thresholding serves to process the original signal, but, to this point, no actual compression of data has occurred". This explains that the wavelet analysis does not actually compress a signal, it simply provides information about the signal which allows the data to be compressed by standard entropy coding techniques, such as Huffman coding. Huffman coding is good to use with a signal processed by wavelet analysis, because it relies on the fact that the data values are small and in particular zero, to compress data. It works by giving large numbers more bits and small numbers fewer bits. Long strings of zeros can be encoded very efficiently using this scheme. Therefore an actual percentage compression value can only be stated in conjunction with an entropy coding technique. To compare different wavelets, the number of zeros is used. More zeros will allow a higher compression rate, if there are many consecutive zeros, this will give an excellent compression rate.

13.[infoscience.epfl.ch/record/33854/files/ChangYV00a.pdf]

2.9 Principles of using transform as source encoder:

We will go ahead and describe few terms most commonly used in this report. Mathematical transformations are applied to signals to obtain further information from that signal that is not readily available in the raw signal. Various wavelet transforms.

2.9.1 Haar wavelet

In mathematics, the Haar wavelet is a certain sequence of functions. It is now recognised as the first known wavelet.

This sequence was proposed in 1909 by Alfred Haar. Haar used these functions to give an example of a countable orthonormal system for the space of square integrable functions on the real line. The study of wavelets, and even the term "wavelet", did not come until much later.

The Haar wavelet is also the simplest possible wavelet. The technical disadvantage of the Haar wavelet is that it is not continuous, and therefore not differentiable.

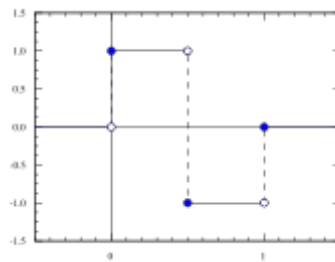


Figure 2-2 : Haar wavelet.

The Haar wavelet's mother wavelet function $\psi(t)$ can be described as

$$\psi(t) = \begin{cases} 1 & 0 \leq t < 1/2, \\ -1 & 1/2 \leq t < 1, \\ 0 & \text{otherwise.} \end{cases}$$

and its scaling function $\phi(t)$ can be described as

$$\phi(t) = \begin{cases} 1 & 0 \leq t < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Wavelets are mathematical functions that were developed by scientists working in several different fields for the purpose of sorting data by frequency. Translated data can then be sorted at a resolution which matches its scale. Studying data at different levels allows for the development of a more complete picture. Both small features and large features are discernable because they are studied separately. Unlike the discrete cosine transform, the wavelet transform is not Fourier-based and therefore wavelets do a better job of handling discontinuities in data.

The Haar wavelet operates on data by calculating the sums and differences of adjacent elements. The Haar wavelet operates first on adjacent horizontal elements and then on adjacent vertical elements. The Haar transform is computed using:

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

2.9.2 Symlets

The symlets are nearly symmetrical wavelets proposed by Daubechies as modifications to the db family. The properties of the two wavelet families are similar.

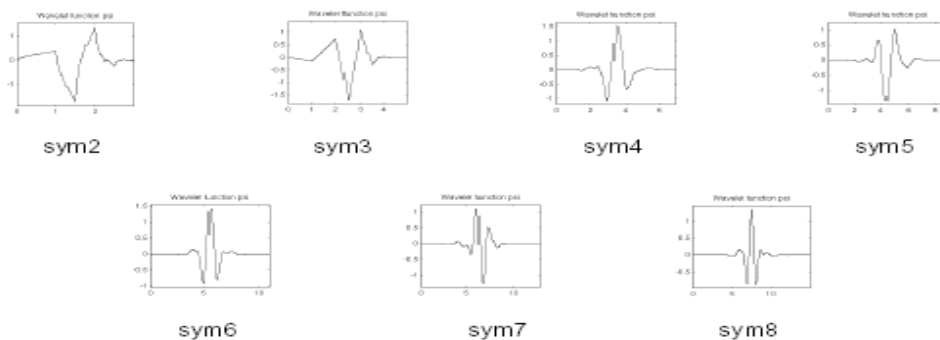


Figure 2-3 : symlets wavelet families.

2.9.3 Coiflets:

The wavelet function has $2N$ moments equal to 0 and the scaling function has $2N-1$ moments equal to 0. The two functions have a support of length $6N-1$.

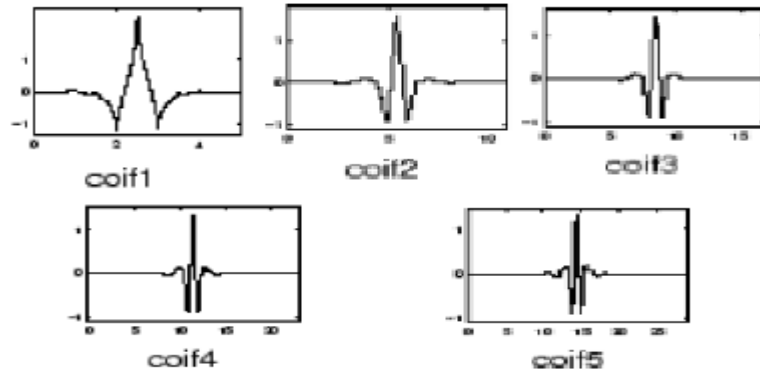


Figure 2-4 : Coiflets wavelet families.

2.9.4 Daubechies:

Ingrid Daubechies, one of the brightest stars in the world of wavelet research, invented what are called compactly supported orthonormal wavelets — thus making discrete wavelet analysis practicable. The names of the Daubechies family wavelets are written dbN , where N is the order, and db the -surname of the wavelet. The $db1$ wavelet, as mentioned above, is the same as Haar wavelet. Here is the wavelet

functions ψ of the next nine members of the family:

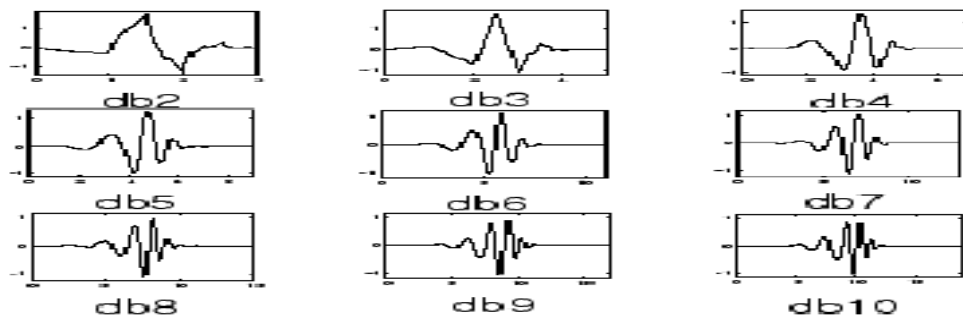


Figure 2-5 : Daubechies wavelet families.

2.9.5 Dmey:

This wavelet is a FIR based approximation of the Meyer wavelet, allowing fast wavelet coefficients calculation using DWT.

You can obtain a survey of the main properties of this wavelet by typing `waveinfo('dmey')` from the MATLAB command line.

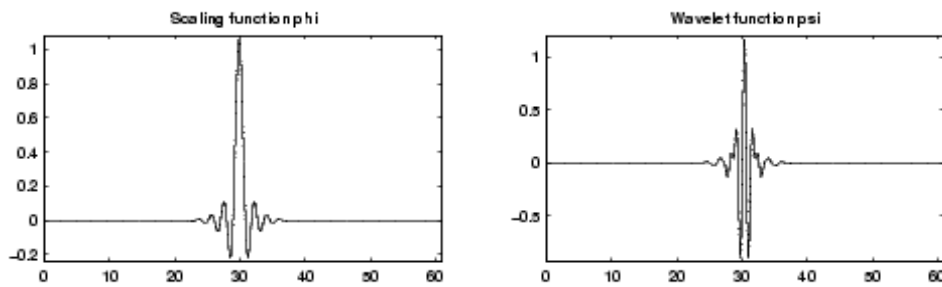


Figure: 2.6 Dmey wavelet families.

Chapter: 3

Theory of JPEG compression

3.1 JPEG

JPEG is an image compression standard used for storing images in a compressed format. It stands for Joint Photographic Experts Group. The remarkable quality of JPEG is that it achieves high compression ratios with little loss in quality.

JPEG format is quite popular and is used in a number of devices such as digital cameras and is also the format of choice when exchanging large sized images in a bandwidth constrained environment such as the Internet.

The JPEG algorithm is best suited for photographs and paintings of realistic scenes with smooth variations of tone and color. JPEG is not suited for images with many edges and sharp variations as this can lead to many artifacts in the resultant image. In these situations it is best to use lossless formats such as PNG, TIFF or GIF. It is for this reason that JPEG is not used in medical and scientific applications where the image needs to reproduce the exact data as captured and the slightest of errors may snowball into bigger ones.

A JPEG image may undergo further losses if it is frequently edited and then saved. The operation of decompression and recompression may further degrade the quality of the image.

To remedy this, the image should be edited and saved in a lossless format and only converted to JPEG format just before final transmittal to the desired medium. This ensures minimum losses due to frequent saving.

The JPEG compression algorithm is designed to compress image files created using the Joint Photographic Experts Group (JPEG) standard. JPEG files are inherently difficult to compress because of their built-in compression based on a combination of run-length and entropy coding techniques. The algorithm is lossless and reversible so that when the file is decompressed, the original entropy coding can be reapplied resulting in a bit for bit match with the original.

15.{en.wikipedia.org/wiki/JPEG}

The following diagram illustrates the process:

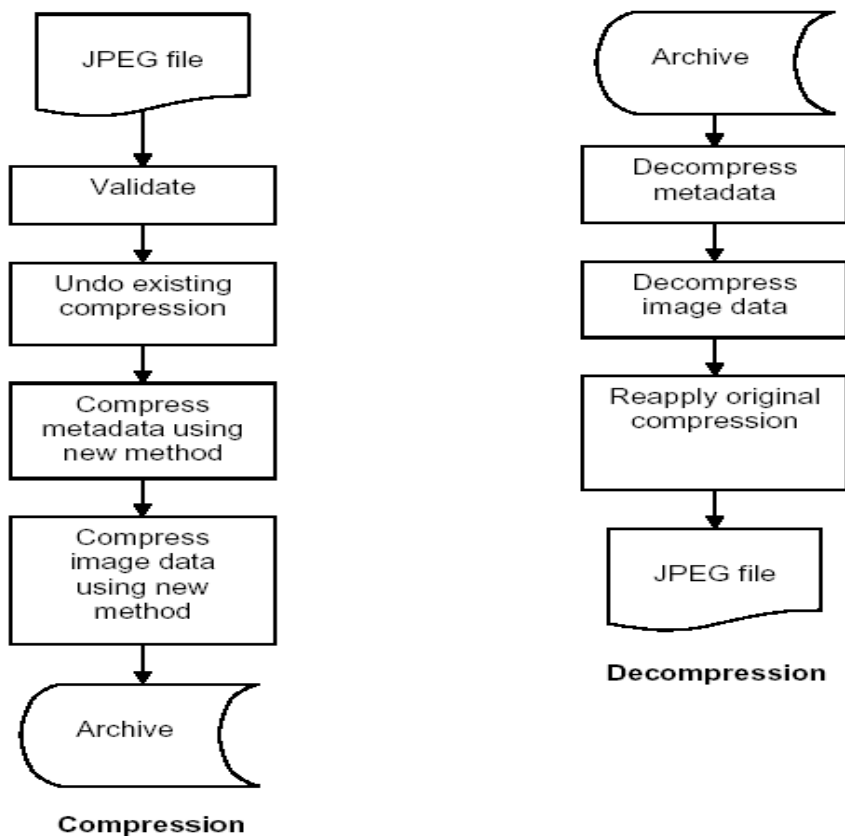


Figure 3-7: Compressed and Decompressed Process.

3.2 JPEG compression

The compression method is usually [lossy](#), meaning that some original image information is lost and cannot be restored, possibly affecting image quality. There is an optional [lossless](#) mode defined in the JPEG standard; however, this mode is not widely supported in products. There is also an [interlaced](#) "Progressive JPEG" format, in which data is compressed in multiple passes of progressively higher detail. This is ideal for large images that will be displayed while downloading over a slow connection, allowing a reasonable preview after receiving only a portion of the data. However, progressive JPEGs are not as widely supported, and even some software which does support them.

There are also many medical imaging and traffic systems that create and process 12-bit JPEG images, normally grayscale images. The 12-bit JPEG format has been part of the JPEG specification for some time, but again, this format is not as widely supported.

JPEG files are normally either JPEG File Interchange Format (JFIF) files or Exchangeable Image File Format (Exif) files with the latter being used by most digital cameras. Both formats are based on the JPEG Interchange Format (JIF) as specified in Annex B of the standard. The differences between the two are small, relating to a subset of markers. The marker differences are inconsequential to the compression algorithm so both formats are readily supported.

The JPEG compression algorithm supports the following image types:

- Baseline and extended (sequential) encoding
- 8 or 12 bits/sample
- Scans with 1, 2, 3 or 4 components
- Interleaved and non-interleaved scans

3.3 Effects of JPEG compression:

JPEG compression artifacts blend well into photographs with detailed non-uniform textures, allowing higher compression ratios. Notice how a higher compression ratio first affects the high-frequency textures in the upper-left corner of the image, and how the contrasting lines become more fuzzy. The very high compression ratio severely affects the quality of the image, although the overall colors and image form are still recognizable. However, the precision of colors suffer less (for a human eye) than the precision of contours (based on luminance). This justifies the fact that images should be first transformed in a color model separating the luminance from the chromatic information, before sub sampling the chromatic planes (which may also use lower quality quantization) in order to preserve the precision of the luminance plane with more information bits.

3.4 Compression algorithm

The JPEG standard makes use of the Discrete Wavelet Transform (DWT). It also supports a number of features such as multi-resolution representation, Region Of Interest (ROI) coding, error resilience and a flexible file format. The fundamental building blocks of JPEG are: pre-processing, DWT, uniform quantizer with dead-zone, adaptive binary arithmetic coder, and bit stream organization.

In the pre-processing stage, an inter-component transformation is used to decorrelate the color data. There are two possible transforms. Both transforms operate on the first three components of an image tile with the implicit assumption that these components correspond to Red, Green, and Blue (RGB). One transform is the Irreversible Color Transform (ICT), which is identical to the traditional RGB to $YCbCr$ transformation and can only be used for lossy coding. The other transform is the Reversible Color Transform (RCT), which is a reversible integer-to-integer transform that approximates the ICT for color decorrelation and can be used for both lossless and lossy coding. After color conversion, the DWT is applied to the processed

samples. The DWT produces a multi-resolution image representation. Furthermore, it achieves good compression due to its energy compaction and the ability to de-correlate the image across a large scale. The resulting wavelet coefficients are quantized using a uniform quantizer with a central dead-zone. It is shown that this quantizer is optimal for a continuous signal with a Laplacian distribution such as DCT or wavelet coefficients. The coefficients are gathered in subbands. Each subband is partitioned into small rectangular blocks called codeblocks and each independently coded by an Adaptive Binary Arithmetic encoder. Finally, the output of the arithmetic encoder is organized as a compressed bit-stream which offers a significant degree of flexibility. This enables features such as random access, Region of Interest coding, and scalability. This flexibility is achieved partly through the various structures of components, tiles, subbands, resolution levels, and code block.

JPEG Wizard Compression

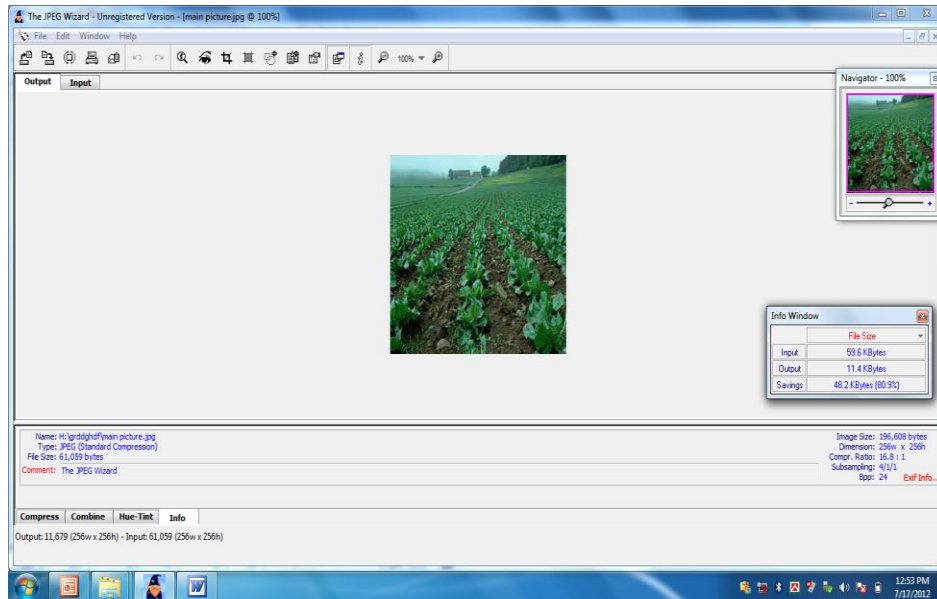


Fig: 3.2 JPEG Wizard Compression

JPEG wizard image compression:

By using JPEG Wizard software we again compress natural image. Then we measure the compress image size from original image. We also calculate peak signal to noise ratio (PSNR), mean square error (MSE), root mean square error (RMSE), bit per pixel (BPP) and compression ratio (CR). For compare between JPEG and wavelet image compression we use a particular ratio (which is 49.1:1) of JPEG compression. Then we compare between wavelet transform and JPEG wizard software for good compression which software is better.

JPEG Compression Table: 1

Input	Output	Saving	Com. Ratio	Ratio	B.P.P
59.6 kb	11.4 kb	48.2%	80.9%	16.8:1	24
59.6 kb	10 kb	49.7%	83.3%	19.3:8	24
59.6 kb	9.3 kb	50.3%	84.4%	20.6:1	24
59.6 kb	8.6 kb	51.1%	85.6%	24.4:1	24
59.6 kb	7.4 kb	51.8%	86.9%	24.7:1	24

3.5 Applications of JPEG

- Consumer applications such as multimedia devices (e.g., digital cameras, personal Digital assistants, 3G mobile phones, color facsimile, printers, scanners, etc.)

- Client/server communication (e.g., the Internet, Image database, Video streaming, Video server, Second Life, etc.)

- Military/surveillance (e.g., HD satellite images, Motion detection, network Distribution and storage, etc.)

- Medical imagery, esp. the DICOM specifications for medical data interchange.

- Remote sensing

- High-quality frame-based video recording, editing and storage.

- Digital cinema

Chapter 4

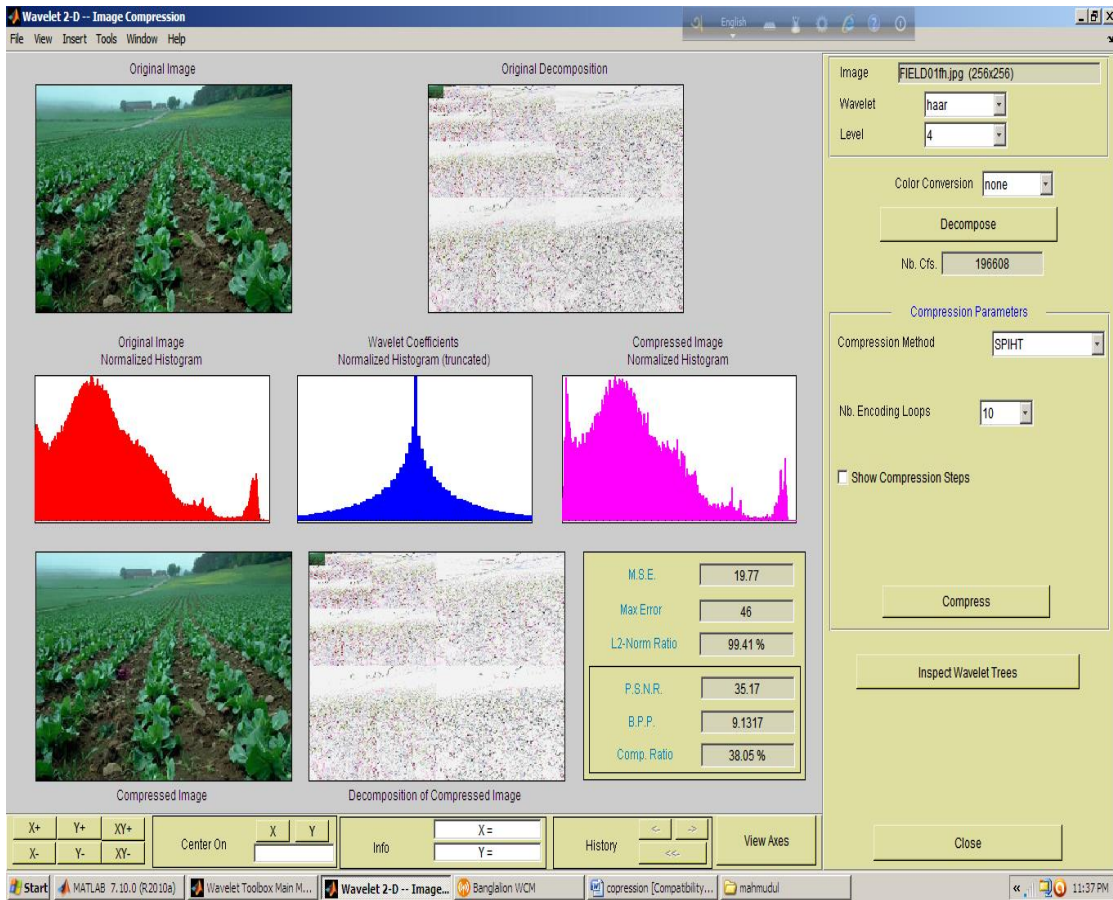
Applying Wavelet Compression Selected Natural image

4.1 Wavelet Image Compression:

4.1.1 Natural Image:

The goal of this paper is to investigate the effect of wavelet compression and also to compare with JPEG compression standards by using two software called Wavelet Compression Engine, and JPEG Wizard. Using wavelet compression 2-D technique we compress image into several wavelets. There are Haar, Daubechies, Symlet, Coiflet. Then we selected two images: one is natural image and another is artificial image. We compress these two images by using several wavelets (Haar, Daubechies, Symlet, Coif let) from wavelet toolbox 2-D true image compression. To compress we use EZW method and number of encoding loops is 11. For natural image we find out which wavelet is better compress. We compare these image by peak signal to noise ratio (PSNR) , mean square error (MSE) , bit per pixel (BPP) and compression ratio (CR). Our project screen print are shown in below.

3.1.1.1 For True Compression 2-D haar-4:



Wavelet	Level	M.S.E.	Max. Error	L2-Norm Ratio	P.S.N.R.	B.P.P.	Comp. Ratio
Haar	4	19.77	46	99.41%	35.17	9.1317	38.05%

Fig: Wavelet 2-D

Image Compression Table: 4.1

Wavelet	Level	M.S.E.	Max. Error	L2-Norm Ratio	P.S.N.R.	B.P.P.	Comp. Ratio
Haar	4	19.77	46	99.41%	35.17	9.1317	38.05%
db2	4	17.82	65	99.20%	35.62	8.9269	37.20%
Sym2	4	17.82	65	99.20%	35.62	8.927	37.20%
coif11	4	17.68	40	99.21%	35.66	8.8682	36.95%
Dmey	4	17.93	23	99.22%	35.59	8.6504	36.04%

Compression 2-D Haar

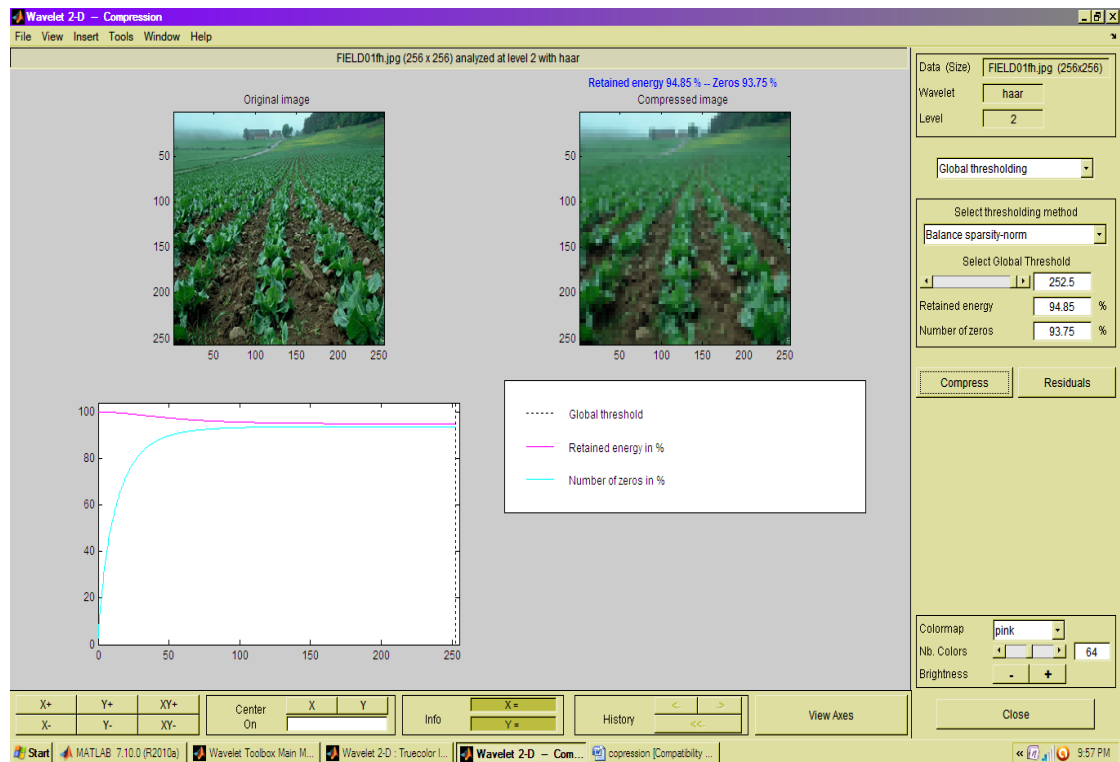


Fig: 4.2 Wavelet 2-D Transform

Global Threshold=252.5,

Retained energy=94.85%,

Number of Zeros=93.75%

Wavelet 2-D Transform Table: 2

Wavelet	Level	Thresoldig Method	Global thresold	Retained energy	Number of Zeros
Haar	4	Blance sparsity-norm	252.5	94.85%	93.75%
Db2	4	Blance sparsity-norm	80.46	97.35%	97.35%
Sym2	4	Blance sparsity-norm	80.46	97.35%	97.35%
Coilf1	4	Blance sparsity-norm	89.25	97.59%	97.59%
Dmey	4	Blance sparsity-norm	833.9	98.08%	95.05%

Wavelet 2-D image denoising:

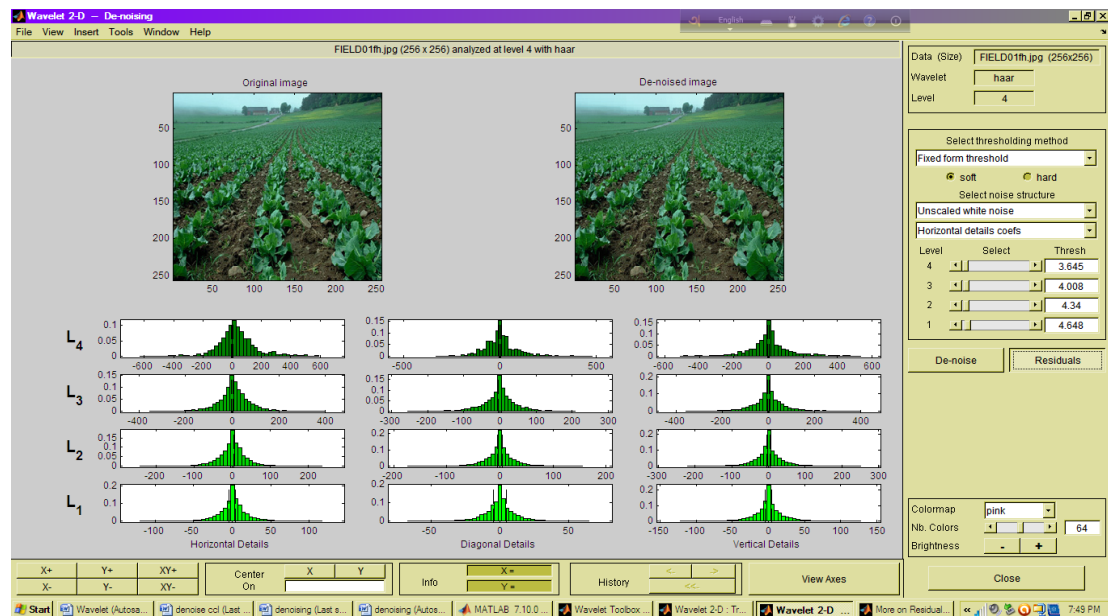


Fig: 4.3 Wavelet 2-D Denoise Haar- 4

Wavelet 2-D Residual:

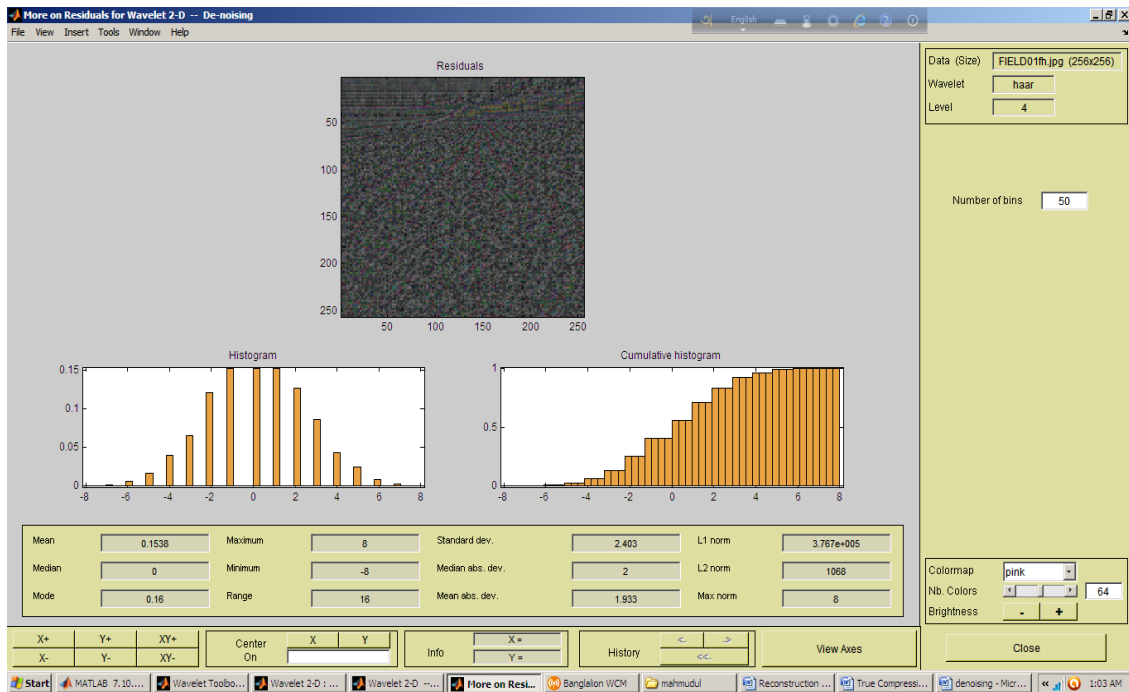
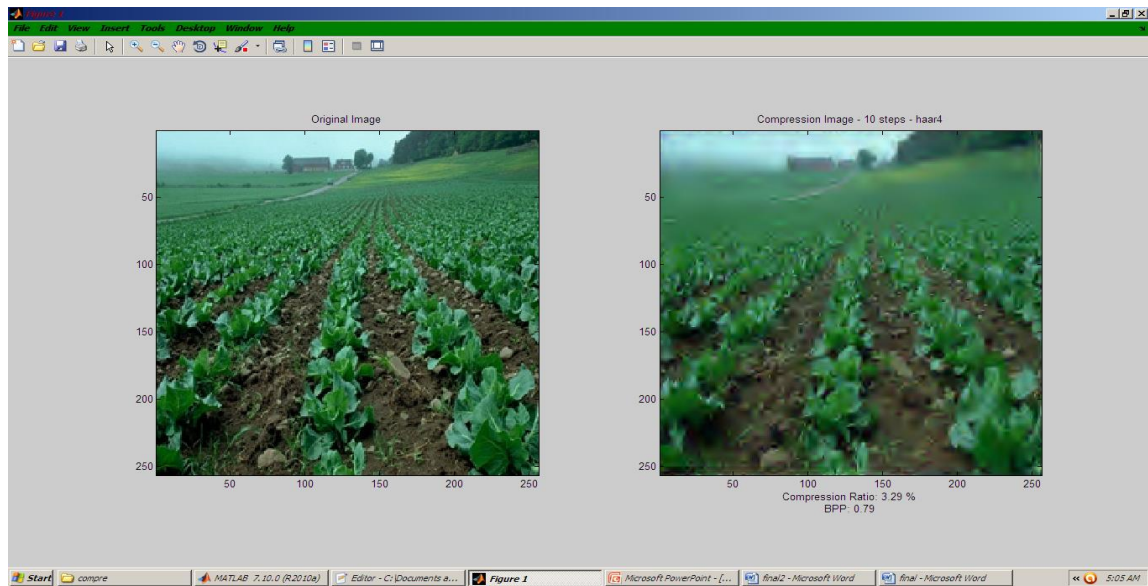


Fig: Wavelet 2-D Residual Haar- 4

Wavelet 2-D image denoising Table: 3

Wavelet	Mean	Mode	Max	Min	Range	Standard Deviation	Mean abs.dev.	Max norm
Haar	0.1538	0.16	8	-8	15	2.403	1.933	8
Db2	0.1496	-0.13	11	-10	21	24	1.864	11
Sym2	0.1496	-0.13	11	-10	21	24	1.864	11
Colif1	0.1499	-0.19	12	-11	23	2.367	1.848	12
Dmey	0.149	0.24	12	-12	24	2.423	1.905	12

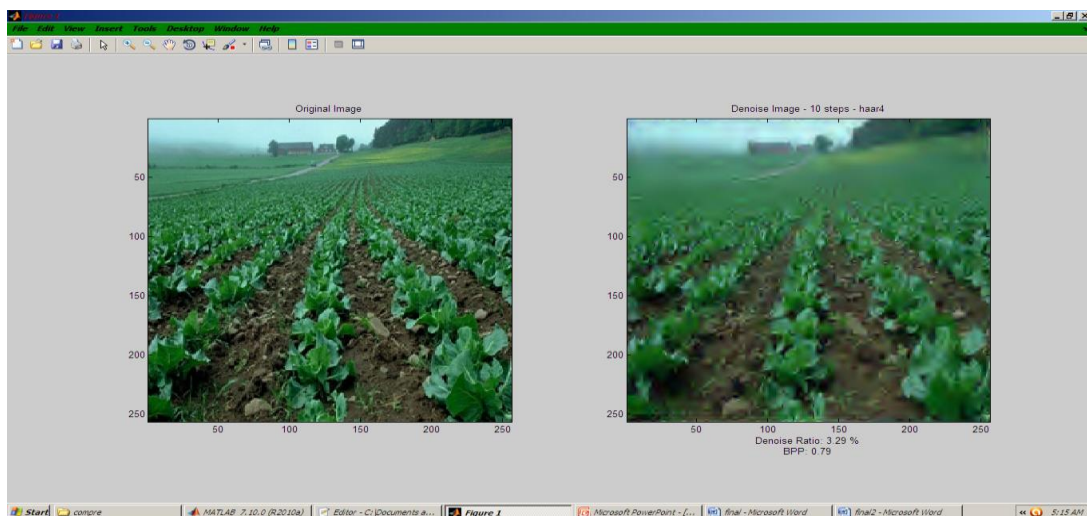
Matlab Compression Image :



Matlab Program Compression Table: 4

Wavelet	M.S.E	R.M.S.E	B.P.P	P.S.N.R	Com. Ratio
Haar	55.61	7.4573	4.8751	30.67	22.9741%
Db2	53.85	7.3384	3.9146	30.89	21.5971%
Sym2	53.85	7.3384	3.9146	30.89	21.5970%
Coif1	53.64	7.3245	3.8756	30.92	20.9724%
Dmey	53.93	7.3442	3.7552	30.87	20.5063%

Matlab De Noise Image:



Matlab Program De noise Table: 5

Wavelet	M.S.E	R.M.S.E	B.P.P	P.S.N.R	Com. Ratio
Haar	60.85	7.9006	0.78	40.75	3.2913%
Db2	59.48	7.7123	0.63	38.55	2.6862%
Sym2	59.48	7.7123	0.73	38.55	2.6862%
Coif1	58.94	7.6772	0.51	38.67	2.1599%
Dmey	58.63	7.6570	0.59	38.87	2.0524%

Chapter 5

Result and discussion

5.1 Discussion:

5.1.1 For Wavelet Transform:

The wavelet transform gives the better compression ratio in our simulations. We have applied our method to one natural image. The natural image frame is 256 by 256 pixel (jpeg format) in size. But we need to resize these image is 256 by 256 pixels in size. From these image we calculate peak signal to noise ratio (PSNR) , mean square error (MSE) , bit per pixel (BPP) and compression ratio (CR) by wavelet toolbox 2-D true image compression to compare between natural image. Comparisons of results for natural based on various performance parameters.

Mean Squared Error (MSE) is defined as the square of differences in the pixel values between the corresponding pixels of the two images. For a better compression of image required as lower mean square error (MSE) and higher peak signal to noise ratio (PSNR). And also need to maximum compression ratio when image quality will not be damage. The bit per pixel (BPP) is the another factor to compare two images. We use different types of wavelet in our experiment. We see from that the jpeg image has lower mean square error (MSE) than wavelet image. Lower mean square error is better for compressed image. The peak signal to noise ratio (PSNR) , bit per pixel (BPP) and compression ratio (CR) are also higher than jpeg image. Which is also better compressed image. We measure natural wavelet image, Haar wavelet can be better compress. So we will compare between haar wavelet compression and JPEG compression. Then we see natural image compress ratio using haar wavelet 38.05% and jpeg image compress ratio is 86.9%. Now we see that jpeg image is better compress (48.85%) than wavelet image.

Conclusion

Wavelet analysis is very powerful and extremely useful for compressing data such as images. Its power comes from its multi resolution. Although other transforms have been used, for example the DCT was used for the JPEG format to compress images, wavelet analysis can be seen to be far superior, in that it doesn't create blocking artifacts. This is because the wavelet analysis is done on the entire image rather than analyzing section by section. The image itself has a dramatic effect on compression. This is because it is the images pixel values that determine the size of the coefficients, and hence how much energy is contained within each sub signal. Furthermore, it is the changes between pixel values that determine the percentage of energy contained within the detail sub signals, and hence the percentage of energy vulnerable to thresholding. Therefore, different images will have different compressibility. From the results of this study we conclude that the JPEG can be used at higher compression ratios before information loss than Wavelet compression for natural image. The Wavelet algorithm introduces high image errors, which yields higher PSNR for high bit rate. We have shown that in terms of image quality, the JPEG algorithm is better than Wavelet for these images. Furthermore we also observe that by using Wavelet, for wavelet image the PSNR values obtained were higher than those achieved by using JPEG compression. We have briefly discussed these image compression methods and observed those elements of JPEG that directly relate to the wavelet transform. We have applied WT and JPEG compression technique to the selected a natural image and compared their performances. We also observed the Haar performance better for our several wavelet. Compress with wavelet by using Haar perform less efficiently than JPEG.

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