

PERFORMANCE ANALYSIS OF LS ESTIMATED OFDM SYSTEM OVER RAYLEIGH FADING CHANNEL

Md. Noor -A- Rahim¹, Sheikh Md. Rabiul Islam², Md. Nashid Anjum³

*Department of Electronics and Communication Engineering
Khulna University of Engineering and Technology, Bangladesh*

Telephone #+88041-769471-Ext 253

Mobile#+8801714087378, Fax # +88-041-774403

Email: ¹mash_0409@yahoo.com, ²robi_kuet@yahoo.com, ³nashidzone@gmail.com

Abstract: This paper investigates the bit error rate analysis of orthogonal frequency division multiplexing (OFDM) system over Rayleigh fading channel. We have compared the performances of the system by measuring the bit error rate with varying system parameters (No. of subcarriers, Doppler shift, and modulation scheme) where the channel estimation is done with block type pilot based Least square (LS) algorithm.

Index Terms : Channel estimation, Rayleigh fading channels, OFDM, pilots.

Introduction

Recently orthogonal Frequency Division Multiplexing (OFDM) has been applied widely in wireless communication systems due to high speed transmissions in a frequency-selective fading environment. By converting a wideband signal into an array of properly-spaced narrowband signals for parallel transmission, each narrowband OFDM signal suffers from frequency-flat fading and, thus, needs only a one-tap equalizer to compensate for the corresponding multiplicative channel distortion [1]. It has been used in wireless LAN standards such as American IEEE802.11a and the European equivalent HIPERLAN/2 and in multimedia wireless services such as Japanese Multimedia Mobile Access Communications [2]. Channel estimation is required for the OFDM receiver to perform detection, when transmit signal goes through multipath environment. In practice, Channel estimation can be performed at the receiver by inserting training (a.k.a. pilot) symbols at the transmitter. Pilot symbol assisted channel estimation is especially attractive for wireless links, where the channel is time-varying [2]. Channel estimation using pilot for OFDM symbols is investigated in [3] and [4]. In this

paper we will show the BER approximations for pilot assisted OFDM transmissions with M-ary quadrature amplitude modulation (QAM) over Rayleigh-fading channels. This paper also presents the BEP performance of OFDM receivers which is functions of the average bit signal energy to noise ratio (SNR), and some parameters like No. of sub carriers, Doppler spread.

The rest of this paper is organized as follows. Section II gives a brief descriptions of an OFDM system model in Rayleigh fading, we discuss Pilot based channel estimators in Section II. We analyze the Simulations and bit-error rate (BER) performance in Section IV, and conclusions are drawn in Section V.

SYSTEM DESCRIPTION

Figure 1 shows the OFDM system based on pilot channel estimation.

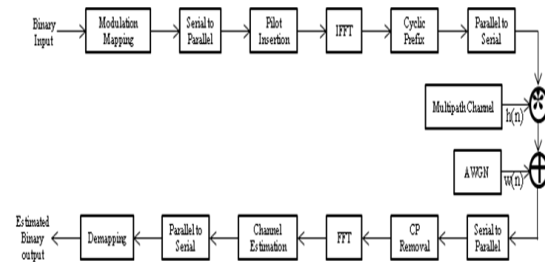


Fig.1 OFDM system model

Let $\{X(k), k = 1, \dots, N\}$ represent a block of N complex data symbols chosen from an appropriate signal constellation like quadrature amplitude modulation (QAM). The IFFT of the data block is

$$x(n) = \sum_{k=0}^{N-1} X(k) \exp\left(\frac{j2\pi kn}{N}\right), n = 0, 1, 2, \dots, N-1$$

(1) where $x(n)$ is the time-domain sequence and N is the FFT length.

To mitigate the effects of ISI caused by channel delay spread, each block of N IFFT coefficients is typically preceded by a cyclic prefix (CP) or a guard interval consisting of N_g samples, such that the length of the CP is at least equal to the channel delay spread. The cyclic prefix is simply a repetition of the last N_g IFFT coefficients. The resultant OFDM symbol is given as follows [2]:

$$x_{cp}(n) = \begin{cases} x(N+n), & n = -N_g, -N_g + 1, \dots, -1 \\ x(n), & n = 0, 1, 2, \dots, N-1 \end{cases} \quad (2)$$

The transmitted signal will go through the frequency selective time varying fading channel with additive white Gaussian noise. The received signal is given by:

$$y_r(n) = x_{cp}(n) * h(n) + w(n) \quad (3)$$

where $w(n)$ Additive White Gaussian Noise (AWGN) and $h(n)$ is the channel impulse response. The channel response is shown in [5]. After removal of cyclic prefix at receiver, $y(n)$ is sent to FFT block and the resulting signal given by:

$$Y(k) = \frac{1}{N} \sum_{n=0}^{N-1} y(n) \exp\left(\frac{-j2\pi kn}{N}\right) \quad (4)$$

The relation of the resulting $Y(k)$, $H(k)$, $I(k)$ and $W(k)$ is shown in [8], with the following equation:

$$Y(k) = X(k)H(k) + I(k) + W(k) \quad (5)$$

Where $H(k)$ and $W(k)$ is IFFT of $h(n)$ and $w(n)$ respectively and $I(k)$ is ICI because of Doppler frequency.

I. Channel Estimation

In block-type pilot-based channel estimation OFDM channel estimation symbols are transmitted periodically, and all subcarriers are used as pilots. The block-type pilot-based LS Estimation is performed in [2][4][9].

If we ignore the inter carrier interference, then equation (5) can be written as

$$Y = XFh + W \quad (6)$$

$$\text{Where } \begin{aligned} X &= \text{diag}\{X(0), X(1), \dots, X(N-1)\} \\ Y &= [Y(0)Y(1) \dots Y(N-1)]^T \\ W &= [W(0)W(1) \dots W(N-1)]^T \end{aligned}$$

$$W_N^{nk} = \frac{1}{\sqrt{N}} \exp\left(\frac{-j2\pi nk}{N}\right) \quad (7)$$

If $H_e(k)$ is estimated channel response in frequency domain then LS estimation is given by [9]:

$$H_e = H_{LS} = FQ_{LS}F^H X^H Y \quad (8)$$

Where

$$Q_{LS} = (F^H X^H X F)^{-1}$$

This estimated channel response is used to find the estimated transmitted signal $\{X_e(k)\}$

$$X_e(k) = \frac{Y(k)}{H_e(k)} \quad (9)$$

Then the signal is obtained by demapping $X_e(k)$.

$$H = [H(0)H(1) \dots H(N-1)]^T$$

II. **Simulation Model And BER Analysis**
OFDM system parameters used in the simulation are given in Table I.

$$F = \begin{bmatrix} W_N^{00} & \dots & \dots & \dots & \dots & \dots & W_N^{0(N-1)} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ W_N^{(N-1)0} & \dots & \dots & \dots & \dots & \dots & W_N^{(N-1)(N-1)} \end{bmatrix}$$

Table I. OFDM system Parameters

Parameters	Specifications
FFT Size (N)	256, 512, 1024
Total number of Pilots (N/8)	32, 64, 128
Guard Type	Cyclic Prefix
Guard Interval (N/4)	64, 128, 256
Sample Rate	100KHz
Channel Type	Rayleigh Fading Channel
No. Of Path	5
Path Delays	1 μ s, 2 μ s, 6 μ s, 1 μ s, 3.5 μ s, 6 μ s
Modulation Scheme	Quadrature Amplitude Modulation (QAM)

We have consider the guard interval greater than the maximum delay spread in order to avoid inter symbol interference. The channel estimation at pilot frequencies is performed by LS algorithm which is discussed in section III. Through simulation we carried out the BER for different signal to noise ratio (SNR) while varying the system parameters. Each time a transmission took place the SNR variable changed. The lowest

S/N ratio was decided to have the value 0 dB and the highest 27 dB.

A. Varying M-Ary QAM

Fig. 2 gives the BER performance of LS channel estimation algorithm for different modulation order and for Rayleigh fading channel, with $N = 256$ and Doppler frequency 40 Hz and other OFDM parameters given in Table I. During mapping the binary data, chosen a different QAM order (M) to simulate. Through investigating Fig. 2 we can notice, for a particular value of SNR the more we increase the modulation order M , the BER increases.

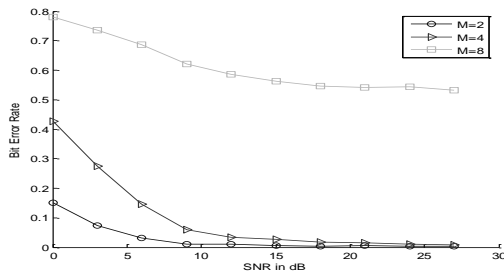


Fig. 2. BER Vs SNR with modulation order ($M=2,4,8$)

B. Varying Number of Sub Carriers

The transmission was simulated for 3 sets of carriers 256, 512 and 1024 carriers. For each set of carriers a BER curve as a function of S/N ratio was plotted. There are two plots. In the first the echoes have high level and in the second low levels. Here we selected modulation order $M = 2$ as modulation scheme and a Doppler shift of 40 Hz. We observed that the performance is superior for less number of sub carriers.

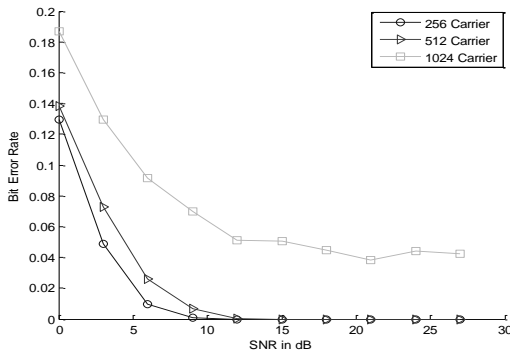


Fig. 3. BER Vs SNR with number of sub carriers ($N=256, 512, 1024$).

C. Varying Doppler Shift

Fig. 4 shows the performance of LS channel estimation for 8-QAM modulation, Rayleigh fading channel and 256 FFT for different Doppler frequencies (40 Hz, 80Hz, 120 Hz). For a specific value of SNR the behavior of the plots is that BER increases as the Doppler spread increases. This is because the higher Doppler shifts cause higher ICI i.e. degradation in system performances.

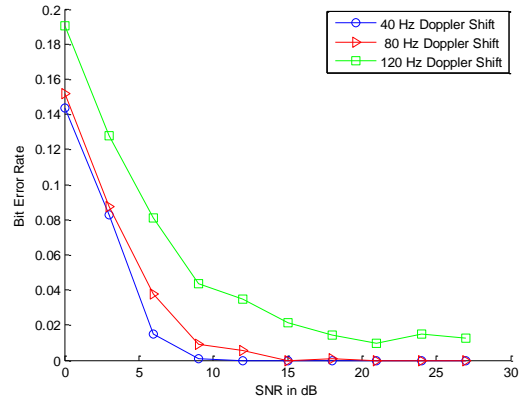


Fig. 4. BER Vs SNR with Doppler shift ($f_d = 40$ Hz, 80Hz, 120 Hz)

V. Conclusion

In this paper we have presented, bit error rate performance of OFDM with QAM modulation over Rayleigh-fading channels, in the presence of channel estimation. For a high S/N ratio, we can notice that the less the number of carriers, lower modulation order and low Doppler shift provides the more immunity in the Multipath environment.

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Md. Noor -A- Rahim is currently surveying as a Lecturer in the Department of Electronics and Communication Engineering at Khulna University of Engineering and Technology (KUET), Bangladesh. He received his B.Sc from KUET in March 2009.



Sheikh Md. Rabiul Islam received the B.Sc.in Engg. (ECE) from Khulna University, Khulna, Bangladesh in December 2003, and M.Sc. in Telecommunication Engineering from the University of Trento, Italy, in October 2009.

He joined as a Lecturer in the department of electronics and communication engineering of Khulna University of Engineering & Technology, Khulna, in 2004, where he is currently an Assistant Professor in the same department in the effect of 2008. He has published two Journal and four conferences in national and International (Specially USA) level. Since 2004 to 2006 he has acted as a visiting lecturer of ECE and CSE discipline in Khulna University, Bangladesh. His research interests include antennas and propagation, wireless communications & signal processing, and biomedical engineering.

Mr. Islam is an Associate Member of Institute of Engineers Bangladesh (IEB), Life member of Bangladesh Electronic Society (BES).