Short Communication

EFFECT OF TRANSPORTATION PROBLEM IN BUSINESS AND ECONOMIC SECTOR

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Keywords: *Transportation table, cell, rim requirement, added capacity, optimal solution.*

1. Transportation Problem

Transportation problem is one of the sub-class of Linear Programming (L.P) problems in which the objective is to transport various amount of a homogeneous commodity, that are initially stored at various origins, to different destinations in such a way that the total transportation cost is minimum. Its major application in solving problems involving several product sources and several destinations of products, this type of problem is frequently called the Transportation Problem (T.P). It gets its name from its application to problems involving transporting products from several sources to several destinations. Although the formation can be used to represent more general assignment and scheduling problems as well as transportation and distribution problems. The two common objectives of such problems are either (1) minimize the cost of shipping m units to n destinations or (2) maximize the profit of shipping m units to n destinations [1].

Let us illustrate by formulating a typical transportation problem involving m origins and n destinations.

A milk manufacturing concern has m different cities of Bangladesh. The total supply potential of the manufactured product is absorbed by n retail shops in n different cities of the country. Determine the transportation schedule that minimizes the total cost of transporting milk from various plant locations retail shops.

Mathematical formulation of the problem: Let us identify the m plant locations (origins) as $O_1, O_2, O_3, \dots\dots O_m$ and n retail shops (destinations) as $D_1, D_2, D_3, \dots\dots D_n$ respectively. Let $a_i > 0, j = 1, 2, \dots m.$ be the amount of milks available at the ith plant O_i and let the amount of milks required at the jth retail shops D_j be $b_j > 0,\ j = 1, 2, \dots\dots n$.

Let the cost of transporting one unit of milks from plant O_i to retail shop D_j be $c_{ij} (i = 1, \dots\dots m, j = 1, \dots\dots n)$. If $x_{ij} \geq 0$ be the amount of milks to be transported from O_i to destination D_j, then the problem is to determine x_{ij} so as to

$$\textit{Minimize} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} \, c_{ij}$$

subject to the constraints:

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1, 2, \dots\dots m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1, 2, \dots\dots n$$

$$x_{ij} \geq 0 \qquad \textit{forall} \quad i \textit{ and } j.$$

The above formulation looks likes an L.P. problem. This special L.P. problem will be called a Transportation Problem (T.P) [2].

2. Matrix Form of T.P

Consider the T.P. discuss in the last section. The set of constraints $\sum_{j=1}^{n} x_{ij} = a_i$ and $\sum_{i=1}^{m} x_{ij} = b_j$ represent $m+n$ equations in mn non-negative variables x_{ij}. Each variable x_{ij}. appears in exactly two constraints, one associated with the ith origin O_i and the other with the jth destination D_j. In the above ordering of constraints–the origin equations first, then the destination equations- the T.P. can be restated in the matrix form as

Date of submission : 03. 08..2011 Date of acceptance : 16. 04. 2012

$$Minimize\ Z = c^T xc,\ x \in R^{mn}$$

subject to the constraints

$$Ax = b,\ x \geq 0 \qquad b \in R^{m+n}$$

where $x = [x_{11},....x_{1n}, x_{21},...x_{2n},....x_{m1},.......x_{mn}]$, the cost vector, $b = [a_1,........a_m, b_1,.....b_n]$ and A is an $(m+n)\times mn$ real matrix containing the coefficients of constraints. The reader should note that the elements of A are either 0 or +1. Thus the general L.P.P can be reduced to a T.P. if (i) the a_{ij}'s are restricted to the values 0 and +1, and (ii) the units among the constraints are homogeneous. For a T.P. involving 2 origins and 3 destinations (m=2, n=3) the matrix A is given by

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} e_{23}^1 & e_{23}^2 \\ I_3 & I_3 \end{bmatrix}$$

and, therefore in general, for an m-origin, n-destination T.P., we may write

$$A = \begin{bmatrix} e_{mn}^1 & e_{mn}^2 \cdots e_{mn}^m \\ I_n & I_n \cdots I_n \end{bmatrix}$$

Were e_{mn}^j *is* $m\times n$ matrix having a row of unit elements as its jth row and 0's everywhere else, and I_n is the $n\times n$ identity matrix. If a_{ij} denotes the column vector of A associated with any variable x_{ij}, then it is easily verify that $a_{ij} = e_i + e_{m+j}$ where e_i, $e_{m+j} \in R^{m+n}$ are unit vectors and it was devoted by [2].

3. The Transportation Table

Since the T.P. is just a general L.P.P., the application of simplex method would give an optimum solution to the problem. However, due to some special structure of the T.P. shortcuts in simplex method are available, which enable us to solve a transportation problem much more easily. The special structure of the T.P. enable us to represent the problem in the form of a rectangular table, called Transportation Table-the computational major vehicle of the transportation method. A specimen of the transportation table for an m-origin, n-destination T.P. is given below [2]:

The mn large square are called the cells each corresponding to a variable x_{ij}. Each row

	D_1	D_2		D_n	
O_1	c_{11}	c_{12}	....	c_{1n}	a_1
O_2	c_{21}	c_{22}	....	c_{2n}	a_2 Availability
	...	......	....	..	
O_m	c_{m1}	c_{m2}	....	c_{mn}	a_m
	b_1	b_2		b_n	

corresponds to one of the m constraints $\sum x_{ij} = a_i$ and each column corresponds to one the n constraints $\sum x_{ij} = b_j$. The per unit cost c_{ij} of transporting from the ith origin O_i to the jth destination D_j is displayed in the lower right position of the *(i, j)*th cell. Any feasible solution to the T.P. is displayed in the table by entering the value of x_{ij} in the small square at the upper position of the *(i, j)*th cell. The various origin capacities and destination requirements are listed in the rightmost (outer) column and bottom row respectively. This are called rim requirements. For feasibility of the solution, one can verify by summing the values of x_{ij} across the rows and down the columns.

4. Effective Methods of Solving T.P

A. The Initial Basic Feasible Solution: (Vogel's Approximation Method (VAM)):

It is always possible to assign an initial feasible solution to a T.P. in such a manner that the rim requirements are satisfied. This can be achieved either by inspection or following some simple rules. We begin by imagining that the transportation table is blank, that is initially all $x_{ij} = 0$. The simplest procedures for initially allocation are given below:

The Vogel's Approximation Method takes into account not only the least cost c_{ij} but also the costs that *just exceed* c_{ij}. The steps of the method are given below:

Step 1: For each row of the transportation table identity the *smallest* and *next to smallest* costs. Determine the difference between them for each row. Display them alongside the transportation table by enclosing them in parenthesis against the respective rows. Similarly, compute the difference for each column.

Step 2: Identify the row or column with the largest difference among all the rows and column. If a tie occurs, use any arbitrary tie breaking choice. Let the greatest difference correspond to ith row and let c_{ij} be the smallest cost in the ith row. Allocate the maximum feasible amount $x_{ij} = \min.(a_i, b_j)$ in the $(i, j)th$ cell and cross off the ith row of the jth column in the usual manner.

Step 3: Recomputed the column and row difference for the reduced transportation table and go to *step 2*. Repeat the procedure until all the requirements are satisfied.

Remarks: 1. A row or column "difference" indicates the minimum unit penalty incurred by failing to make an allocation to the smallest cost cell in that row or column.

2. It will seen later that VAM determines an initial basic feasible solution which is very close to the optimum solution, that is the number of iterations required to reach the optimum solution is smaller in this case.

B. Stepping –Stone Method of Solving of Transportation Problem:

An algorithm used to find the optimal solution to a Transportation Problem. Given the transportation table all with an initial solution and *m+n-1* occupied cells, the Stepping-Stone method is as follows [3].

Step 1: For each unoccupied cell, identify its Stepping-Stone path through the transportation tableau.

Step 2: Compute the per unit change c_{ij} from adding one unit to each unoccupied cell or follows,

(a) Label the starting or unoccupied cell under consideration or cell 1 and number sequentially 2, 3, 4,........the occupied cells on the corners of its Stepping-Stone path.

(b)The per unit change from adding one unit to the unoccupied cells is found by adding the unit shipping costs of all odd numbered cells on the path and subtracting the unit shipping costs of all even numbered cells on the path.

Step 3: In a minimization problem, if the per unit changes for all unoccupied cells are non-negative, the solution is optimal. However, if negative per unit change exist, identify the best cell and

Step 4: For best cell, identify the sequentially numbered occupied cells on the corners of its Stepping-Stone path as under *step* 2. Determine the even numbered Stepping-Stone cell and over which the smallest quantity is being shipped add this quantity to the new cell and all odd numbered cells. Subtract this quantity from all even numbered cells return to *step* 1.

5. Numerical Example

Consider the following transportation problem as

	D_1	D_2	D_3	D_4	Available
O_1	6	3	5	4	22
O_2	5	9	2	7	15
O_3	5	7	8	6	8
Required	7	12	17	9	

At first we find the initial basic feasible solution of the given problem by using Vogel's Approximation Method (VAM) [2].

The transportation table of the given T.P. has 12 cells. The difference between the smallest and next to smallest costs in each row and each column are computed and displayed inside the parenthesis against the respective columns and rows. The largest of this difference is (4) and is associated with the second column of the transportation table. The minimum cost in the second column is $c_{12} = 3$. Accordingly we allocate $x_{12} = \min(22, 12) = 12$ in the cell (1, 2). This exhausts the availability of second column.

Cross off the second column. The row and column differences are now calculated for the resulting reduced transportation table; the largest of these is (3) and is associated with second row as well as also second column. We arbitrarily choose second row. Thus the second allocation of magnitude $x_{22} = \min.(15, 17) = 15$ is made in the cell (2, 2). Cross off the second row from the table. Continuing n the manner, the subsequent reduced transportation tables and differences for the surviving rows and columns are shown in

Table 2: Stepping-Stone Table

6	(12) 3	5	4	22 (1)
5	9	2	7	15 (3)
5	7	8	6	8 (1)
7	12	17	9	
(1)	(4)	(3)	(2)	

Table 3: Stepping-Stone table

6	5	4	10 (1)
5	(15) 2	7	15 (3)
5	8	6	8 (1)
7	17	9	
(1)	(3)	(2)	

Table 4: Stepping-Stone table

6	(2) 5	4	10 (1)
5	8	6	8 (1)
7	2	9	
(1)	(3)	(2)	

Table 5:

6	(8) 4	8 (1)
5	6	8 (1)
7	9	
(1)	(2)	

Table:6

(7) 5	(1) 6	8 (1)
7	1	
(1)	(2)	

Here the initial basic solution table is

Table 7: Initial basic solution table

(0) 6	(12) 3	(2) 5	(8) 4
(0) 5	(0) 9	(15) 2	(0) 7
(7) 5	(0) 7	(0) 8	(1) 6

Hence

$Min.\ Z = 3\times12+5\times2+4\times8+2\times15+5\times7+6\times1$

$= 149$

Now we try to solve the problem by Stepping – Stone Method.

Here we see that

$$m+n-1 = o.c$$

$$\Rightarrow 4+3-1 = 6$$

where m= number of column

n= number of row

$o.c$= occupied cell and non-zero vertices are six. So we may use Stepping –Stone Method algorithm.

Table 8: Initial Basic Solution Table

	D_1	D_2	D_3	D_4
O_1	(0) 6	(12) 3	(2) 5	(8) 4
O_2	(0) 5	(0) 9	(15) 2	(0) 7
O_3	(7) 5	(0) 7	(0) 8	(1) 6

To justify the optimality of initial feasible solution, we need to find out opportunity cost of each empty cell by the following way:

(i) $(O_1, D_1)\to(O_1, D_4)\to(O_3, D_4)\to(O_3, D_1)$

$= 6-4+6-5 = 3 =$ *positive*

(ii) $(O_2, D_1)\to(O_2, D_3)\to(O_1, D_3)\to(O_1, D_4)\to(O_3, D_4)\to(O_3, D_1)$

$=5-2+5-4+6-5=5=$ *positive*

(iii) $(O_2, D_2)\to(O_1, D_2)\to(O_1, D_3)\to(O_2, D_3)$

$=9-3+5-2=9=$ *positive*

(iv) $(O_2, D_4)\to(O_2, D_3)\to(O_1, D_3)\to(O_1, D_4)$

$=7-2+5-4=6=$ *positive*

$$(vi)(O_3, D_3) \rightarrow (O_3, D_4) \rightarrow (O_1, D_4)$$
$$\rightarrow (O_1, D_3) = 8-6+4-5=1= positive$$

Here we see that there is no empty cell with non-negative opportunity cost. Therefore determination of the optimal transportation cost

$$= 12\times3+2\times5+8\times4+15\times2+7\times5$$
$$+1\times6=149$$

which is the optimal solution of the given problem.

6. Effects of Added Capacity

A direct consequence of such strong and clear relationships among residential density, household size, income, and auto ownership, is the multi-co linearity that exists among these variables that have traditionally been considered to most strongly influence household trip generation[4]. Being defined as a function of land use and interzonal travel time variables, accessibility measures are also multi-collinear with the other contributing factors. As a result, it is extremely difficult to determine the independent effect of each contributing factor. Consequently, it has not been possible to produce definitive answers to such seemingly rudimentary questions as: Does an increase in capacity induce trips?" or "Can we decrease automobile ownership and increase transit use by increasing residential density?" The problem is further compounded due to the endogeneity of these "explanatory" variables.[5]

Summarizing the discussion of this article, economic formulations of trip making offer unambiguous indications that added capacity, which implies decreased cost of transport would lead to more trips and additional Vehicle miles traveled (VMT). Travel budgets, or travel expenditures to be more precise, are clearly determined by households; although no models reviewed here attempt to model the process of determining a travel budget endogenously. The most desirable level of travel expenditure of either time or money will vary from household to household or from situation to situation [7]. The notion of forecasting future travel demand based on the assumption that the travel expenditure of a household remains constant over time is not well founded and appears to produce results that cannot be theoretically supported. Then how does a travel expenditure, or trip making in general, change in response to changes in capacity and resulting changes in generalized travel costs? The discussion here pointed out the multi-co linearity among the factors that contribute to trip making, which is a consequence of ecological correlation that prevails in an urban area and according to our above discussed method and applying the concept of transportation added capacity, we can minimize

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