

AUDIO, IMAGE AND MRI SIGNAL PROCESSING USING WAVELET

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This Report is presented in Partial Fulfillment of the Requirements for the Degree
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APPROVAL

This Project titled “**Audio, Image and MRI Signal Processing Using Wavelet**”, submitted by Md. Ekhlalur Rahman and B.M Nahid Hasan Bappy to the Department of Electronics and Telecommunication Engineering, Daffodil International University, has been accepted as satisfactory for the partial fulfillment of the requirements for the degree of B.Sc. in Electronics and Telecommunication Engineering and approved as to its style and contents. The presentation has been held on august 27, 2012.

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DECLARATION

We hereby declare that except for the contents where specific references have been made to the work of others, the studies contain in this thesis are the results of investigation carried out by the authors under supervision of Assistant Professor Mrs. Shahina Haque, Department of Electronics & Telecommunication Engineering, and Daffodil International University.

No part of this thesis has been submitted to any other university or other educational establishment for a degree, diploma or other qualification.

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ABSTRACT

In this project, we have studied the use of wavelet transform (WT) for processing audio, image and MRI signal. For audio signal we used different wavelets such as Haar, Daubechies, Biorthogonal, Symmlet and Coiflet for decomposing and reconstructing selected signal. We measured the effect of several wavelets for evaluating the quality of the reconstructed signal. The parameters of evaluations are Peak Signal to Noise Ratio (PSNR), Normalized Root Mean Square Error (NRMSE), Signal to Noise Ratio (SNR), Retained energy (RE). We observed that Haar gives highest value of PSNR but NRMSE, SNR is low. For coiflet gave lowest value of NRMSE, SNR but PSNR is highest as compare to haar . Therefore haar is best for our audio data. For image, we used Biorthogonal 4.4 wavelets and different type of compression methods such as Embedded Zero tree Wavelet (EZW), Set Partitioning in Hierarchical Trees (SPIHT) Coding, Wavelet Difference Reduction (WDR), Adaptively scanned (ASWDR), Spatial-orientation Tree Wavelet (STW), Set Partitioning in Hierarchical Trees-3D (SPIHT-3D) Coding, and Wavelet level metropolis Monte Carlo (LVL-MMC) etc. We measured the effect of several compression methods for evaluating the quality of the reconstructed image. The parameters evaluations are Mean square Error (M.S.E), are Peak Signal to Noise Ratio (PSNR),Max Error, and Bit per pixel (BPP),Compression ratio. We observed that EZW gave lowest value of M.S.E, Max. Error but PSNR, for STW gave almost similar results like EZW. So, EZW method is best for our image. For MRI signal, we used different wavelets such as Haar, Deubachies, Biorthogonal, Symmlet and Coiflet for decomposing and reconstructing signal. We measured the effect of several wavelets for evaluating the quality of the reconstructed signal. We calculated the error between the original and the reconstructed image. We observed that Haar gives lowest value of Error where PSNR is high. Coiflet gave largest value of Error as compare to haar. So, Haar is best for our MRI signal.

TABLE OF CONTENT

| | |
|-------------------|------|
| Approval | II |
| Declaration | III |
| Acknowledgments | IV |
| Abstract | V |
| Table of contents | VI |
| List of Figures | VIII |
| List of Tables | IX |
| List of graphs | IX |

CHAPTER 1

| | |
|-------------------------|---|
| Introduction | 1 |
| 1.1 Aims and objectives | 2 |
| 1.2 Backgrounds | 2 |
| 1.3 Results | 4 |

CHAPTER 2

Theoretical Part of Wavelet Analysis

| | |
|------------------------------------|---|
| 2.1 What Is Wavelet Analysis? | 6 |
| 2.2 The Discrete Wavelet Transform | 6 |
| 2.3 The Fast Wavelet Transform | 8 |
| 2.3.1 Fourier analysis | 9 |
| 2.3.2 Fourier Transforms | 9 |

| | |
|--|----|
| 2.3.3 Discrete Fourier Transforms | 9 |
| 2.3.4 Fast Fourier Transforms | 9 |
| 2.3.5 Wavelet vs. Fourier Transforms | 10 |
| 2.3.6 Dissimilarities between Fourier and Wavelet Transforms | 10 |
| 2.3.7 What Is Wavelet Tool? | 11 |
| 2.3.8 Wavelet Families | 12 |
| 2.3.8.1 Haar | 13 |
| 2.3.8.2 Daubechies | 13 |
| 2.3.8.3 Biorthogonal | 13 |

CHAPTER 3

Wavelet Performance Analysis

| | |
|---|----|
| 3.1 One Dimensional Signal Decomposition & Reconstruction | 15 |
| 3.1.1 1D Signal Analysis (Decomposition) using Different Wavelets | 15 |
| 3.2 2D Image Analysis (compression) at Different compression method | 21 |
| 3.2.1 Error Metrics | 23 |
| 3.2.2 Embedded Zero tree Wavelet (EZW) | 23 |
| 3.2.3 Set Partitioning in Hierarchical Trees (SPIHT) Coding | 24 |
| 3.2.4 Wavelet Difference Reduction (WDR) | 25 |
| 3.2.5 ASWDR | 26 |
| 3.2.5.1 Features of ASWDR | 27 |
| 3.2.6 Spatial-orientation Tree Wavelet (STW) | 27 |
| 3.2.7 Set Partitioning in Hierarchical Trees-3D (SPIHT-3D) Coding | 27 |
| 3.2.8 Wavelet level metropolis Monte Carlo (LVL-MMC) | 28 |

| | |
|--|----|
| 3.3 Three Dimensional Decomposition & Reconstruction | 31 |
|--|----|

CHAPTER 4

| | |
|----------------|----|
| 4.1 Conclusion | 39 |
|----------------|----|

CHAPTER 5

| | |
|------------|----|
| References | 40 |
|------------|----|

List of Figure

| | |
|--|----|
| Fig.2.1 wavelet tool | 11 |
| Fig.3.1 flowchart of One Dimensional Signal Decomposition & Reconstruction | 15 |
| Fig. 3.2.1 original audio speech of /e/ & /i/ | 16 |
| Fig. 3.2.2 Decomposition audio speech of /e/ & /i/ | 16 |
| Fig. 3.2.3 reconstruction audio speech of /e/ & /i/ | 16 |
| Fig.3.1.1.1 wavelet haar | 17 |
| Fig.3.1.1.2 wavelet Daubechies | 17 |
| Fig.3.1.1.3 wavelet symm | 18 |
| Fig.3.1.1.4 wavelet Biorthogonal | 18 |
| Fig.3.1.1.5 wavelet coif | 19 |
| Fig.3.2 flowchart of two Dimensional images compresses | 21 |
| Fig.3.2.2.1 compression EZW for image 1 and 2 | 24 |
| Fig.3.2.3.1 compression SPIHT for image 1 and 2 | 25 |
| Fig.3.2.4.1 compression WDR for image 1 and 2 | 26 |
| Fig.3.2.5.1 compression ASWDR for image 1 and 2 | 26 |

| | |
|--|----|
| Fig.3.2.6.1 compression STW for image1 and 2 | 27 |
| Fig.3.2.7.2 compression SPIHT-3D for image 1 and 2 | 27 |
| Fig.3.2.8..1compression LVL-MMC for image 1 and 2 | 28 |
| Fig.3.3 flowchart of three Dimensional Decomposition &Reconstruction | 32 |
| .3.3.1 some slices z, y-orientation, decomposed and reconstruction | 33 |
| Fig.3.3.2 wavelet haar decomposed & reconstruction | 34 |
| Fig.3.3.3wavelet Deubachies decomposed & reconstruction | 35 |
| Fig.3.3.4 wavelet coif let decomposed & reconstruction | 36 |
| Fig.3.3.5 wavelet symmlet decomposed & reconstruction | 36 |

List of Table

| | |
|--|----|
| Table 1: Quantitative Analysis of Different Wavelets Voice Signal at level 5 | 19 |
| Table 2: Different compression method on 2D 512*512*3 image for 1 | 29 |
| Table 3: Different compression method on 2D 512*512*3 image for 2 | 30 |
| Table 4: Quantitative Analysis of Different Wavelets on MRI data | 37 |

List of graph

| | |
|---|----|
| Graph -1: Quantitative Analysis of Different Wavelets PSNR | 20 |
| Graph -2: Quantitative Analysis of Different Wavelets retained energy | 20 |
| Graph 3: Different compression method on 2D 512*512*3 image for 1 | 29 |
| Graph 4: Different compression method on 2D 512*512*3 image for 2 | 31 |

Chapter I

Introduction

Introduction

1.1 Aims and Objectives

Every section needs audio signals, image and MRI signal that can be analyzed for developing. There are seismic tremors, human speech, engine vibrations, medical images, financial data, music, and much other type of signals. Wavelet analysis is a new and promising set of tools and techniques for analyzing these signals.

1.2 Background

The fundamental idea behind wavelets is to analyze according to scale. Indeed, some researchers in the wavelet field that, by using wavelets, one is adopting a whole new mindset or perspective in processing data. Wavelets are functions that satisfy certain mathematical requirements and are used in representing data or other functions. Many speech/voice processing tasks, like speech and word recognition, reached satisfactory performance levels on specific applications, and although a variety of commercial products were launched in the last decade, many problems remain an open research area, and absolute solutions have not been found out yet [16].

Speech and Audio Signal Processing published in 1999, it stood out from its competition in its breadth of coverage and its accessible, intuition-based style. This book was aimed at individual students and engineers excited about the broad span of audio processing and curious to understand the available techniques. Since then, with the advent of the iPod in 2001, the field of digital audio and music has exploded, leading to a much greater interest in the technical aspects of audio processing.

First generation wavelets transform essentially needs the Fourier transform and the basic functions which are dynamically scalable with translation property of one particular mother basis function. These are the first nontrivial wavelets developed around 1980s. These include the Daubechies wavelet, Haar wavelet, Shannon Wavelet, Coif lets Wavelet and the Meyer wavelet. The major drawback of the first generation wavelet is that it can be deployed for infinite or periodic signals and cannot be optimized in the bounded domain. These wavelets transforms (WTs) are used in identifying pure frequencies, de-noising signals, detecting discontinuities and breakdown points, detecting self-similarity and compressing images. [15]

Second generation wavelets transform originates with concept of Lifting scheme to maintain the time-frequency localization and fast algorithms instead of Fourier domain to deploy in geometrical applications. This should replace translation and dilation as well as any Fourier analysis. The basic algorithm of the lifting schemes is to split up even samples then are adjusted to serve the coarse version of the original signal data in even set and odd set dilation as well as any Fourier analysis.

Image compression is one of the most visible applications of wavelets. The rapid increase in the range and use of electronic imaging justifies attention for systematic design of an image compression system and for providing the image quality needed in different applications. The basic measure for the performance of a compression algorithm is compression ratio (CR), defined as a ratio between original data size and compressed data size. In a lossy compression scheme, the image compression algorithm should achieve a tradeoff between compression ratio and image quality [17].

The basic algorithm of the lifting scheme is to split even samples then are adjusted to serve the coarse version of the original signal data in even set and odd set then predict odd signal using even part to detect the missing parts called details and update even samples for adjustment to serve the coarse version of the original signal. These WTs are extensively used for lossy data compression, in geographical data analysis, computer graphics and efficient coding in compression algorithm.

Third generations wavelets transform are the complex wavelet transform (CWT) with the complex-valued extension to the standard discrete wavelet transform (DWT). It is typically two-dimensional wavelet transform deployed for the multi-resolution, sparse representation, and useful feature characterization based on the structure of an image. The major pros are that these WTs do not exhibit oscillations, lack of directivity, aliasing and degree of shift-variance in its magnitude. But, the major cons are that it exhibits two dimension of the signal being transformed and yields the redundancy compared to a separable. [15]

Next generation wavelets transform optimize the PSNR, error free, lossless and advanced multi level resolution. These wavelets will be more advanced in terms of efficiency and performance. These WTs are still under research and they will focus specific applications such as human audio

voice, image and bio-medical signal analysis and so on. Multi-scale analysis represents the hierarchy of structural implementation to enhance the physical characteristics of the signal (1D, 2D and 3D). When the multi-scale stage (level) is increased then it provides the fine resolution from coarse resolution.

In other words, it is the systematic process to analyze signal at lower multi-scale stage with coarse resolution and then higher multi-scale stage with fine resolution [1-2]. This is deployed using different kinds of wavelets in audio signal, image and bio-medical signal decomposition and reconstruction to investigate their performance. The Haar wavelet, Daubechies wavelet, Morlet wavelet, Cauchy wavelet, Shannon wavelet, DCT, FFT, Biorthogonal wavelet, Symmlet wavelet and Coiflet wavelet are deployed in audio signal and image and MRI signal in this project.

1.3 Results:

We used wavelet transform (W.T) for processing audio, image and MRI signal. We used different wavelets such as Haar, Daubechies, Biorthogonal, Symmlet and coiflet for decomposing and reconstructing selected audio and MRI signal. We used Biorthogonal 4.4 wavelets and different type of compression methods such as EZW, SPHIT, WDR, ASWDR, STW, SPHIP-3D, and LVL-MMC etc. We found best wavelet for audio and MRI signal and compression method for image.

Chapter II

Theoretical Part of Wavelet Analysis

2.1 What Is Wavelet Analysis?

Wavelet analysis represents the next logical step: a windowing technique with variable-sized regions. Wavelet analysis allows the use of long time intervals where we want more precise low-frequency information, and shorter regions where we want high-frequency information. A wavelet is waveform of effectively limited duration that has an average value of zero.

Comparing wavelets with sine waves; which are the basis of Fourier analysis, it is found that sinusoids do not have limited duration – they extend from minus to plus infinity. And where sinusoids are smooth and predictable, wavelets tend to be irregular and asymmetric.

Fourier analysis consists of breaking up a signal into sine waves of various frequencies. Similarly, wavelet analysis is the breaking up of a signal into shifted and scaled versions of the original (or mother) wavelet.

Just looking at pictures of wavelets and sine waves, we can see intuitively that signals with sharp changes might be better analyzed with an irregular wavelet than with a smooth sinusoid, just as some foods are better handled with a fork than a spoon.

It also makes sense that local features can be described better with wavelets that have local extent.

2.2 The Discrete Wavelet Transform

Dilations and translations of the “Mother function,” or “analyzing wavelet” $\phi(x)$ define an orthogonal basis, our wavelet basis:

$$\phi_{(s,l)}(x) = \frac{2^{-s}}{2} \phi(2^{-s}x - l)$$

The Variables s and l are integers that scale and dilate the mother function [symbol] to generate wavelets, such as a Daubechies wavelet family. The scale index s indicates the wavelet’s width, and the location index l gives its position. Notice that the mother functions are rescaled, or “dilated” by powers of two, and translated by integers. What makes wavelet bases especially interesting is the self-similarity caused by the scales and dilations. Once we know about the

mother functions, we know everything about the basis. To span our data domain at different resolutions, the analyzing wavelet is used in scaling equation:

$$W(x) = \sum_{k=-1}^{N-2} (-1)^k C_{k+1} \phi(2x+k)$$

Where $W(x)$ is the scaling function for the mother function [symbol], and [symbol] is the wavelet coefficient. The wavelet coefficients must satisfy linear and quadratic constraints of the form

$$\sum_{k=-1}^{N-2} (-1)^k = 2$$

Where δ is the delta function and l is the location index. One of the most useful features of wavelets is the ease with which a scientist can choose the defining coefficients for a given wavelet system to be adapted for a given problem. In Daubechies' original paper, she developed specific families of wavelet systems that were very good for representing polynomial behavior. The Haar wavelet is even simpler, and it is often used for educational purposes.

It is helpful to think the coefficients $\{C_0, \dots, C_n\}$ as a filter. The filter or coefficients are placed in a transformation matrix, which is applied to a raw data vector. The coefficients are ordered using two dominant patterns, one that works as a smoothing filter (like a moving average) and one pattern that works to bring out the data's "detail" information. These two orderings of the coefficients are called a quadrature mirror filter pair in signal processing parlance.

To complete our discussion of the DWT, let's look at how the wavelet coefficient matrix is applied to the data vector. The matrix is applied in a hierarchical algorithm, sometimes called a pyramidal algorithm. The wavelet coefficients are arranged so that odd rows contain an ordering of wavelet coefficients that act as the smoothing filter, and the even rows contain an ordering of wavelet coefficients with different signs that act to bring out the data's detail. The matrix is first applied to the original, full-length vector. Then the vector is smoothed and decimated by half and the matrix is applied again. Then the smoothed, halved vector is smoothed, and halved again, and the matrix is applied once more. This process continues until a trivial number of "smooth-smooth-smooth..." data remain. That is, each matrix application brings out a higher resolution of

the data while at the same time smoothing the remaining data. The output of the DWT consists of the remaining “smooth (etc.)” components all of the accumulated “detail” components.

2.3 The Fast Wavelet Transform

The DWT matrix is not sparse in general, so we face the complexity issues that we had previously faced for Discrete Fourier transform. We solve it as we did for the FFT, by factoring the DWT into a product of a few sparse matrices using self-similarity properties. The result is an algorithm that requires only order n operations to transform an n -sample vector. This is the ‘fast’ DWT of Mallat and Daubechies.

2.4 What can we do with Wavelet Analysis?

With the help of wavelet analysis we are able to perform different task which are impossible to do with the Fourier transform. The most important feature of this is we can analyze a particular area of a very big signal.

If we consider a signal with a very small discontinuity that can be barely visible. This kind of signal can be generated in real life. In such case if we plot Fourier coefficients of this signal using ‘fft’ the signal will simply show a flat spectrum with two –peaks whose frequency is single. If we do same thing using wavelet then we will see the exact location in time of discontinuity.

2.3.1 Fourier analysis

Fourier’s representation of function as a superposition of sins and cosines has become ubiquitous for both the analytic and numerical solution of differential equations and for the analysis and treatment of communication signals. Fourier and wavelet analysis have some very strong links.

2.3.2 Fourier Transforms

The Fourier transform’s utility lies in its ability to analyze a signal in the time domain for its frequency content. The transform works by first translating a function in the time domain into function in the frequency domain. The signal can then be analyzed for its frequency content because the Fourier coefficients of the transformed function represent the

contribution of each sine and cosine function at each frequency. An inverse Fourier transform does just what expected; transform data from the frequency domain into the time domain.

2.3.3 Discrete Fourier Transforms

The Discrete Fourier Transform (DFT) estimates the Fourier transform of a function from a finite number of its sampled points. The sampled points are supposed to be typical of what the signal looks like at all other times.

The DFT has symmetry properties almost exactly the same as the continuous Fourier transform. In addition, the formula for the inverse discrete Fourier transform is easily calculated using the one for the discrete Fourier transform because the two formulas are almost identical.

2.3.4 Fast Fourier Transforms

To approximate a function by samples, and to approximate the Fourier integral by the discrete Fourier transform, requires applying a matrix whose order is the number sample points n . Since multiplying a $n \times n$ matrix by a vector costs on the order of [symbol] arithmetic operations, the problem gets quickly worse as the number of sample points increases. However, if the samples are uniformly spaced, then the Fourier matrix can be factored into a product of just a few sparse matrices, and the resulting force can be applied to a vector in a total order of $n \log n$ arithmetic operations. This is the so-called fast Fourier transform or FFT.

2.3.5 Wavelet vs. Fourier Transforms

The fast Fourier transform (FFT) and the discrete wavelet transform (DWT) are both linear operations that generate a data structure that contains [symbol] segments of various lengths, usually filling and transforming it into a different data vector of length.

The mathematical properties of the matrices involved in the transforms are similar as well. The inverse transform matrix for both the FFT and the DWT is the transpose of the original. As a result, both transforms can be viewed as a rotation in function space to a different domain. For the FFT, this new domain contains basis functions that are sines and cosines. For the wavelet transform, this new domain contains more complicated basis functions called wavelets, mother wavelets, or analyzing wavelets.

Both transforms has another similarity. The basic functions are localized in frequency, making mathematical tools such as power spectra (how much power is contained in a frequency interval) and scale grams useful at picking out frequencies and calculating power distributions.

2.3.6 Dissimilarities between Fourier and Wavelet Transforms

The most interesting dissimilarity between these two kinds of transforms is that individual wavelet functions are localized in space. Fourier Sine and Cosine functions are not. This localizing feature, along with the wavelets' localization of frequency, makes many functions and operators using wavelets “sparse” when transformed into the wavelet domain. This sparseness, in turn, results in a number of useful applications such as data compression, detecting features in images, and removing noise from time series.

One way to see the time-frequency resolution differences between the Fourier Transform and the wavelet transform is to look at the basis function coverage of the time-frequency plane. Figure 1 show a windowed Fourier Transform, where the window is simply a square wave. The square wave window truncates the sine or cosine function to fit a window of a particular width. Because a single window is used for all frequencies in WFT, the resolution of the analysis is the same at all locations in the time time-frequency plane.

An advantage of wavelet transforms is that the windows *vary*. In order to isolate signal discontinuities, one would like to have some very short basis functions. At the same time, in order to obtain detailed frequency analysis, one would like to have very long basis functions. A way to achieve this is to have short high-frequency basis functions and long-low frequency ones. This happy medium is exactly what comes with wavelet transforms. Figure 2 shows the coverage in the time frequency plane with one wavelet function, the Daubechies wavelet. One thing to remember is that wavelet transforms do not have a single set of basic functions like the Fourier transform, which utilizes just the sine and cosine functions. Instead, wavelet transforms have an infinite set of possible basis functions. Thus wavelet analysis provides immediate access to information that can be obscured by other time-frequency methods such as Fourier analysis.

2.3.7 What Is Wavelet Tool?

The Wavelet Toolbox is a collection of functions built on the MATLAB® Technical Computing Environment. It provides tools for the analysis and synthesis of signals and images, and tools for statistical applications, using wavelets and wavelet packets within the framework of MATLAB.

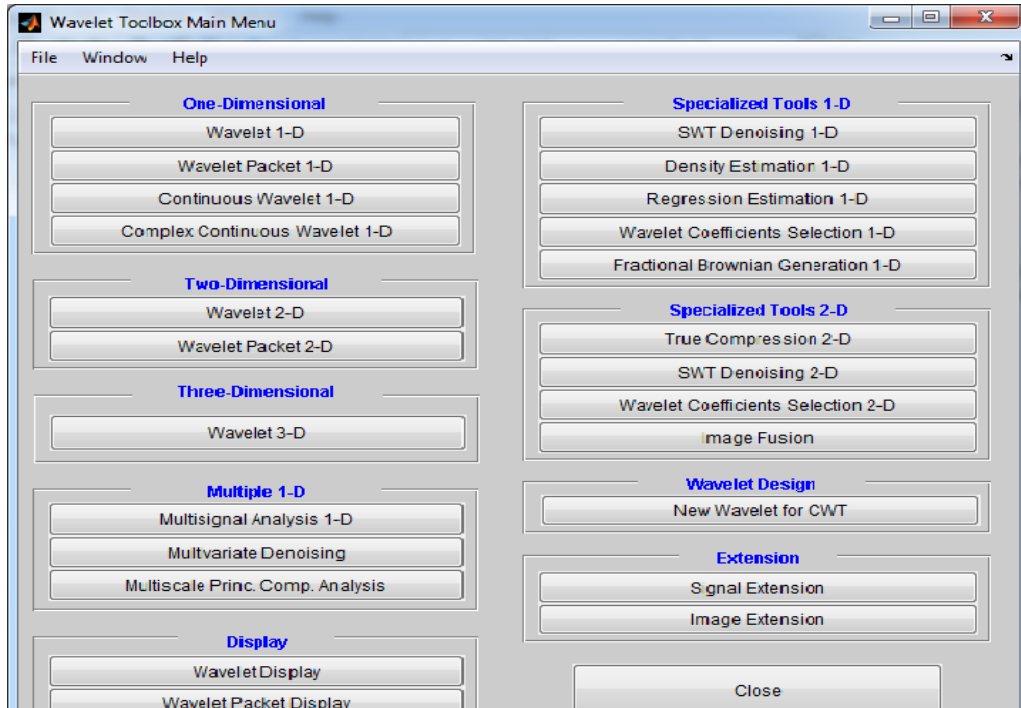


Fig.2.1 wavelet tool

The Math Works provides several products that are relevant to the kind of tasks that can be performed with the Wavelet Toolbox.

- ▶ Command line functions

- ▶ Graphical interactive tools

2.3.8 Wavelet Families

Several families of wavelets that have proven to be especially useful are included in this toolbox. What follows is an introduction to some wavelet families.

| Wavelet Family Short Name | Wavelet Family Name |
|---------------------------|--|
| 'HAAR' | Haar wavelet |
| 'DB' | Daubechies wavelets |
| 'SYM' | Symlets |
| 'COIF' | Coiflets |
| 'BIOR' | Biorthogonal wavelets |
| 'RBIO' | Reverse biorthogonal wavelets |
| 'MEYR' | Meyer wavelets |
| 'DMEY' | Discrete approximation of meyer wavelets |
| 'GAUS' | Gaussian wavelets |
| 'MEXH' | Mexican hat wavelets |
| 'MORL' | Morlet wavelets |
| 'CGAU' | Complex Gaussian wavelets |
| 'SHAN' | Shannon wavelets |
| 'FBSP' | Frequency B-Spline wavelets |
| 'CMOR' | Complex Morlet wavelets |

Type
menu
the

wave
from

- MATLAB command line. The Wavelet Toolbox Main Menu appears.
- Click the Wavelet Display menu item the Wavelet Display tool appears.
- Select a family from the wavelet menu at the top right of the tool.
- Click the Display button. Pictures of the wavelets and their associated filters appear.

5. Obtain more information by clicking the information buttons located at the right.

2.3.8.1 Haar

Any discussion of wavelets begins with Haar wavelet, the first and the simplest. Haar wavelet is discontinuous, and resembles a step function. It represents the same wavelets as Daubechies db1.

2.3.8.2 Daubechies

Ingrid Daubechies, one of the brightest stars in the world of wavelet research, invented what are called compactly supported orthonormal wavelets – thus

2.3.8.3 Biorthogonal

This family of wavelets exhibits the property of linear phase, which is needed for signal and image reconstruction. By using two wavelets, one for decomposition (on the left side) and the other for reconstruction (on the right side) instead of the same single one, interesting properties are derived.

Chapter III

Wavelet Performance Analysis

3.1 One Dimensional Signal Decomposition and Reconstruction

One dimensional discrete wavelet transform is used for 1D signal decomposition and reconstruction in time-scale (frequency) representation of non-stationary signals. It is based on multi-resolution approximation in which a function uses scaling function at various resolutions so that the lost details can be recovered using wavelet functions and the original signal is reconstructed by adding approximation and detail coefficient. It is deployed by a sequence of low pass and high pass filters. Low pass (LP) filters are associated with the scaling function and provide approximation whereas high pass (HP) filters are associated with the wavelet function and provide detail lost in approximating the signal. The basic scheme for Decomposition and Reconstruction is shown in Figure 1 below.

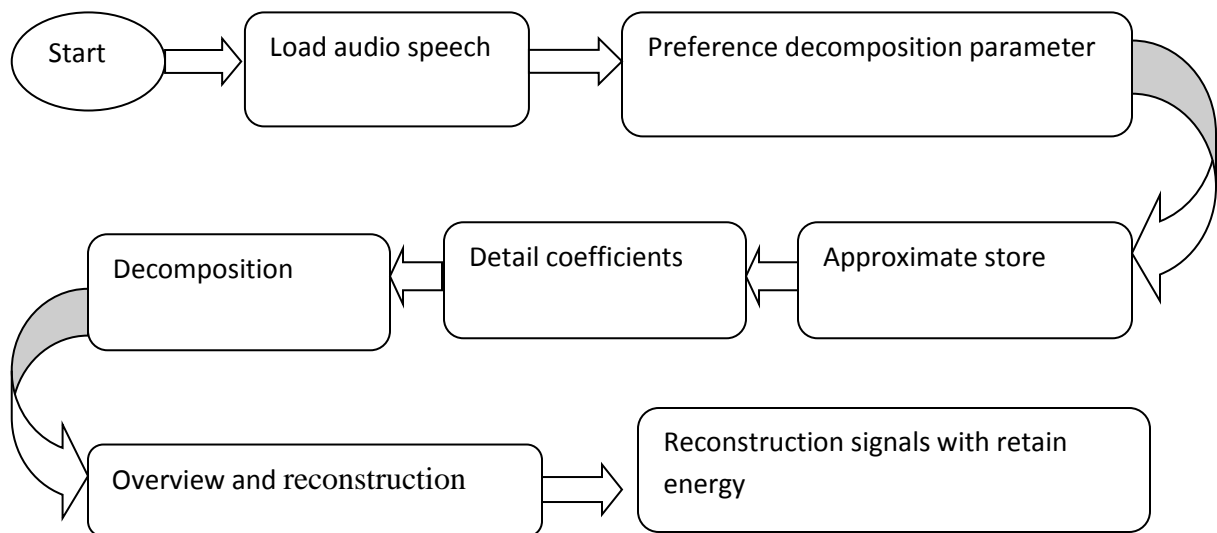


Fig.3.1flowchart of One Dimensional Signal Decomposition and Reconstruction

3.1.1 1D Signal Analysis (Decomposition) using Different Wavelets

1D signal decomposition is done using a sequence of LP and HP filter banks at five different wavelets. There are two 1sec audio speech signals such as /e/ and/ i/. There are original signal, decomposition and reconstruction below.

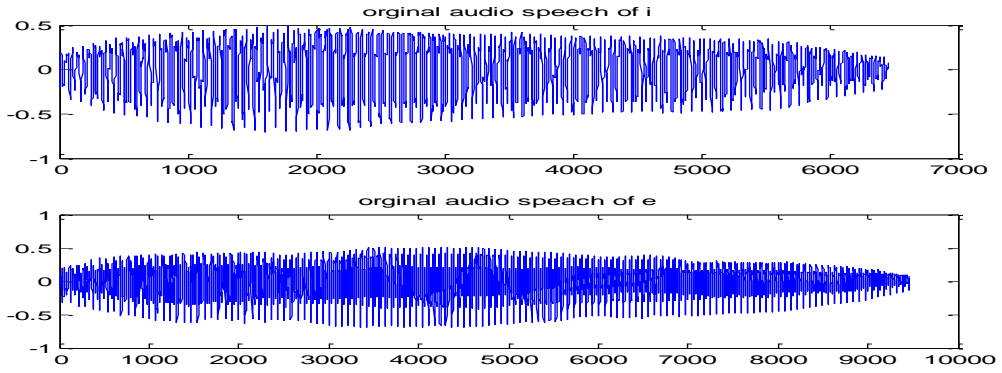


Fig. 3.2.1 original audio speech of /e/ and /i/

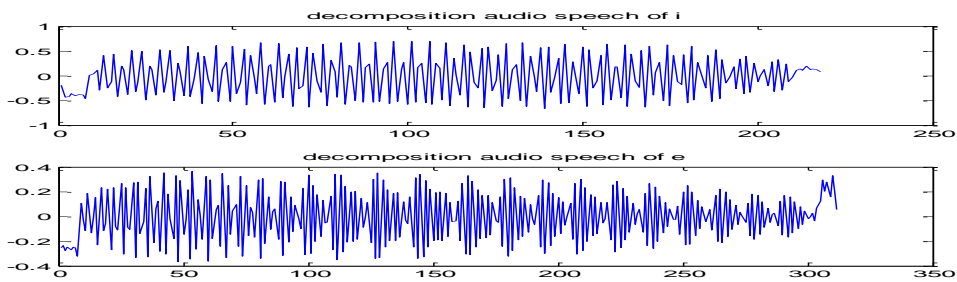


Fig. 3.2.2 Decomposition audio speech of /e/ and /i/

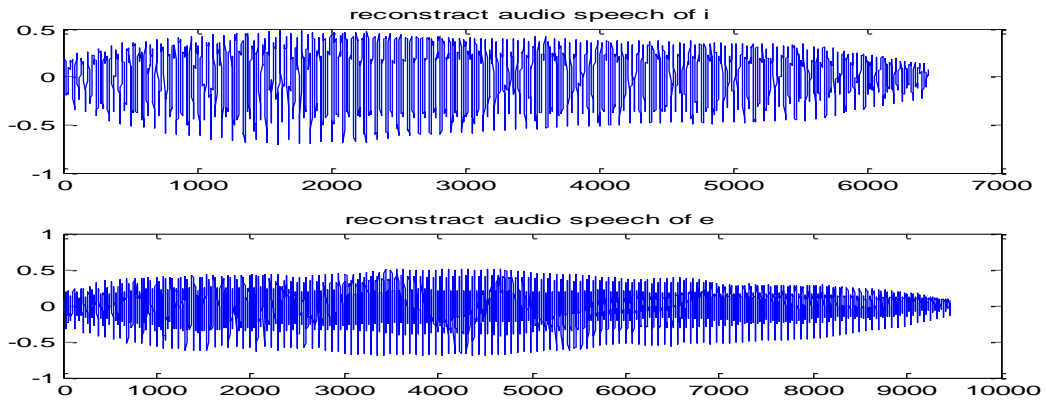
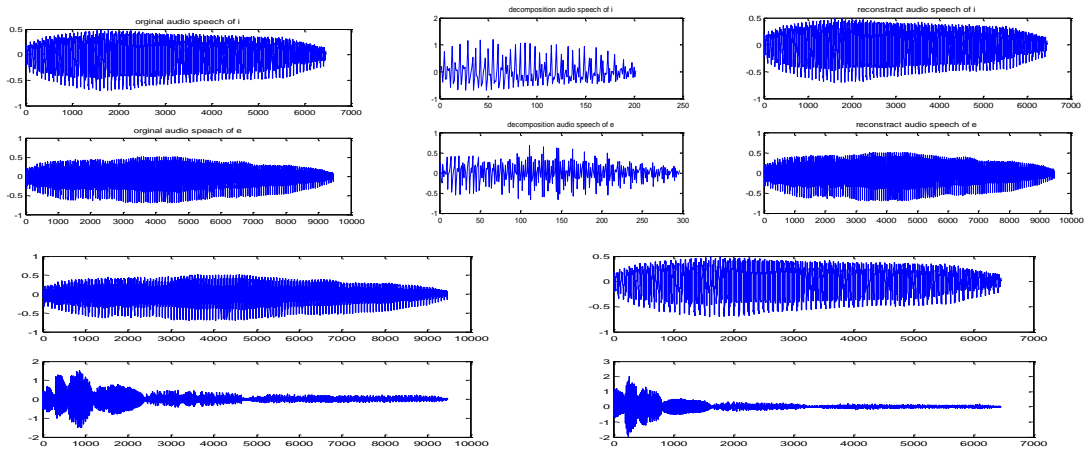


Fig. 3.2.3 reconstruction audio speech of /e/ and /i/

1D voice signal is decomposed and reconstructed different wavelets such as Haar, Daubechies, Biorthogonal, Symmetry and Coif let as illustrated in fig. 4 to fig. 8.

Wavelet haar

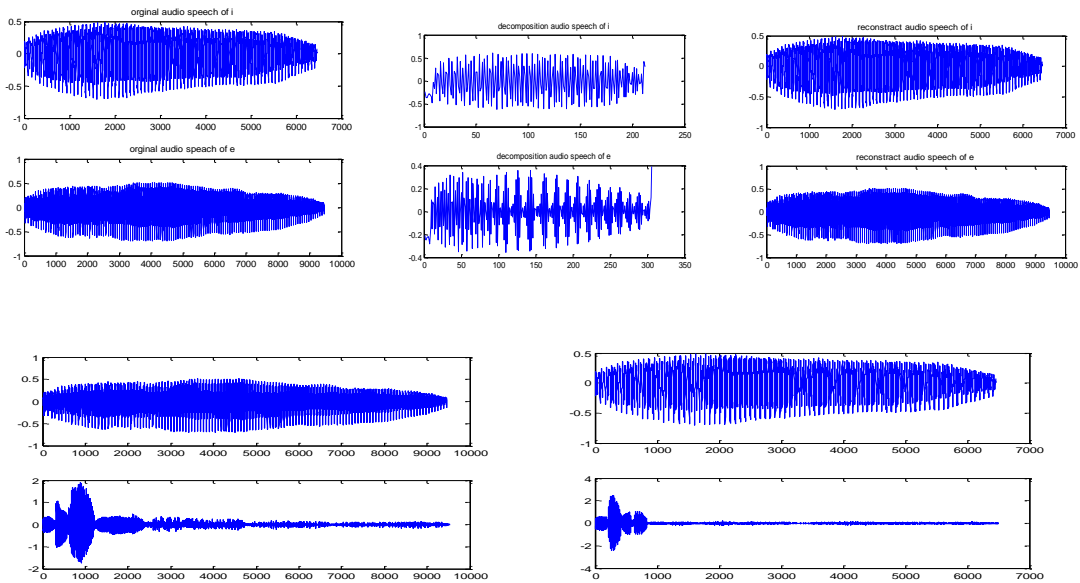


Retained energy for $-/e/$

Retained energy for $-/i/$

Fig.3.1.1.1 wavelet haar

Wavelet Daubechies

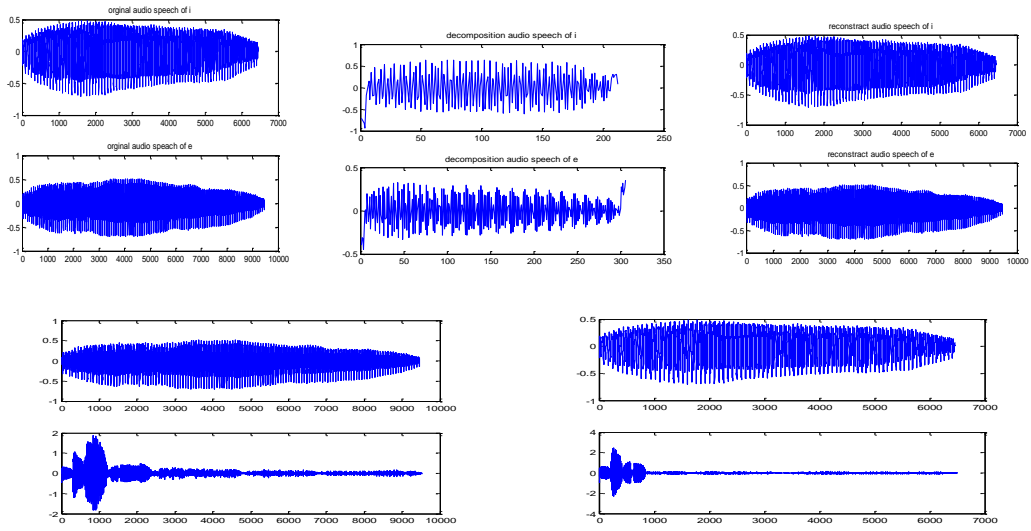


Retained energy for $-/e/$

Retained energy for $-/i/$

Fig.3.1.1.2 wavelet Daubechies

Wavelet symm

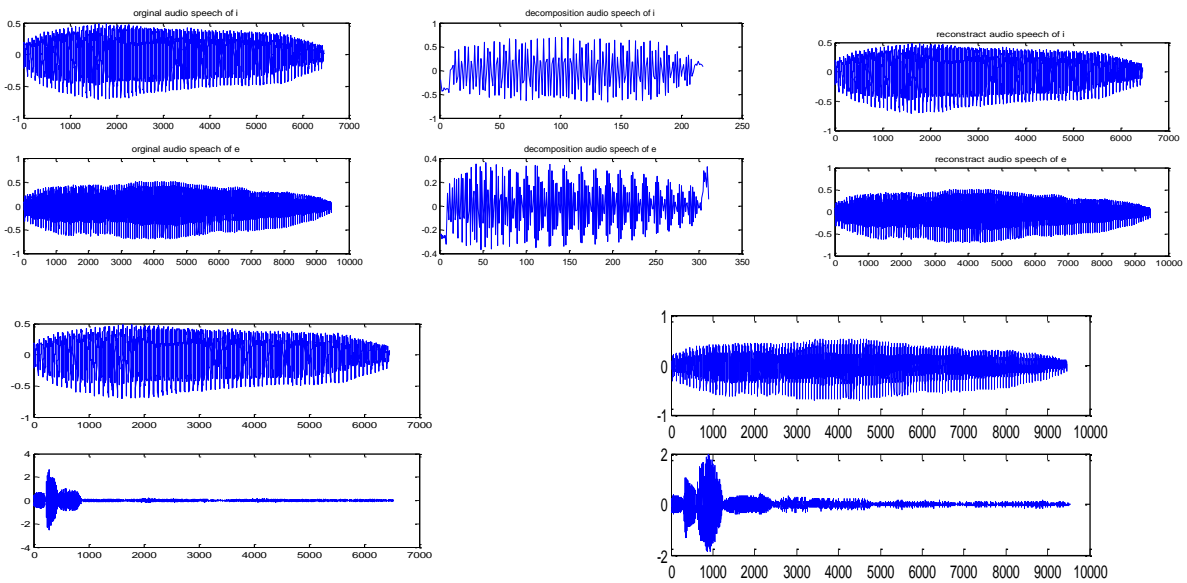


Retained energy for $-/e/$

Retained energy for $-/i/$

Fig.3.1.1.3 wavelet symm

Wavelet Biorthogonal



Retained energy for $-/e/$

Retained energy for $-/i/$

Fig.3.1.1.4 wavelet Biorthogonal

Wavelet coif -5

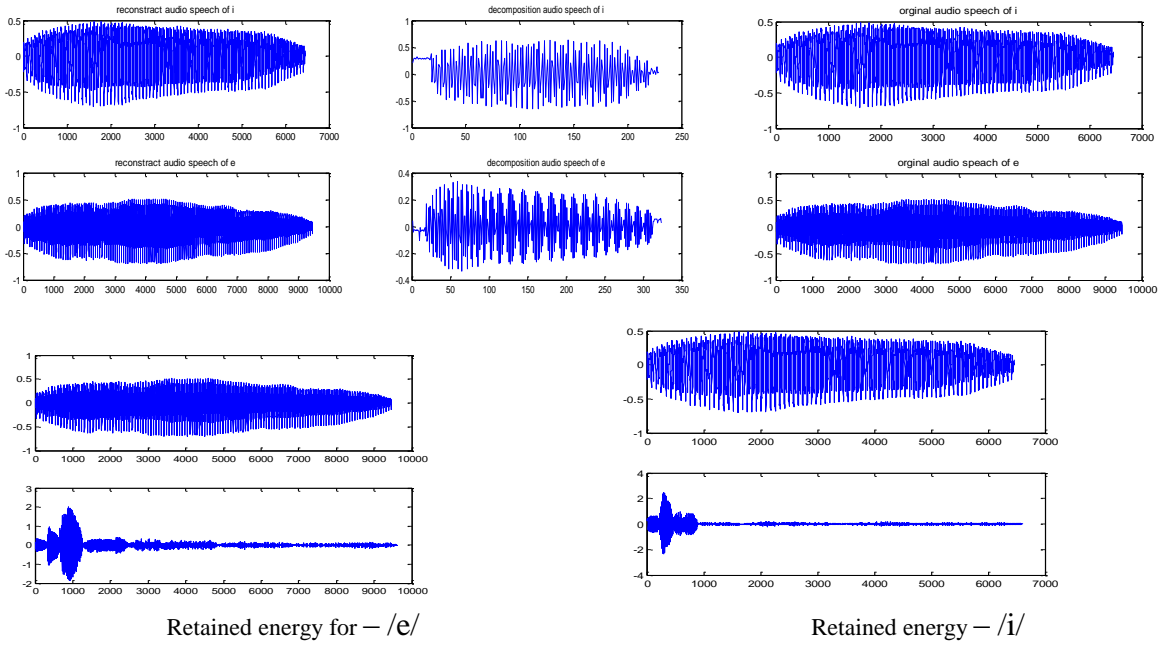


Fig.3.1.1.5 wavelet coif

Quantitative Analysis of Different Wavelets on 1D Voice Signal at level 5-

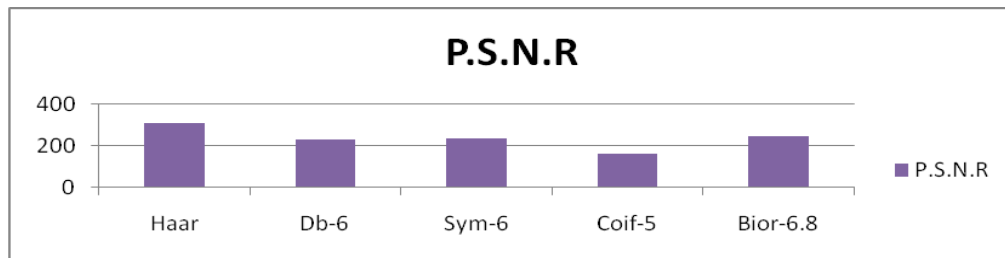
| Wavelet type | Haar | Db-6 | Sym-6 | Coif-5 | Bior-6.8 |
|--------------|------------------------|------------------------|------------------------|------------------------|-----------------------|
| P.S.N.R | 309.3840 | 232.8580 | 238.3702 | 163.1178 | 248.9091 |
| NRMSE | 6.3426e ⁻¹⁶ | 4.2518e ⁻¹² | 2.2540e ⁻¹² | 1.3049e ⁻⁰⁸ | 6.6991e ¹³ |
| SNR | 1.4932e ⁰² | 1.1098e ⁰² | 1.1349e ⁰² | 87.4582 | 118.7763 |
| RE | 96.2521 | 98.4730 | 98.4575 | 98.6002 | 98.8496 |
| Error | 1.0e ⁻⁰¹⁵ | 1.0e ⁻⁰¹¹ | 2.2540e ⁻¹² | 1.349e ⁻⁰⁸ | 1.0e ⁻⁰¹² |

Table 1: Quantitative Analysis of Different Wavelets Voice Signal at level 5

From the table 1 represents that the total analysis of different types of wavelet transform where some parameter like as PSNR, NRMS, SNR, RE, ERROR is analyzed. From the quantitative analysis on voice signal, it is found that Haar provides highest PSNR, as well as lower NRMSE

and SNR. Similarly coif provides lowest PSNR as well as higher NRMSE and SNR. Haar WT does not have overlapping windows, and reflects only changes between adjacent sample pairs. The Haar wavelet uses only two scaling and wavelet function coefficients thus calculates pair wise averages and differences. That's provide lowest error. That's why; Haar is found best WT for voice signal decomposition and reconstruction.

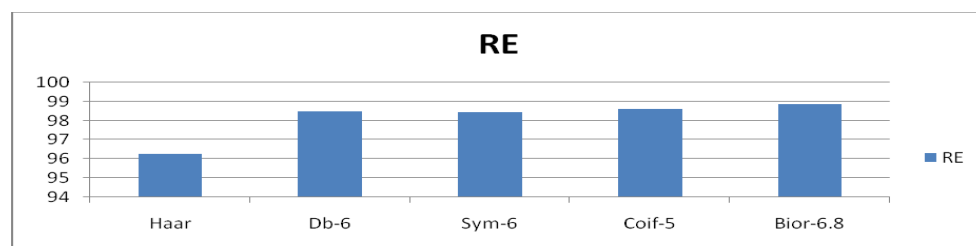
| Wavelet type | Haar | Db-6 | Sym-6 | Coif-5 | Bior-6.8 |
|--------------|----------|----------|----------|----------|----------|
| P.S.N.R | 309.3840 | 232.8580 | 238.3702 | 163.1178 | 248.9091 |



Graph -1: Quantitative Analysis of Different Wavelets PSNR

From this graph represents that Peak signal to noise ratio is higher than the other method, and it almost above 300 .

| Wavelet type | Haar | Db-6 | Sym-6 | Coif-5 | Bior-6.8 |
|--------------|---------|---------|---------|---------|----------|
| RE | 96.2521 | 98.4730 | 98.4575 | 98.6002 | 98.8496 |



Graph -2: Quantitative Analysis of Different Wavelets retained energy

From this graph represented that the RE is lower in Haar but in other case it is higher ,so here it has been decided that in haar method perform better than the other wavelet transform.

3.2 Two dimensional Image Analysis (compression) at Different compression method:

2D image decomposition is done using a sequence of combination of LP and HP filter banks in rows and Columns (LL, LH, HL, HH) at different wavelet compression. The basic scheme for compressing images is shown in Figure 9 below. Compression consists of two steps to generate a compressed bit stream.

The first step is a wavelet transform of the image and the second step is the compressed encoding of the image's wavelet transform. Decompression simply consists of reversing these two steps, decoding the compressed bit stream to produce an (approximate) image transform.

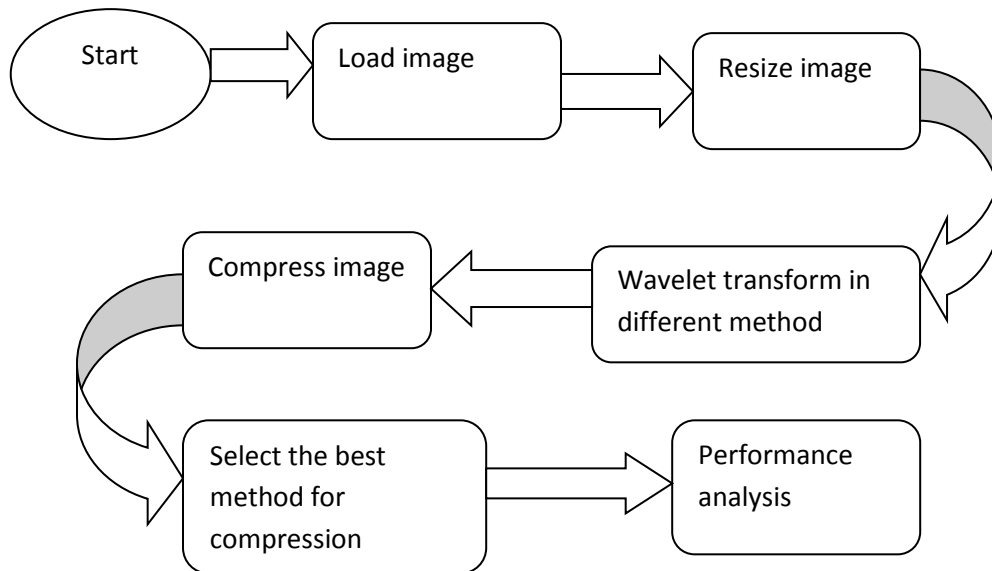


Fig.3.2 flowchart of two Dimensional image compresses

At first we should take an image from the camera, and then it's going through the MATLAB basement and then resizes it to 512*512*3 format because we know that for true compression, it is necessary to keep the size of rows and columns in the power of 2. Then take the wavelet Biorthogonal4.4 level-9 for compression then apply different types of method like as EZW, SPHIT, WDR, ASWDR, STW, SPHIP-3D, and LVL-MMC etc.

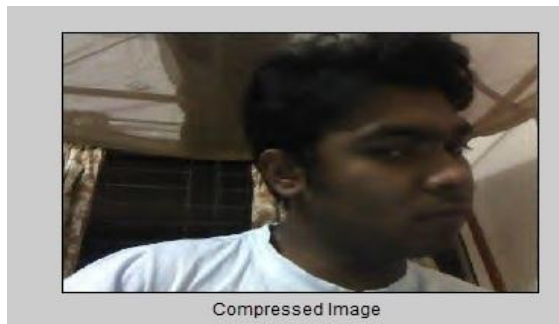
We compare between different method and select the best method for compression here we take 2 photo and then analyst it.



Picture- original Image 1



Picture- original Image 2



Picture- compressed Image 1



Picture- compressed Image 2

Wavelet-based coding provides substantial improvements in picture quality at higher compression ratios. A variety of novel and sophisticated wavelet-based image coding schemes have been developed. These include Embedded Zero tree Wavelet (EZW), Set-Partitioning in Hierarchical Trees (SPIHT), Set-Partitioning in Hierarchical Trees (SPIHT-3D), Wavelet Difference Reduction (WDR), Wavelet level metropolis Monte Carlo (LVL-MMC), Adaptively Scanned Wavelet Difference Reduction (ASWDR), Compression with Reversible Embedded Wavelet (CREW), Embedded Predictive Wavelet Image Coder (EPWIC) , Embedded Block Coding with Optimized Truncation (EBCOT), and Stack- Run (SR) . This list is by no means exhaustive and many more such innovative techniques are being developed. A few of these algorithms are briefly discussed here.

3.2.1 Error Metrics

Two of the error metrics used to compare the various image compression techniques are the Mean Square Error (MSE) and the Peak Signal to Noise Ratio (PSNR). The MSE is the cumulative squared error between the compressed and the original image, whereas PSNR is a measure of the peak error.

The mathematical formulae for the two are

$$\text{Error } E = \text{Original image} - \text{Reconstructed image} \dots\dots\dots (1)$$

$$\text{MSE} = E / (\text{SIZE OF IMAGE}) \dots\dots\dots (2)$$

$$PSNR = 20 * \log_{10} \left(\frac{255}{\sqrt{MSE}} \right) \dots\dots\dots(3)$$

A lower value for MSE means lesser error, and as seen from the inverse relation between the MSE and PSNR, this translates to a high value of PSNR. Logically, a higher value of PSNR is good because it means that the ratio of Signal to Noise is higher. Here, the 'signal' is the original image, and the 'noise' is the error in reconstruction. So, a compression scheme having a lower MSE (and a high PSNR), can be recognized as a better one.

3.2.2 Embedded Zero tree Wavelet (EZW):

The EZW algorithm was one of the first algorithms to show the full power of wavelet-based image compression.

Features of EZW:

- Better performance than the other method.
- Employs progressive and embedded transmission.
- Uses zero tree concept
- Tree coded with single symbol
- Used predefined scanning order
- Good result without pre-stored tables codebooks, training.

Demerits of EZW:

- Transmission of coefficient position is missing
- No real compression
- Followed by arithmetic encoder

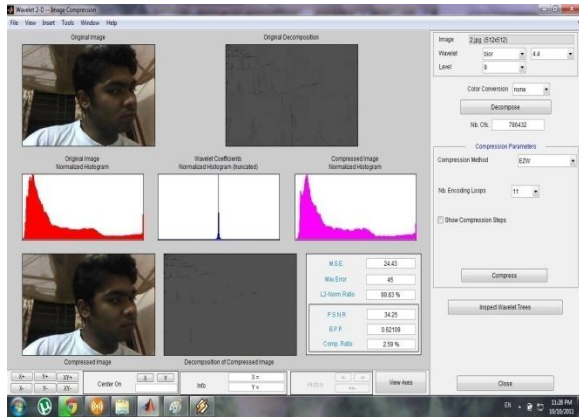


Fig.3.2.2.1 compression EZW for image 1

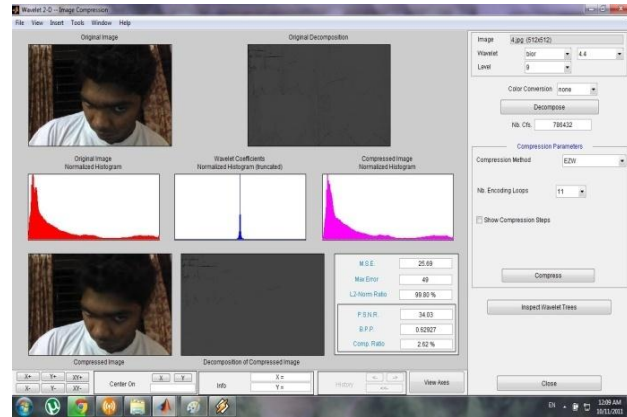


Fig.3.2.2.2 compression EZW for image 2

3.2.3 Set Partitioning in Hierarchical Trees (SPIHT) Coding:

SPIHT is a wavelet-based image compression coder. It first converts the image into its wavelet transform and then transmits information about the wavelet coefficients. The decoder uses the received signal to reconstruct the wavelet and performs an inverse transform to recover the image.

SPIHT displays exceptional characteristics over several properties all at once [12] including:

- Good image quality with a high PSNR
- Fast coding and decoding
- A fully progressive bit-stream
- Can be used for lossless compression
- May be combined with error protection

Demerits of SPIHT:

- Only implicitly locates position of significant coefficient
- More memory required due to 3 lists
- Suits variety of natural images

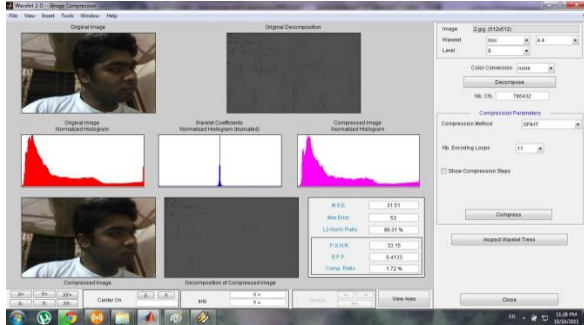


Fig.3.2.3.1compression SPIHT for image 1

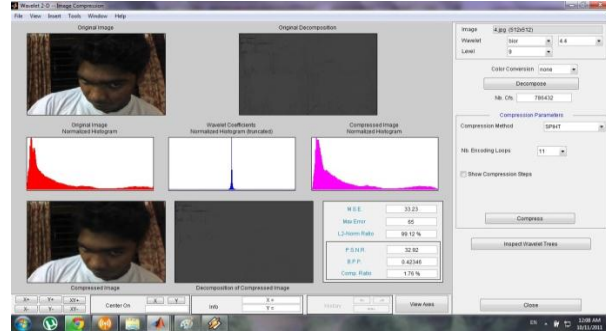


Fig.3.2.3.2 compression SPIHT for image 2

3.2.4 Wavelet Difference Reduction (WDR):

One of the defects of SPIHT is that it only implicitly locates the position of significant coefficients. This makes it difficult to perform operations which depend on the position of significant transform values, such as region selection on compressed data.

Features of WDR:

- Uses ROI concept
- Introduced by Tian and Wells.
- Better perceptual image quality than SPIHT
- No searching through quad trees as in SPIHT
- Less complex than SPIHT
- Better preservation of edges than SPIHT

Demerits of WDR:

- PSNR is not higher than SPIHT.

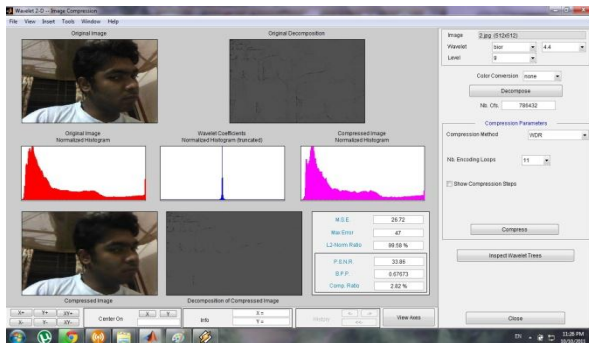


Fig.3.2.4.1compression WDR for image 1

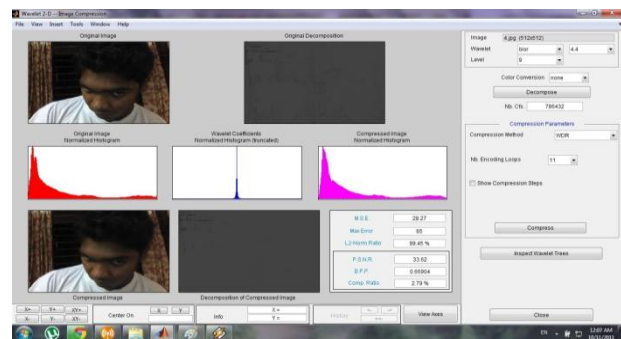


Fig.3.2.4.2compression WDR for image 2

3.2.5 ASWDR:

The ASWDR algorithm aims to improve the subjective perceptual qualities of compressed images and improve this result of objective distortion measures. We shall treat two distortion measures, PSNR and edge correlation, which we shall define in the section or experimental results. PSNR is a commonly used measure of error, while edge correlation is a measure that we have found useful in quantifying the preservation of edge details in compressed images, and seems to correspond well to subjective impressions of the perceptual quality of the compressed images.

3.2.5.1 Features of ASWDR:

- Modified scanning order compared to WDR
- Prediction of locations of new significant values
- Dynamically adapts to the locations of edge details
- Encode s more significant values than

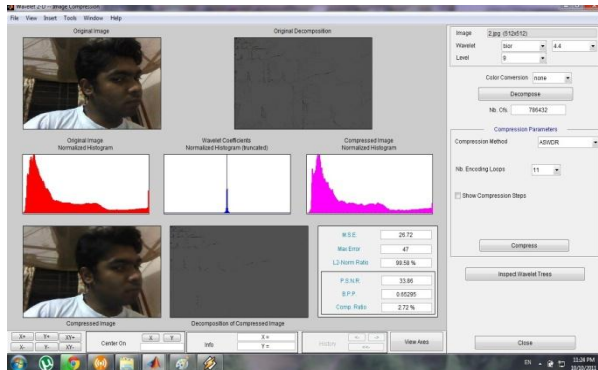


Fig.3.2.5.1 compression ASWDR for image 1

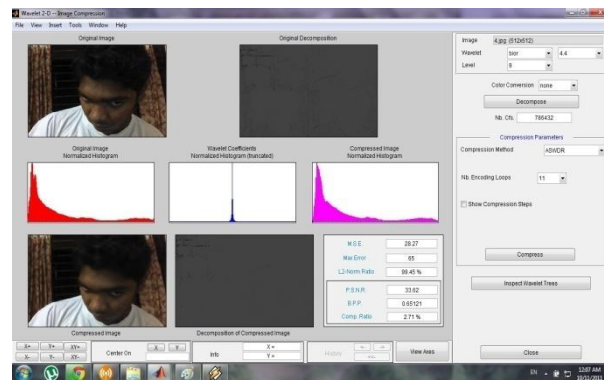


Fig.3.2.5.2 compression ASWDR for image 2

3.2.6 Spatial-orientation Tree Wavelet (STW):

STW is essentially for the SPIHT algorithm. The only difference is that SPIHT is slightly more careful in its organization of coding output. Second, we describe the SPIHT algorithm. It is easier to explain SPIHT using the concepts underlying STW. Third, we see how well SPIHT

compresses images. . The only difference between STW and EZW is that STW uses a different approach to encoding the zero tree information. STW uses a state transition model.

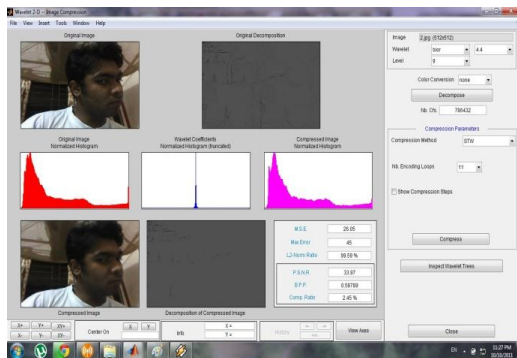


Fig.3.2.6.1 compression STW for image 1

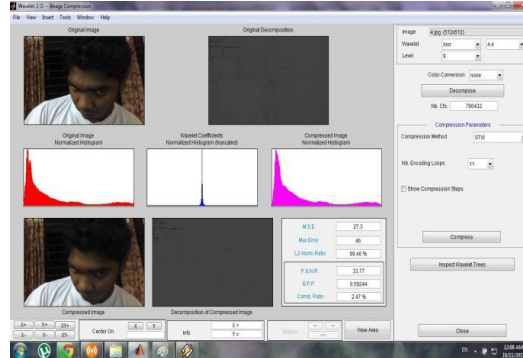


Fig.3.2.6.2 compression STW for image 2

3.2.7 Set Partitioning in Hierarchical Trees-3D (SPIHT-3D) Coding:

SPIHT-3D is a wavelet-based image compression coder as like as SPIHT.

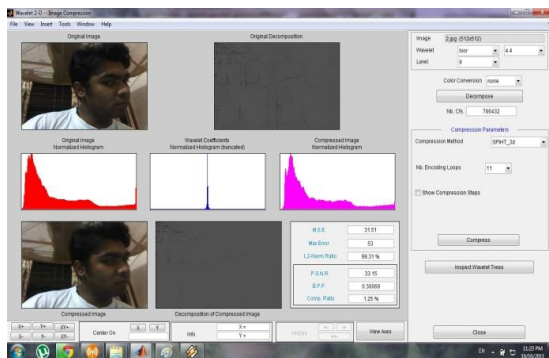


Fig.3.2.7.1compression SPIHT-3D for image 1

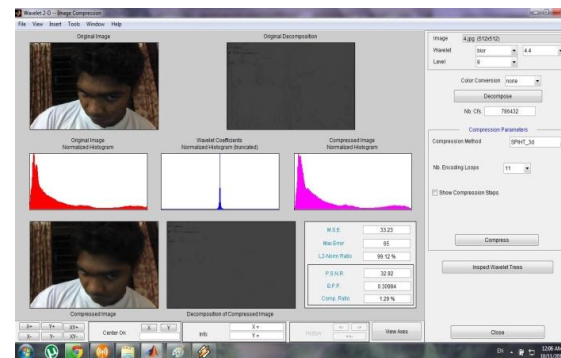


Fig.3.2.7.2 compression SPIHT-3D for image 2

3.2.8 Wavelet level metropolis Monte Carlo (LVL-MMC):

The MMC algorithm can be used to estimate the thermodynamic properties of finite Using systems. By this well-known method, a recurrent, irreducible, a periodic Markov chain is constructed so that its detailed balance condition satisfies the Boltzmann distribution. Then by periodicity, temporal averages of any state dependent function, over a single time series, converge asymptotically to the ensemble average.

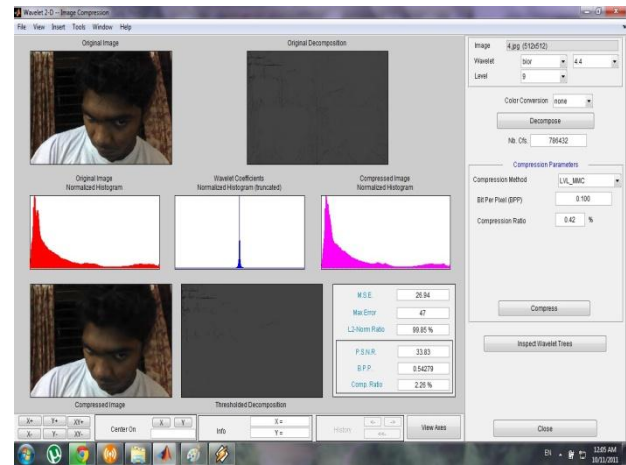
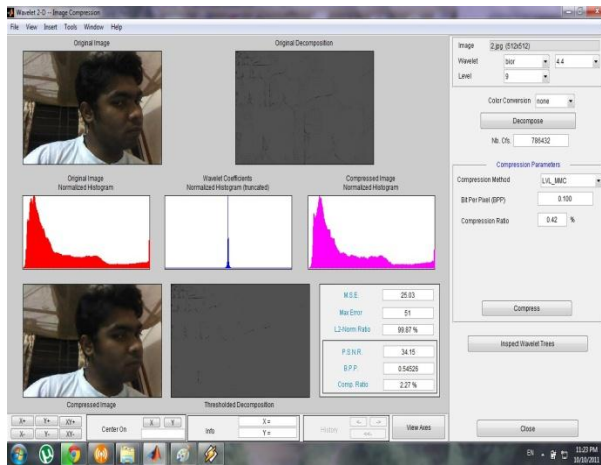


Fig.3.2.8..1compression LVL-MMC for image 1

Fig.3.2.8.2 compression LVL-MMC for image 2

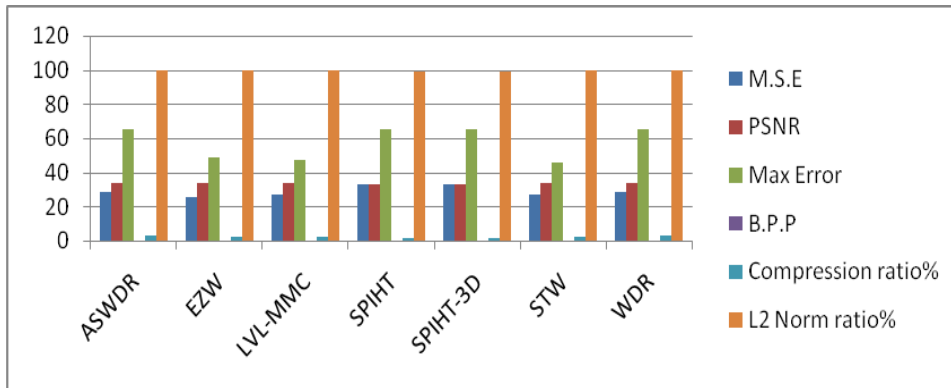
Quantitative Analysis of Different compression method on 2D for 512*512*3 image1:

| Compression method | ASWDR | EZW | LVL-MMC | SPIHT | SPIHT-3D | STW | WDR |
|--------------------|---------|---------|---------|---------|----------|---------|---------|
| M.S.E | 28.27 | 25.69 | 26.94 | 33.23 | 33.23 | 27.30 | 28.27 |
| PSNR | 33.62 | 34.03 | 33.83 | 32.92 | 32.92 | 33.77 | 33.62 |
| Max Error | 65 | 49 | 47 | 65 | 65 | 46 | 65 |
| B.P.P | 0.65121 | 0.62927 | 0.54279 | 0.42346 | 0.30984 | 0.59244 | 0.66904 |
| Compression ratio% | 2.71 | 2.62 | 2.26 | 1.76 | 1.29 | 2.47 | 2.79 |
| L2 Norm ratio% | 99.45 | 99.80 | 99.85 | 99.12 | 99.12 | 99.46 | 99.45 |

Table 2: Quantitative Analysis of Different compression method on 2D 512*512*3 image for 1

From this report it have been said that the overall performance of the two images are shown in the below table. Here it have been said that, in case of EZW method M.S.E is 25.69 and P.S.N.R is 34.03 and B.P.P is 0.62927. For SPIHT method, M.S.E is 33.23 and P.S.N.R is 32.92 and B.P.P is 0.423. For STW method, M.S.E is 27.30 and P.S.N.R is 33.77 and B.P.P is .59244. For WDR method, M.S.E is 28.27 and P.S.N.R is 33.62 and B.P.P is 0.66904. For ASWDR method, M.S.E is 28.27 and P.S.N.R is 33.62 and B.P.P is 0.65121. For LVL-MMC method, M.S.E is 26.94 and P.S.N.R is 33.83 and B.P.P is 0.54279. For ASWDR method, M.S.E is 33.23 and P.S.N.R is 32.92

and B.P.P is. 0.30984. Among all these methods, EZW is best performed though STW is averagely good as compared to other method.



Graph 3: Quantitative Analysis of Different compression method on 2D 512*512*3 image for 1

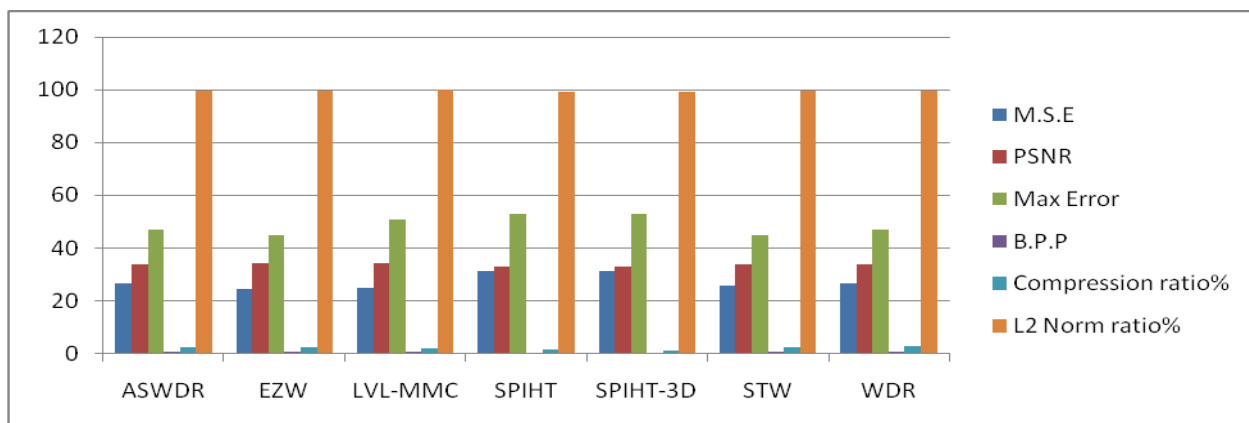
From the figure it have been seen that the total performance analysis of the Image compression here we see that in lowest Bit per pixel the EZW method perform better than the other method.

Quantitative Analysis of Different compression method on 2D for 512*512*3 image 2:

| Compression method | ASWDR | EZW | LVL-MMC | SPIHT | SPIHT-3D | STW | WDR |
|--------------------|---------|---------|---------|---------|----------|---------|---------|
| M.S.E | 26.72 | 24.43 | 25.03 | 31.51 | 31.51 | 26.05 | 26.72 |
| PSNR | 33.86 | 34.25 | 34.15 | 33.15 | 33.15 | 33.97 | 33.86 |
| Max Error | 47 | 45 | 51 | 53 | 53 | 45 | 47 |
| B.P.P | 0.65295 | 0.62109 | 0.54526 | 0.41330 | 0.30069 | 0.58789 | 0.67673 |
| Compression ratio% | 2.72 | 2.59 | 2.27 | 1.72 | 1.25 | 2.45 | 2.82 |
| L2 Norm ratio% | 99.58 | 99.83 | 99.87 | 99.31 | 99.31 | 99.59 | 99.58 |

Table 3: Quantitative Analysis of Different compression method on 2D 512*512*3 image for 2

From this report it have been said that the overall performance of the two images are shown in the below table. Here it have been said that, in case of EZW method M.S.E is 24.43 and P.S.N.R is 34.25 and B.P.P is 0.62109. For SPIHT method, M.S.E is 31.51 and P.S.N.R is 33.15 and B.P.P is 0.41330. For STW method, M.S.E is 26.05 and P.S.N.R is 33.97 and B.P.P is 0.58789. For WDR method, M.S.E is 26.72 and P.S.N.R is 33.86 and B.P.P is 0.67673. For ASWDR method, M.S.E is 26.72 and P.S.N.R is 33.86 and B.P.P is 0.62109. For LVL-MMC method, M.S.E is 25.03 and P.S.N.R is 34.15 and B.P.P is 0.30069. For SPIHT-3D method, M.S.E is 31.51 and P.S.N.R is 33.15 and B.P.P is 0.30069. Among all these methods, EZW is best performed though STW is averagely good as compared to other method.



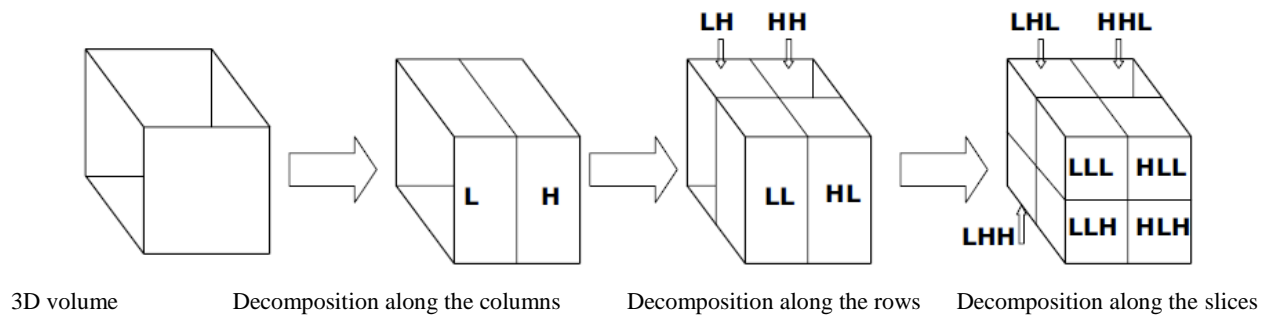
Graph 4: Quantitative Analysis of Different compression method on 2D 512*512*3 image for 2

From the figure it have been seen that the total performance analysis of the Image compression here we see that in lowest Bit per pixel the EZW method perform better than the other method.

3.3 Three Dimensional Decomposition and Reconstruction

Three-dimensional Discrete Wavelet Transform is a separable, sub-band transform. 3-D wavelets can be constructed as separable products of 1-D wavelets by successively applying a 1-D analyzing wavelet in three spatial directions (x, y, and z). One-level separable 3-D discrete wavelet decomposition of an image volume. The volume $F(x, y, z)$ is firstly filtered along the x-dimension, resulting in a low-pass image $L(x, y, z)$ and a high-pass image $H(x, y, z)$.

Both L and H are then filtered along the y-dimension, resulting in four decomposed sub volumes: LL, LH, HL and HH. Then each of these four sub volumes is filtered along the z-dimension, resulting in eight sub-volumes: LLL , LLH , LHL , LHH , HLL , HLH , HHL and HHH .



The basic scheme for Decomposition and Reconstruction is shown in Figure 17 below.

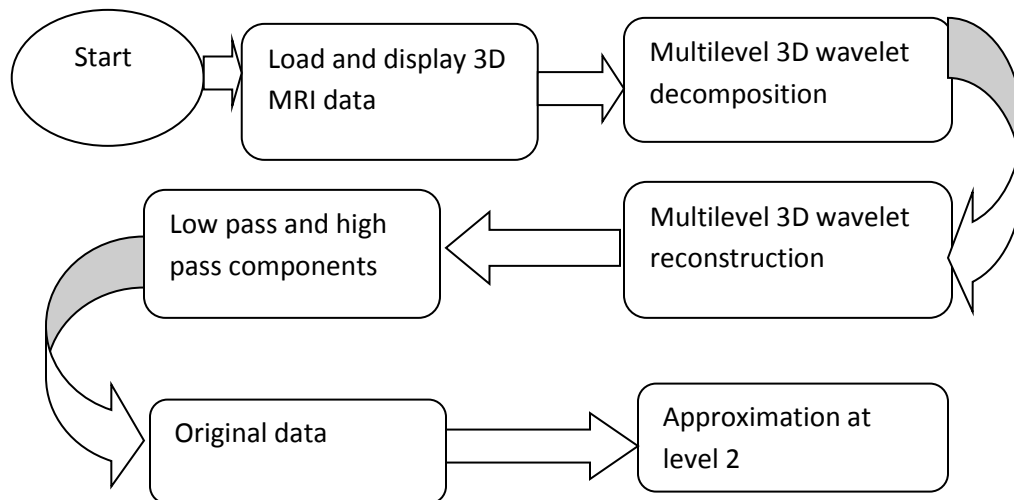
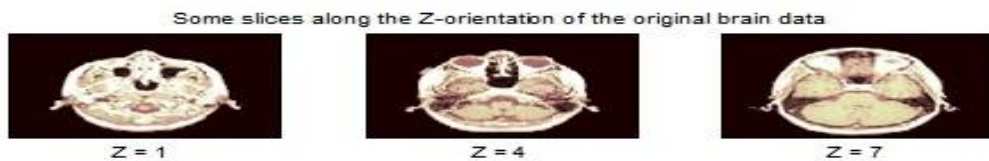


Fig.3.3 flowchart of three Dimensional Decomposition and Reconstruction

First we should take MRI an image from MATLAB building function, and then it's going through the MATLAB basement and that size it to 128*128*27. There are decomposition, reconstruction, original data and approximation level 2 below.



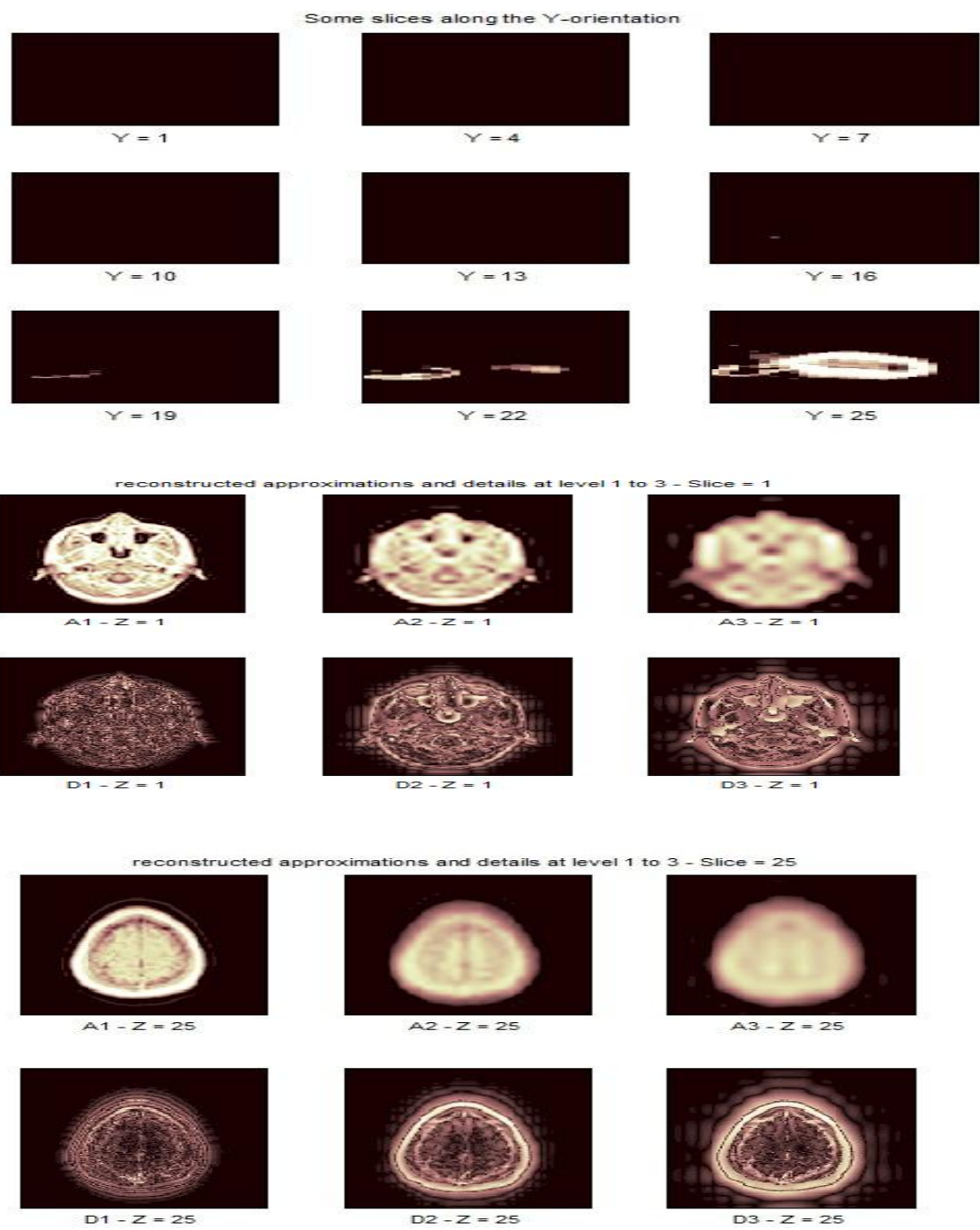


Fig.3.3.1 some slices z, y-orientation, decomposed and reconstruction.

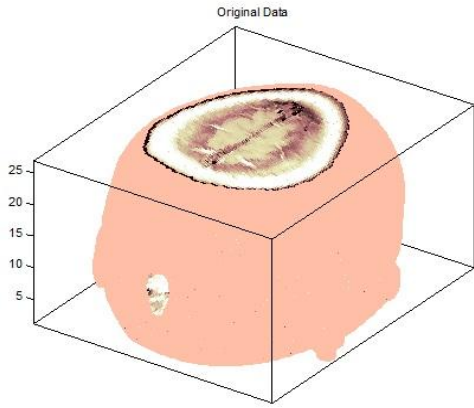


Fig.3.3.1.a original data

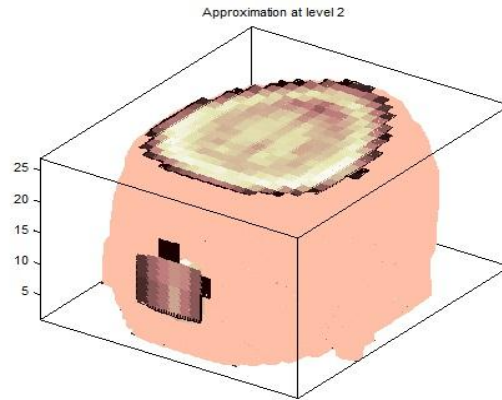


fig.3.3.1.b approximation at level 2

3D MRI data is decomposed and reconstructed different wavelets such as Haar, Daubechies, Symmlet and Coiflet as illustrated in fig. 3.3.1 to fig. 3.3.5.

Wavelet Haar

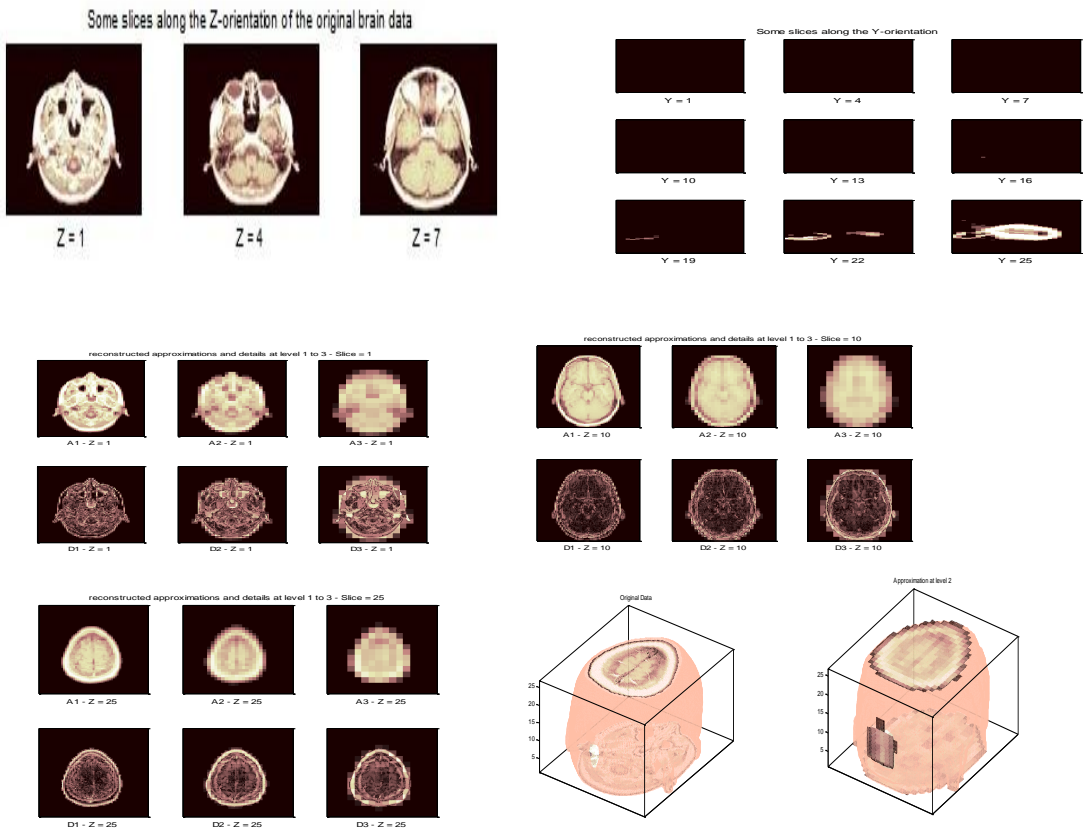


Fig.3.3.2 wavelet haar decomposed and reconstruction

Wavelet Daubechies

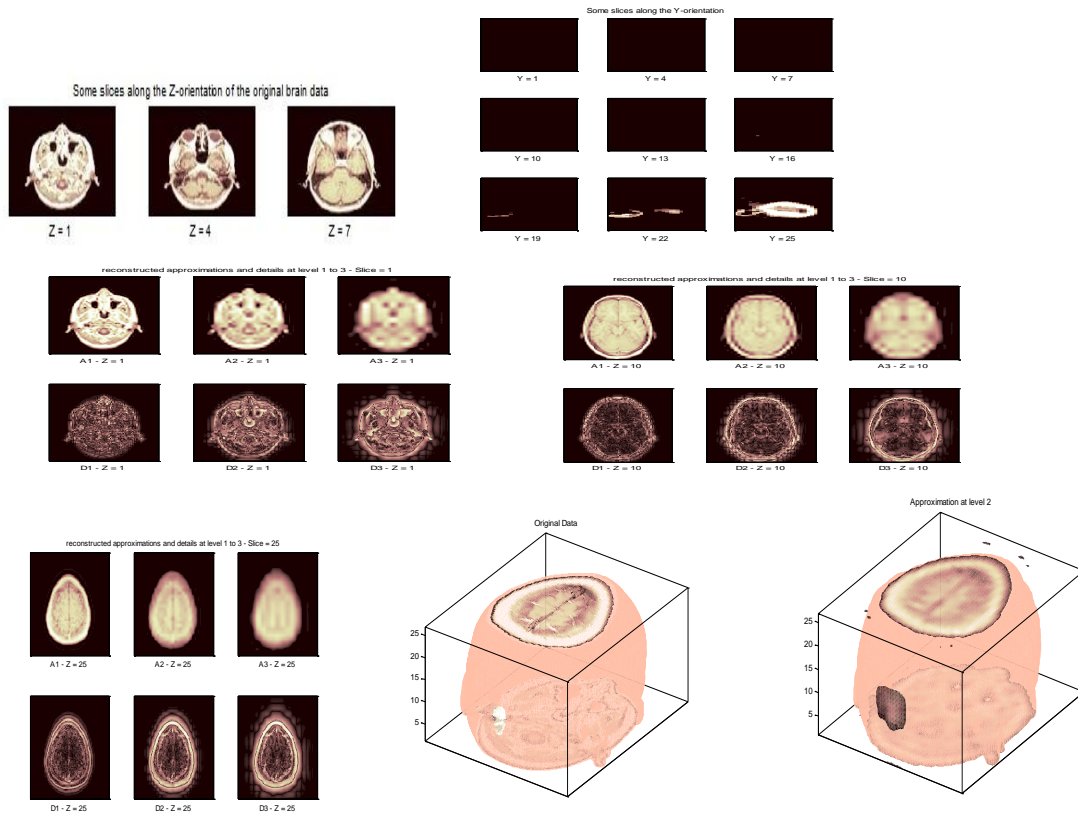


Fig.3.3.3 wavelet Daubechies decomposed and reconstruction

Wavelet Coiflet



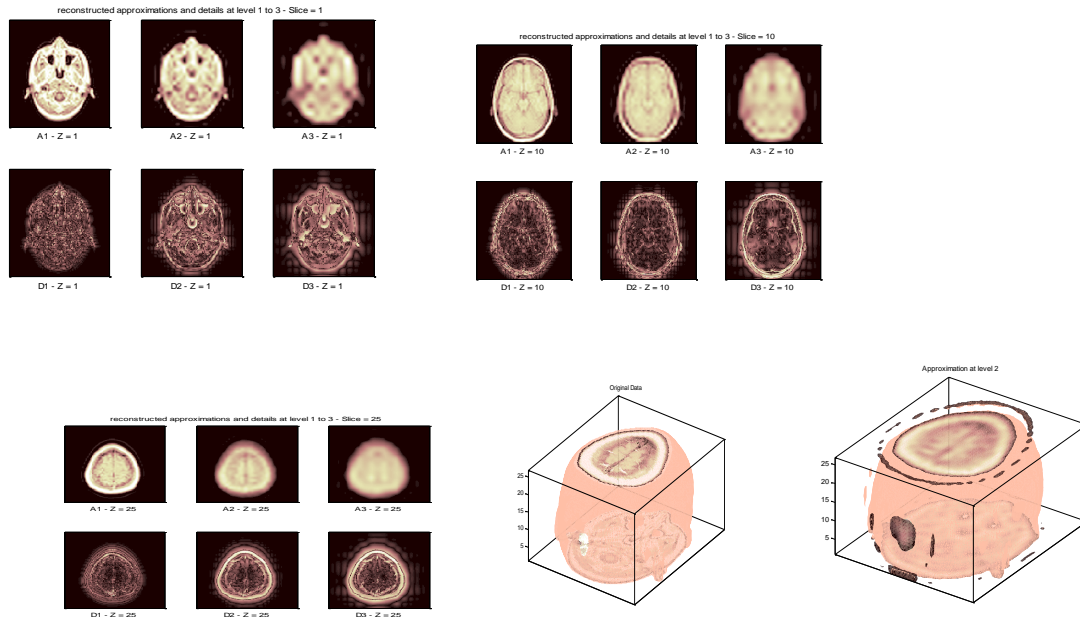


Fig.3.3.4 wavelet coiflet decomposed and reconstruction

Wavelet Symmlet

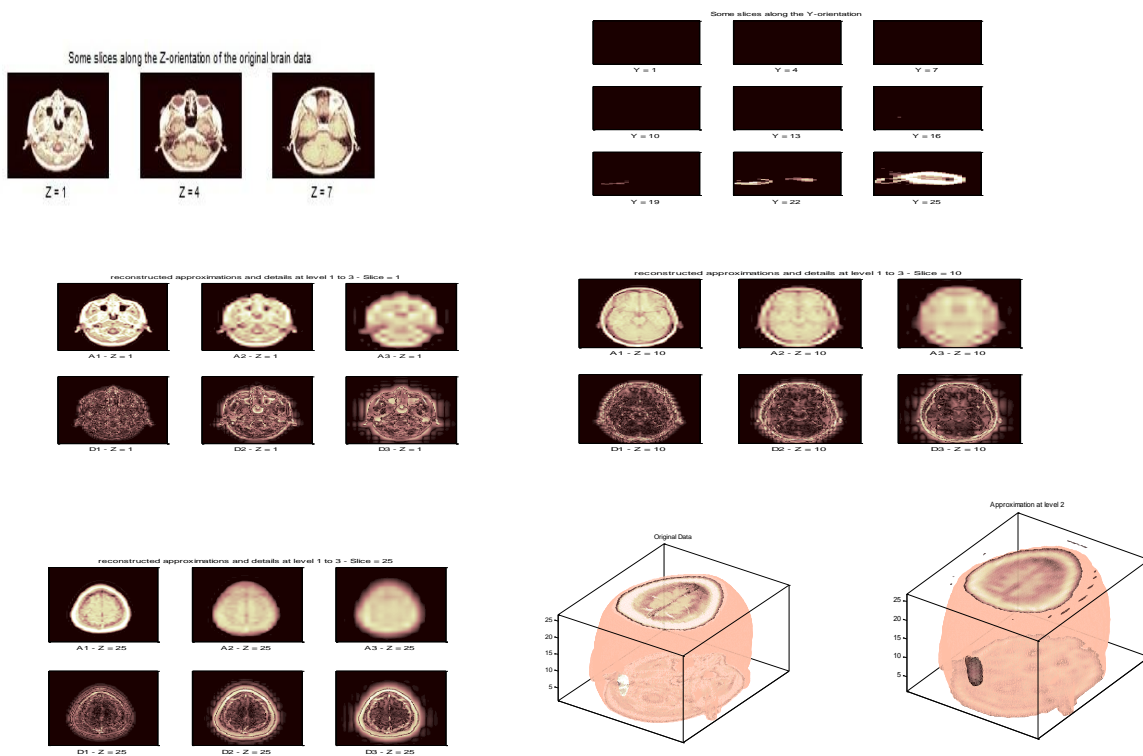


Fig.3.3.5 wavelet symmlet decomposed and reconstruction

Quantitative Analysis of Different Wavelets on MRI data:

| Wavelet type | Haar | Db-4 | Sym-4 | Coif-4 |
|--------------|----------|----------|----------|----------|
| Error | 1.0e-012 | 1.0e-009 | 1.0e-009 | 1.0e-008 |

-

Table 4: Quantitative Analysis of Different Wavelets on MRI data

From the table 4 represents that the total analysis of different types of wavelet transform where one parameter ERROR is analyzed. From the quantitative analysis on MRI data, it is found that Haar provides lowest ERROR. Coif provides highest ERROR. Db and symmlet provides ERROR that is more Haar. That's why; Haar is found best WT for MRI data decomposition and reconstruction.

Chapter IV

Conclusion

4.1 Conclusion

In this project, audio speeches were compared using different wavelets. The types of wavelets we used for analysis are Haar, Daubechies, Biorthogonal, Symmlet and Coif let. Using different wavelets on different parameters like P.S.N.R, NRMSE, SNR, RE and Error. The experimental result shows that Haar is better than other method. In the case of Haar wavelet we gained retained energy is above 98%.

In case of image signal analysis we took images from different position of human face which we compared with the different types of wavelet-based image compression techniques. The effects of different wavelet functions filter orders, number of decompositions, image contents and compression ratios were examined. The results of the above techniques WDR, ASWDR, STW, SPIHT, EZW etc, were compared by using the parameters such as PSNR, MSE, BPP values from the reconstructed image.

From the experimental results, it is identified that the PSNR values from the compressed images by using EZW and SEW compression is higher than other compression. And also it is shown that the MSE values from the reconstructed images by using STW compression are lower than other compression. Finally, it is identified that EZW and STW compression performs better when compare to WDR, ASWDR and other compression.

In case of MRI signal were compared by the different wavelet. The types of wavelets we used for analysis are Haar, Daubechies, Biorthogonal, Symmlet and Coif let. Using different wavelets on only one parameters like Error. The experimental results show that Haar is better than other method where as Error.

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